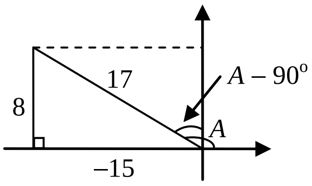
2015 S3 EOY IM1 Solutions

No.

1

(i) $\sin A = \dfrac{8}{17}$



(ii) $\tan(180° - A) = -\tan A$

$= \dfrac{8}{15}$

(iii) $\cos(A - 90°) = \sin A$

$= \dfrac{8}{17}$

2

	Y	X	m	c
$y = \dfrac{a}{\sqrt{x}} - b\sqrt{x}$	$y\sqrt{x}$ (given)	x	$-b$	a
$y = 2 + ab^{x}$	$\lg(y-2)$ $\ln(y-2)$	x	$\lg b$ $\ln b$	$\lg a$ $\ln a$

$y = \dfrac{a}{\sqrt{x}} - b\sqrt{x}$

$y\sqrt{x} = -bx + a$

$y = 2 + ab^{x}$

$\lg(y-2) = \lg ab^{x}$

$\lg(y-2) = x\lg b + \lg a$

3 (a)

$\sin\angle AEC = \dfrac{8\sqrt{3}}{16} = \dfrac{\sqrt{3}}{2}$

$\sin\angle CED = \dfrac{\sqrt{3}}{2}$

3 (b)

$\angle CED = 180° - \sin^{-1}(\dfrac{\sqrt{3}}{2}) = 120°$ (take obtuse $\angle$)

$\Rightarrow \angle ECD = 30°$ ($\angle$ sum of Δ)

OR

$\angle CEA = \sin^{-1}(\dfrac{\sqrt{3}}{2}) = 60°$

$\Rightarrow \angle ECD = 30°$ (ext. $\angle$ = sum of int. opp. $\angle$)

OR

$\angle BCA = \sin^{-1}(\dfrac{12}{8\sqrt{3}}) = 60°$

$\Rightarrow \angle ECD = 30°$ (adj. $\angle$s on a str. line)

Since $\angle ECD = \angle CDE = 30°$, ΔCED is isosceles

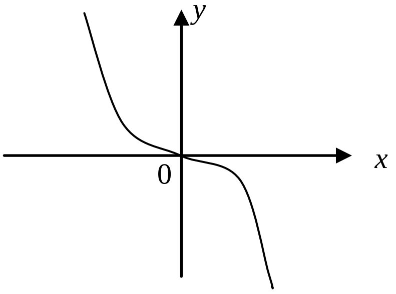
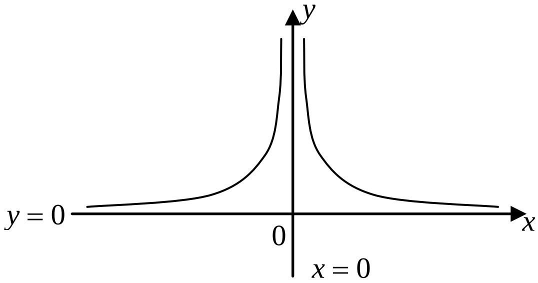
3 (c)

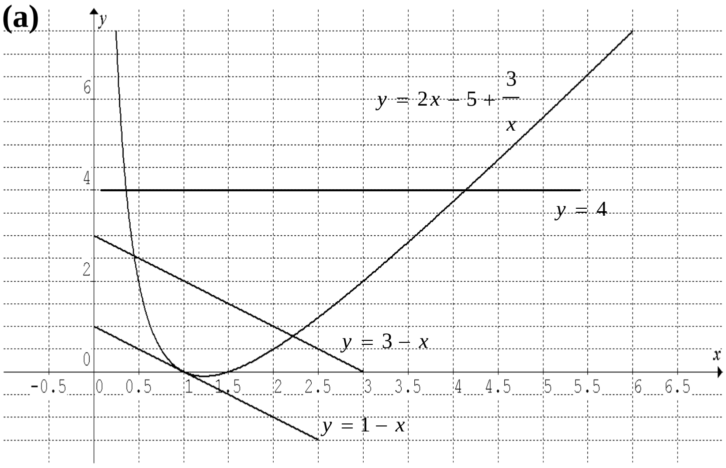
Using the property rt $\angle$ in a semicircle,
AD is the diameter of circle through A, B and D

