Qn	**Working**	**Mark allocated**
1	$3x+4y=54$, $\frac{4}{x}+\frac{3}{y}-1=0$. $3x+4y=54$ $y=\frac{54-3x}{4}$ ...(1) $\frac{4}{x}+\frac{3}{y}=1$ $4y+3x=xy$...(2) sub (1) into (2) $4\left(\frac{54-3x}{4}\right)+3x=x\left(\frac{54-3x}{4}\right)$ $216-12x+12x=54x-3x^2$ $3x^2-54x+216=0$ $x^2-18x+72=0$ $(x-12)(x-6)=0$ $x=12$ or $x=6$ sub $x=12$ into (1), $y=\frac{53-3(12)}{4}$ $=4\frac{1}{2}$ sub $x=6$ into (1), $y=\frac{53-3(6)}{4}$ $=9$ $x=12, y=4\frac{1}{2}$ or $x=6, y=9$	M1 substitution leading to one variable only M1 solve equation A1 for *x* values A1 for *y* values
		4 m
2(a)	$-\sqrt{3}\sin\theta+3\cos\theta=3\cos\theta-\sqrt{3}\sin\theta$ Let $3\cos\theta-\sqrt{3}\sin\theta=R\cos(\theta+\alpha)$ $=R\cos\theta\cos\alpha-R\sin\theta\sin\alpha$ $\tan\alpha=\frac{\sqrt{3}}{3}$ $\alpha=\frac{\pi}{6}$ $R^2=\left(\sqrt{3}\right)^2+3^2$ $R^2=12$ $R=2\sqrt{3}$ $-\sqrt{3}\sin\theta+3\cos\theta=2\sqrt{3}\cos(\theta+\frac{\pi}{6})$	B1 for α or oe Do not award if tan α is -ve M1 for R A1 oe.

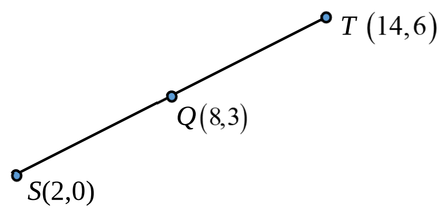
(b)	$\frac{1}{-\sqrt{3}\sin\theta+3\cos\theta-1}=\frac{1}{2\sqrt{3}\cos(\theta+\frac{\pi}{6})-1}$ Minimum value of $y=-1$ y is min when $\cos\left(\theta+\frac{\pi}{6}\right)=0$ $\theta+\frac{\pi}{6}=\frac{\pi}{2}$ $\theta=\frac{\pi}{3}$ Hence, and its corresponding value of θ is $\frac{\pi}{3}$.	B1 answer M1 find angle using their minimum A1 oe.
		6 m
3(a)	$e^{2x}(e^{2x}-3)=28$ $e^{4x}-3e^{2x}-28=0$ $(e^{2x}+4)(e^{2x}-7)=0$ $e^{2x}=7$ or $e^{2x}=-4$ (NA) $2x=\ln 7$ $x=\frac{1}{2}\ln 7\approx 0.973$	B1 quadratic form and RHS=0 B1 factorised form B1 exact or oe
(b)	$\log_4 x-\log_{16} x=(\log_4 x)(\log_{16} x)$ $\log_4 x-\frac{\log_4 x}{\log_4 16}=(\log_4 x)\left(\frac{\log_4 x}{\log_4 16}\right)$ $\log_4 x-\frac{1}{2}\log_4 x=(\log_4 x)\left(\frac{1}{2}\log_4 x\right)$ Let $y=\log_4 x$ $y-\frac{1}{2}y=y\left(\frac{1}{2}y\right)$ $\frac{1}{2}y=\frac{1}{2}y^2$ $y^2-y=0$ $y(y-1)=0$ $y=0$ or $y=1$ $\log_4 x=0$ or $\log_4 x=1$ $x=4^0$ or $x=4^1$ $\therefore x=1$ or 4	M1 change base M1 attempt to express into quadratic equation M1 factorisation M1 change from log to exp form A1
		8 m

