



NANYANG JUNIOR COLLEGE

JC2 Preliminary Examination

Higher 2

FURTHER MATHEMATICS

9649/02

Paper 2

18th September 2025

3 hours

Additional Materials: Printed Answer Booklet
 List of Formulae and Results (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** the questions.

Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.

You must show all necessary working clearly.

The number of marks is given in brackets [] at the end of each question or part question.

This document consists of 7 printed pages and 1 blank page.



NANYANG JUNIOR COLLEGE
Internal Examinations

Section A: Pure Mathematics [50 marks]

- 1 (a) Let f and g be two differentiable functions of two variables x and y . Prove that

(i) $\nabla(f+g) = \nabla f + \nabla g$, [2]

(ii) $\nabla(fg) = f \nabla g + g \nabla f$. [2]

- (b) A structure S is erected on the horizontal x - y plane such that the vertical distance H of any point P on S above the x - y plane is given by

$$H = H(x, y) = 100 - (x^2 + y^2), \quad x^2 + y^2 \leq 100$$

where (x, y) is a point on the x - y plane directly below P .

- (i) Describe geometrically the set of points of S which lie on the x - y plane. [1]

- (ii) Show that the directional derivative of H at the highest point of S in any direction is zero. [2]

- 2 Fund managers observe economic global trends when deciding on how and what to invest in. For example, in this particular dynamic investment model,

$$\frac{d^2y}{dt^2} = -a \left(\frac{dy}{dt} \right)^2 + by, \quad \text{for } a, b > 0,$$

where $y(t)$ is the capital stock (in millions of dollars) at time t (in months), a is the depreciation constant and b is reinvestment feedback constant.

- (a) By letting $x = \frac{dy}{dt}$, show that $x \frac{dx}{dy} = -ax^2 + by$. [2]

- (b) Use the substitution $u = x^2$ to solve for u in terms of y . [4]

- (c) It is given that $a = 0.02$, $b = 0.5$. Given further that $y = 100$ and $\frac{dy}{dt} = 0$ initially, find $\frac{dy}{dt}$ in terms of y only. You may assume $\frac{dy}{dt} > 0$. [2]

3 (a) Do not use graphing calculator in answering this part.

Let ω be a cube root of unity whose imaginary part is non-zero. Show that

$$1 + \omega + \omega^2 = 0. \quad [1]$$

Hence evaluate, exactly, the expression

$$(2 - \omega)(2 - \omega^2)(2 - \omega^4)(2 - \omega^5). \quad [3]$$

- (b)** The complex number z is represented by the point P in an Argand diagram. Let c be a fixed complex number and r be a real constant satisfying

$$(r - c^*)z + (r - c)z^* = r^2 - |c|^2$$

where c^* and z^* denote the conjugates of c and z respectively.

Describe the locus of P . [4]

4 The curve C with pole O has polar equation given by

$$r = 2e^{\frac{1}{k}\theta},$$

where k is a positive constant and $0 \leq \theta < 2\pi$. The points A and B on the curve correspond to $\theta = 0$ and $\theta = \alpha$ respectively, where $0 < \alpha < \pi$. The length of the arc AB is denoted by S and the area of the sector OAB is denoted by T .

- (a)** Find S and T in terms of α and k . [7]
- (b)** Deduce that, for large values of k , $T \approx S$ [2]
- (c)** Sketch the graphs of C on separate diagrams, for the cases where $k = 10$, $k = 50$, $k = 100$ and explain how the result in part **(b)** can be deduced from these sketches. [3]

- 5 The surface S has equation $z = f(x, y)$, where $f(x, y) = x^2 + y^2 - 4x + (ax - y)^2$, and a is a real constant.
- (a) Find f_x , f_y , f_{xx} , f_{yy} and f_{xy} in terms of x , y and a . [3]
- (b) Show that the surface S has exactly one stationary point and determine the nature of this stationary point. [5]
- (c) A point $Q(k, 1+h, 2+2h)$ lies on the surface and is close to the point $P(0, 1, 2)$. Using linear approximation at the point P , show that $h \approx (2+a)k$. [2]
- (d) The plane π has equation $\alpha x + \beta y + \gamma z = 0$, where α , β and γ ($\gamma \neq 0$) are real constants. It intersects the surface S in a curve C that also passes through P . It is given that the tangent line to C at P is parallel to the vector $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$.
- (i) Find the value of a . [2]
- (ii) Find the equation of π . [3]

Section B: Probability and Statistics [50 marks]

- 6 A university lecturer wished to investigate whether there was an association between the students' year of study and their primary study approach. A random sample of 100 students was surveyed. The results are shown in the table.