$\sin 30° = \dfrac{12}{AD}$ **OR** $ED = EC = \sqrt{16^2 - \left(8\sqrt{3}\right)^2} = 8$ cm

$\Rightarrow AD = 24$ cm

$\Rightarrow$ radius = 12 cm

4 (a) (i)	
4 (a) (ii)	
4 (b)	$y = a^{bx} + 1$
5 (i)	$\text{grad} = \dfrac{5-1}{3-1} = 2$ $Y - 1 = 2(X-1)$ $Y = 2X - 1$ $y^2 - x = 2y - 1$ **OR** $y = \dfrac{2 \pm \sqrt{4-4(1-x)}}{2}$ $y^2 - 2y + 1 = x$ $y = \dfrac{2 \pm \sqrt{4x}}{2} = 1 \pm \sqrt{x}$ $(y-1)^2 = x$ $y = \sqrt{x} + 1$ (rej $-\sqrt{x} + 1$)
5 (ii)	sub $x = 9$, $y = 4$
6 (i)	Let l be the perpendicular bisector of PQ grad of $PQ = \dfrac{-12-0}{7-(-5)} = -1 \Rightarrow$ grad of $l = 1$ midpt of $PQ = \left(\dfrac{-5+7}{2}, \dfrac{0-12}{2}\right) = (1,-6)$ $y - (-6) = x - 1$ eqn of l is $y = x - 7$
6 (ii)	At pt C, $3x - 13 = x - 7 \Rightarrow 2x = 6$ $x = 3$, $y = -4$ $C\ (3, -4)$
6 (iii)	$CP = \sqrt{(-5-3)^2 + (0-(-4))^2} = \sqrt{80}$ units eqn of circle is $(x-3)^2 + (y+4)^2 = 80$
7 (a)	**(i)** $\angle AOE = 36°$ ($\angle$ at centre = 2$\angle$ at ⊙^{ce}) **(ii)** $\angle XAE = 18°$ ($\angle$ in alt. seg)

7 (b) (i)	$\angle ABC = 68° + 18°$ (ext. $\angle$ of cyclic quad) $= 86°$ $\angle ABC + \angle BCD = 86° + 94° = 180°$ By interior $\angle$s, // lines, *BA* is parallel to *CX* **OR** $\angle BAD = 180° - 94°$ ($\angle$s in opp. segment) $= 86°$ $\angle BAD = \angle ADX = 86°$ By alternate $\angle$s, // lines, *BA* is parallel to *CX*
7 (b) (ii)	$\angle OAX = 90°$ (tan $\perp$ rad) $\angle AXD = 180° - 90° - 52°$ (interior $\angle$s, *BA* // *CX*) $= 38°$ **OR** $\angle BAY = 90° - 52°$ (tan $\perp$ rad) $= 38°$ $\angle AXD = \angle BAY = 38°$ (corr. $\angle$s, *BA* // *CX*)
7 (c)	*CX* is not a tangent $\Rightarrow \angle ODX \neq 90°$ ($\because$ tan $\perp$ rad) $\Rightarrow \angle OAX + \angle ODX \neq 180°$ Since we can't use the reasoning $\angle$s in opp. segment, *AODX* is not a cyclic quadrilateral
8	**(a)**  **(b)** grad at (2, 0.5) = 1.0 to 1.4 (actual 1.25) **(c)** $x - \frac{9}{2} + \frac{3}{2x} = 0$ $2x - 5 + \frac{3}{x} = 4 \Rightarrow$ add in line $y = 4$ $x = 0.3 - 0.4,\ 4.1 - 4.2$ **(d)** $2x - 5 + \frac{3}{x} = 4 - \frac{3}{2}x$ $\frac{7}{2}x - 9 + \frac{3}{x} = 0$ $7x^2 - 18x + 6 = 0$ **(e)** **(i)** Draw $y = 3 - x$ **(ii)** $c = 1$
9 (a) (i)	$AB^2 = 123^2 + 120^2 - 2(123)(120)\cos 42°$ $AB = 87.128... = 87.1$ m (3 s.f.)