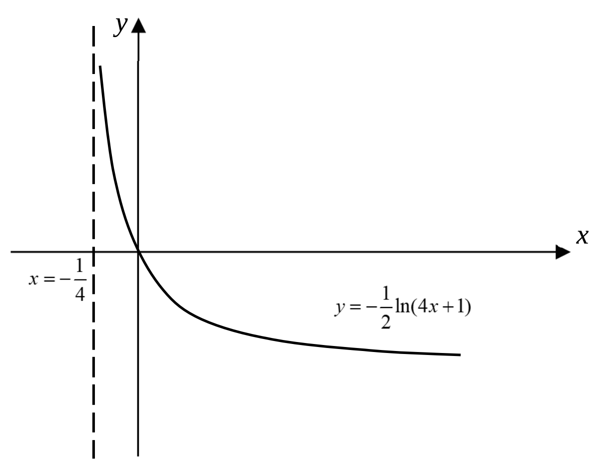
4(a)	$\angle ACD = \angle CAS$ (alternate angles) $\angle ABC = \angle CAS$ (tangent chord theorem) $\therefore \angle ACD = \angle ABC$ $\angle CAB = \angle DAC$ (common angle) $\therefore$ Triangle ADC is similar to Triangle ACB. (AAA)	B1 for statement with property B1 for statement with property (-1m for missing concluding statement; name of test not required)
(b)	$\frac{AC}{AB} = \frac{AD}{AC}$ $AC^2 = AB \times AD$ $= CS \times AB$ ($AD = SC$ as $SADC$ is a parallelogram)	B1 ratio o.e B1 $AD = SC$ with reason and leading to answer AG
(c)	Method 1: $\angle BCA = \angle ABT$ (tangent chord theorem) $\angle CBA + \angle CAD + \angle BCA = 180°$ (Angles sum of triangle) $\therefore \angle CBA + \angle CAD + \angle ABT = 180°$ Method 2: $\angle CAS = \angle CBA$ (tangent chord theorem) $\angle ABT = \angle DAT$ (tangents from external point) $\angle CAS + \angle CAD + \angle DAT = 180°$ (adjacent angles on a straight line) $\therefore \angle CBA + \angle CAD + \angle ABT = 180°$	B1 statement and property B1 statement and property AG B1 statement and property B1 statement and property AG
		6m
5(a)	$6 + 2x - 4x^2 > 0$ $2x^2 - x - 3 < 0$ $(2x-3)(x+1) < 0$ $\therefore -1 < x < 1\frac{1}{2}$	B1 form inequality B1 factorise (-1m if factors are not in integral form) B1
(b)	Since $(m-8)x^2 + 6x + m \geq 0$, therefore $b^2 - 4ac \leq 0$. $6^2 - 4(m-8)m \leq 0$ $36 - 4m^2 + 32m \leq 0$ $4m^2 - 32m - 36 \geq 0$ $m^2 - 8m - 9 \geq 0$ $(m+1)(m-9) \geq 0$ $m \leq -1$ or $m \geq 9$ Also,	B1 discriminant B1 ≤ 0 M1 factorisation A1

	$m-8>0$ $m>8$ Hence, $m \geq 9$. The smallest integer value of m = 9.	A1 only if $m-8>0$ is accounted for
		8 m
6(a)	$\left(x^2+\frac{a}{x}\right)^9 = x^{18} + \binom{9}{1}\left(x^2\right)^8\left(\frac{a}{x}\right) + \binom{9}{2}\left(x^2\right)^7\left(\frac{a}{x}\right)^2 + \ldots$ $= x^{18} + 9ax^{15} + 36a^2x^{12} + \ldots$	B1 $x^{18}+9ax^{15}$ B1 $36a^2x^{12}$
6(b)	For $\left(x^2+\frac{a}{x}\right)^9$, $T_{r+1} = \binom{9}{r}\left(x^2\right)^{9-r}\left(\frac{a}{x}\right)^r$ $= \binom{9}{r}\left(x^2\right)^{9-r}\left(\frac{a}{x}\right)^r$ $= \binom{9}{r}(a)^r\left(x^{18-2r}\right)\left(x^{-r}\right)$ $= \binom{9}{r}(a)^r\left(x^{18-3r}\right)$ For x^3, $18-3r=3$ $r=5$ $\binom{9}{5}(a)^5 = -126$ $126a^5 = -126$ $a^5 = -1$ $\therefore a = -1$	M1 skill leading to x^3 M1 their coefficient A1
(c)	For term independent of x in $\left(x^2+\frac{a}{x}\right)^9$, $18-3r=0$ $r=6$ Term independent of $x = \binom{9}{6}(-1)^6 = 84$ $\left(1-\frac{2}{x^3}\right)\left(x^2-\frac{1}{x}\right)^9 = \left(1-\frac{2}{x^3}\right)\left(\ldots.-126x^3+84+\ldots\right)$ Term independent of $x = 1(84)+(-2)(-126)$ $=336$	 M1 finding term independent of x with their 'a' M1 correct pairing A1
		8 m