Primary study approach	Year 1	Year 3
Study Alone	15	30
Group Study	20	10
Online Resources	10	15

- (a) Carry out a chi-square test at 1% significance level on whether there is an association between the students' year of study and their primary study approach. You should include the contribution to the test-statistic. [5]
- (b) Identify the two cells which make the greatest contributions to the test statistic. Provide a possible explanation for these high contributions and explain whether this contradicts your conclusion in part (a). [3]

- 7 An airport has a lost and found service counter for passengers who lost their belongings during their flight. The random variable Y denotes the number of reports being made by passengers who lost their belongings during an interval of one hour.

(a) State a condition needed to model Y by a Poisson distribution. Explain why this condition may not hold in this context. [2]

It is now assumed that Y follows a Poisson distribution with mean 4.5 per hour.

(b) Find the probability that there is at least one report made during an interval of one hour. [1]

(c) Show that the probability that at least one report being made in a period of t hours is given by $1 - e^{-4.5t}$. [1]

(d) Let T be the random variable denoting the time (in hours) between two successive reports. Show that T follows an exponential distribution, stating its parameter clearly. [2]

(e) Find the probability that the next report is made more than 15 minutes given that there is no report being made in the first 10 minutes. [2]

- 8 The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} 0 & x < k, \\ \frac{k}{(x-1)^3} & x \geq k, \end{cases}$$

where $k > 1$.

(a) Show that $k = 2$ and find the exact value of the expectation of X . [3]

(b) Random variable W is the maximum of three independent observations of X . Find the cumulative distribution function of W and hence find $P(W > 4)$. [4]

(c) Consecutive independent observations of W are made until the first observation that exceeds 4 is obtained. The random variable S is the total number of observations that have been made up to and including the observation exceeding 4. Find the value of $P(S > E(S))$. [3]

- 9 A research team collected exact potassium measurements in mg/kg from soil samples at 10 randomly chosen tomato farms to evaluate whether potassium level is below the recommended threshold of 250 mg/kg. The data is as follows:

Farm	1	2	3	4	5	6	7	8	9	10
Potassium Level (mg/kg)	180	220	250	190	210	271	89	242	199	262

- (a) Researcher A decided to carry out a sign test at the 5% significance level. Perform the test and determine the conclusion of the test. [4]
- (b) It is given that the potassium level for Farm 7 was incorrectly recorded in the original data table and the correct value is more than 250.

Assuming a Wilcoxon signed-rank test is conducted at the 5% significance level and that:

- no tied ranks exist in the absolute differences, and
- the conclusion of this Wilcoxon test must differ from the conclusion reached in part (a).

Determine the range of possible correct potassium levels for Farm 7. [3]

Using the correct potassium level of Farm 7, Researcher A carried out the same sign test again and got the same conclusion as before. Researcher B carried out a t -test for the same hypothesis and got a p -value of 3.80×10^{-10} . Researcher B then concluded that there is very strong evidence that the potassium level is below the recommended threshold of 250 mg/kg.

- (c) Comment on which conclusion is more reliable. [1]

- 10** A smartphone, Aphone, was advertised to have battery life of 11.1 hours. After a recent update of its software to Version 2, the users started to complain of lower battery life. Aphone's manufacturer selects a random sample of 8 Aphones that had undergone the software update and tested for their battery life (in hours):

Phone	1	2	3	4	5	6	7	8
Battery life	10.1	11.3	9.8	10.5	11.1	10.6	10.4	10.0

A $q\%$ confidence interval for the mean battery life of smartphones produced by the manufacturer was found to be (9.9206, 11.029).

- (a) State an assumption needed for the confidence interval to be valid. [1]
- (b) Find q , leaving your answer to the nearest integer. [3]
- (c) Without further computation, explain whether there is evidence that Aphone's battery life is lower than what was advertised at 5% significance level. [1]

Aphone's manufacturer updates the software to Version 3 to improve Aphone's battery life. The same sample of 8 Aphones were updated to Version 3 and tested for their battery life (in hours):

Phone	1	2	3	4	5	6	7	8
Battery life	10.9	11.8	10.2	10.3	11.7	9.7	11.0	10.5

- (d) Test, at the 5% significance level, whether the software update from Version 2 to Version 3 improved Aphone's battery life, stating any assumptions needed. [5]

To verify the effect at scale, they use large-scale internal performance monitoring data on battery life (in hours).

	Mean	Standard deviation	Sample size
Battery life (Version 2)	10.2	0.9	120
Battery life (Version 3)	10.6	1.0	140

- (e) Perform an appropriate test, to determine if the software update from Version 2 to Version 3 had improved Aphone's battery life, at 5% significance level. [5]
- (f) Comment on how the manufacturer should use the conclusions obtained from the tests in parts (d) and (e). [1]

BLANK PAGE