1. Let $\left(\begin{array}{ccc|c} a & -2a & -2b & 4 \\ a & 3a & -3b & 2 \\ 0 & -a & 5 & b \end{array}\right)$ be the augmented matrix of a linear system. Using elementary row operations, simplify the augmented matrix and hence

(a) find the values of a and b for the system to have a unique solution, [2]

(b) give a geometrical interpretation for the system of equations when $a \neq 0$ and $b = -5$. [2]

2. In a four-pole electrical network, the output quantities E_1 and I_1 are given in terms of the input quantities E_2 and I_2 by $\begin{pmatrix} E_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} E_2 \\ I_2 \end{pmatrix}$, where a, b, c and d are non-zero.

Denoting $\mathbf{A}$ by $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, show that

$$\begin{pmatrix} E_1 \\ E_2 \end{pmatrix} = \frac{1}{c}\begin{pmatrix} a & -|\mathbf{A}| \\ 1 & -d \end{pmatrix}\begin{pmatrix} I_1 \\ I_2 \end{pmatrix}.$$

[2]

Similarly, show that

$$\begin{pmatrix} E_1 \\ I_2 \end{pmatrix} = \frac{1}{d}\begin{pmatrix} \gamma & |\mathbf{A}| \\ 1 & \delta \end{pmatrix}\begin{pmatrix} I_1 \\ E_2 \end{pmatrix},$$

where γ and δ are constants to be determined in terms of a, b, c and d. [3]

Given $\mathbf{A}$ is orthogonal, that is $\mathbf{A}\mathbf{A}^T = \mathbf{I}$, and $|\mathbf{A}| = 1$, b and d are non-zero, find E_1 and E_2 in terms of b, d, I_1 and I_2.
[5]