7(a)	$\angle BOD = 2\angle BCD$ ($\angle$ at the centre $= 2$ $\angle$ at circumference) $= 2x$ Area of sector BOD $= \frac{1}{2}(10)^2(2x)$ $= 100x$ cm^2 Area of $\Delta COD = \frac{1}{2}(10)(10)\sin\angle COD$ $= 50\sin(\pi - \angle BOD)$ $= 50\sin(\pi - 2x)$ $= 50\sin 2x$ cm^2 $A =$ Area of sector $BOD +$ Area of ΔCOD $= 100x + 50\sin 2x$	B1 sub into formula B1 sub into formula B1 $(\pi - 2x)$ seen leading to answer AG
7(b)	$\frac{dA}{dx} = 100 + 50(2\cos 2x)$ $= 100 + 100\cos 2x$ Sub. $x = \frac{\pi}{6}$, $\frac{dA}{dx} = 100 + 100\cos\left[2\left(\frac{\pi}{6}\right)\right]$ $= 150$ Let the rate of x be $\frac{dx}{dt}$ and rate of change of A be $\frac{dA}{dt}$. $\frac{dx}{dt} = 0.2$ radian s^{-1} (Given) $\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt}$ $= 150 \times 0.2$ $= 30$ cm^2 s^{-1} Hence, the rate of change of A is 30 cm^2 s^{-1}.	M1 attempt to find $\frac{dA}{dx}$ M1 chain rule with substitution (their $\frac{dA}{dx}$) A1

(c)	$50\frac{dx}{dt}=\frac{dA}{dx}\times\frac{dx}{dt}$ $50=100+50(2\cos 2x)$ $1=2+2\cos 2x$ $\cos 2x=-\frac{1}{2}$ basic angle = $\frac{\pi}{3}$ $2x=\frac{2\pi}{3},\frac{4\pi}{3}$ $x=\frac{\pi}{3},\frac{2\pi}{3}$ (NA) $\therefore x=\frac{\pi}{3}$	M1 knowing how to formulate relation M1 solve equation by making $\cos 2x$ the subject A1
		9 m
8(a)	$\frac{d}{dx}\left[\left(x^2+1\right)e^{2x-1}\right]=2xe^{2x-1}+2e^{2x-1}\left(x^2+1\right)$ $=2xe^{2x-1}+2x^2e^{2x-1}+2e^{2x-1}$ $=2e^{2x-1}\left(x^2+x+1\right)$	B1 product rule B1 derivative of e^{2x-1} B1 derivative of x^2+1 AG
8(b)	$\int -2e^{2x-1}\left(x^2+x\right)\,dx$ $=-\left[\int 2e^{2x-1}\left(x^2+x+1\right)\,dx-\int 2e^{2x-1}dx\right]$ $=-\left[\left(x^2+1\right)e^{2x-1}-e^{2x-1}\right]+c$ $=-x^2e^{2x-1}+c$	M1 converse statement M1 expand or split $\int f(x)\pm g(x)\,dx$ $=\int f(x)\,dx\pm\int g(x)\,dx$ A1 $\int 2e^{2x-1}dx=-e^{2x-1}+c$ M1 simplify to answer AG
(c)	$\int_{\frac{1}{2}}^{1}-2xe^{2x-1}\left(x+1\right)\,dx=-\left[x^2e^{2x-1}\right]_{\frac{1}{2}}^{1}$ $=-\left[e^{2-1}-\frac{1}{4}e^0\right]$ $=\frac{1}{4}-e\approx-2.47$ (3s.f) The area between the graph and the *x*-axis in the interval $\frac{1}{2}<x<1$ **lies below *x*-axis.** <u>Alternative:</u> The graph **lies below *x*-axis** in the interval $\frac{1}{2}<x<1$ **.**	 B1 oe. B1 key words in bold and interval $\frac{1}{2}<x<1$
		9 m