9 (a) (ii)	$\dfrac{\sin\angle BAC}{123}=\dfrac{\sin 42°}{87.128}$ $\angle BAC=\sin^{-1}\left(\dfrac{123\sin 42°}{87.128}\right)$ $=70.843°$ (3 d.p.) Bearing of C from A $=070.8°$ (1 d.p.)
9 (b)	let the angle of depression be θ $\tan\theta=\dfrac{65}{123}$ $\theta=\tan^{-1}\dfrac{65}{123}$ $=27.854...°=27.9°$ (1 d.p.)
9 (c)	Let the shortest distance of B from AC be d $\sin 42°=\dfrac{d}{123}$ $d=123\sin 42°$ $=82.303...$ $=82.3$ m (3 s.f.) $\dfrac{1}{2}\times 120\times d$ $=\dfrac{1}{2}(123)(120)\sin 42°$
9 (d)	$\angle ABC=180°-42°-70.843°$ ($\angle$ sum of Δ) $=67.157°$ (3 d.p.) $\angle EAC=67.157°-42°$ (ext. $\angle$ = sum of int. opp. $\angle$) $=25.157°$ (3 d.p.) area of $\Delta AEC=\dfrac{1}{2}(120)(87.128)\sin 25.157°$ $=2220\ \text{m}^2$ (3 s.f.)
10 (i)	$S(-4, 0)$, $R(2, 0)$ x-coord of $T=\dfrac{-4+2}{2}=-1$ sub $x=-1$, $y=-9$ $T(-1, -9)$
10 (ii)	$(x-2)(x+4)=-x+10$ $x^2+2x-8+x-10=0$ $x^2+3x-18=0$ $(x+6)(x-3)=0$ $x=-6$ or $x=3$ $y=16$ or $y=7$ $P(-6, 16)$ $Q(3, 7)$
10 (iii)	Area of quadrilateral $PQRS$ $=\dfrac{1}{2}\begin{vmatrix} 2 & 3 & -6 & -4 & 2 \\ 0 & 7 & 16 & 0 & 0 \end{vmatrix}$ $=\dfrac{1}{2}\left[(14+48)-(-42-64)\right]$ $=84$ sq units

10 (iv)	x-coord of $X = -1$ sub $x = -1$, $y = 11$ $X\,(-1, 11)$
10 (v)	By similar triangles, $QX : XP = 4 : 5$ **<u>OR</u>** $QX = \sqrt{4^2 + 4^2} = 4\sqrt{2}$ units $XP = \sqrt{5^2 + 5^2} = 5\sqrt{2}$ units ΔRQX and ΔRXP share the same height $\dfrac{\text{area of } \Delta RQX}{\text{area of } \Delta RXP} = \dfrac{4}{5}$
11	The circle is tangent to the lines $y = 2x + 10$ and $y = 2x - 10$ at A and B respectively Eqn of diameter is $y = -\frac{1}{2}x$ $2x + 10 = -\dfrac{1}{2}x$ OR $2x - 10 = -\dfrac{1}{2}x$ $x = -4$ OR $x = 4$ $A(-4, 2)$, $B(4, -2)$ radius $= \sqrt{4^2 + 2^2} = \sqrt{20}$ units Area of circle $= \pi\left(\sqrt{20}\right)^2$ $= 20\pi$ sq units $k = 20$