9(a)	Equation of QS: $y=\frac{1}{2}x-1$ Gradient of $PR=-2$ Equation of PR: $y-7=-2(x-3)$ $y=-2x+13$	B1 soi M1 sub in coordinates A1
9(b)	At S, $y=0\Rightarrow x=2$ $\therefore S=(2,0)$ $\frac{1}{2}x-1=-2x+13$ $\frac{5}{2}x=14$ $x=5\frac{3}{5}$ $y=1\frac{4}{5}$ $\therefore R=\left(5\frac{3}{5},1\frac{4}{5}\right)$	B1 M1 equate A1 x or y-coordinate A1 o.e
9(c)	 Horizontal distance between S and Q = $8-2=6$ Horizontal distance between Q and T $=14-8=6$ $\frac{SQ}{QT}=1$ $\therefore$ Q is the **midpoint** of S and T. Or Q is equidistant from S and T.	M1 compare distance soi A1 ratio o.e A1 o.e
9(d)	$V=(0,7)$ Area of $OVPS$ = $\frac{1}{2}\begin{vmatrix} 0 & 2 & 3 & 0 \\ 0 & 0 & 7 & 7 \end{vmatrix}$ $=\frac{1}{2}\left[(14+21)-0\right]$ $=\frac{1}{2}(35)$ $=17\frac{1}{2}$ sq units	B1 soi M1 determinant method or oe. A1
		13 m

10(a)	$\mathrm{f}''(x)=\frac{8}{(4x+1)^2}$ $\mathrm{f}'(x)=\int\frac{8}{(4x+1)^2}\,\mathrm{d}x$ $=\frac{-2}{4x+1}+c$ $\frac{-2}{4(0)+1}+c=-2$ $c=0$ $\mathrm{f}'(x)=\frac{-2}{4x+1}$	B1 ignore +c M1 their $\mathrm{f}'(x)=-2$ A1
10(b)	$\mathrm{f}(x)=\int\frac{-2}{4x+1}\,\mathrm{d}x$ $=-\frac{2}{4}\ln(4x+1)+d$ $=-\frac{1}{2}\ln(4x+1)+d$ Given graph passes through the origin $\Rightarrow d=0$ $\mathrm{f}(x)=-\frac{1}{2}\ln(4x+1)$	B1 for $\frac{1}{2}\ln(4x+1)$ M1 find *d* by substituting *x*=0, *y*=0 A1
(c)	Since $\mathrm{f}(x)$ is a 'ln' function, it is defined for $4x+1>0$ $\Rightarrow x>-\frac{1}{4}$. It is given that $x>a$. Therefore $a=-\frac{1}{4}$	B1 the expression in their ln () >0 B1 AG
(d)	 	G1 Shape and passes through origin G1 Asymptote
		10 m

<table>
<tr><td>11(a)</td><td><table><tr><td>t</td><td>1.0</td><td>1.5</td><td>2.0</td><td>2.5</td><td>3.0</td><td>3.5</td></tr><tr><td>$s\sqrt{t}$</td><td>4.20</td><td>5.70</td><td>7.20</td><td>8.70</td><td>10.20</td><td>11.69</td></tr></table></td><td>G1 Straight line
P1 Plotted points
Values for ln y
in 2 dp</td></tr>
<tr><td>(b)</td><td>$s = a\sqrt{t} + \frac{b}{\sqrt{t}}$
$s\sqrt{t} = at + b$
$a = \text{Gradient} = \frac{11.69 - 1.20}{3.50 - 0}$
$= 2.997 \approx 3.00$
$b = s\sqrt{t} - \text{intercept} = 1.20$</td><td>B1 linear eqn seen

M1
A1 [2.99714]actual
2.80 to 3.19

A1 [1.204]actual
1.10 to 1.30</td></tr>
<tr><td>(c)</td><td>$2s\sqrt{t} + 2t - 10 = at + b$
$2s\sqrt{t} + 2t - 10 = s\sqrt{t}$
$s\sqrt{t} = -2t + 10$
Draw $s\sqrt{t} = -2t + 10$.

t=1.75</td><td>M1 attempt to manipulate equation to make $s\sqrt{t}$ subject.
G1 Straight line

A1 accept [1.70 to 1.80]</td></tr>
<tr><td></td><td></td><td>9 m</td></tr>
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