Secondary 3 Mathematics
WA3 Revision Time Trial — Ahmad Ibrahim (2023)

Duration: 48 minutes **Score: __________ / 32**

Topics tested: Graphs of Functions; Graphs in Practical Situations; Further Trigonometry; Arc Length and Sector Area.

Section A: Graphs of Functions [10 marks]

1. The variables x and y are connected by the equation

$$y = \frac{1}{2x^2} + \frac{1}{10}x^2 - 3.$$

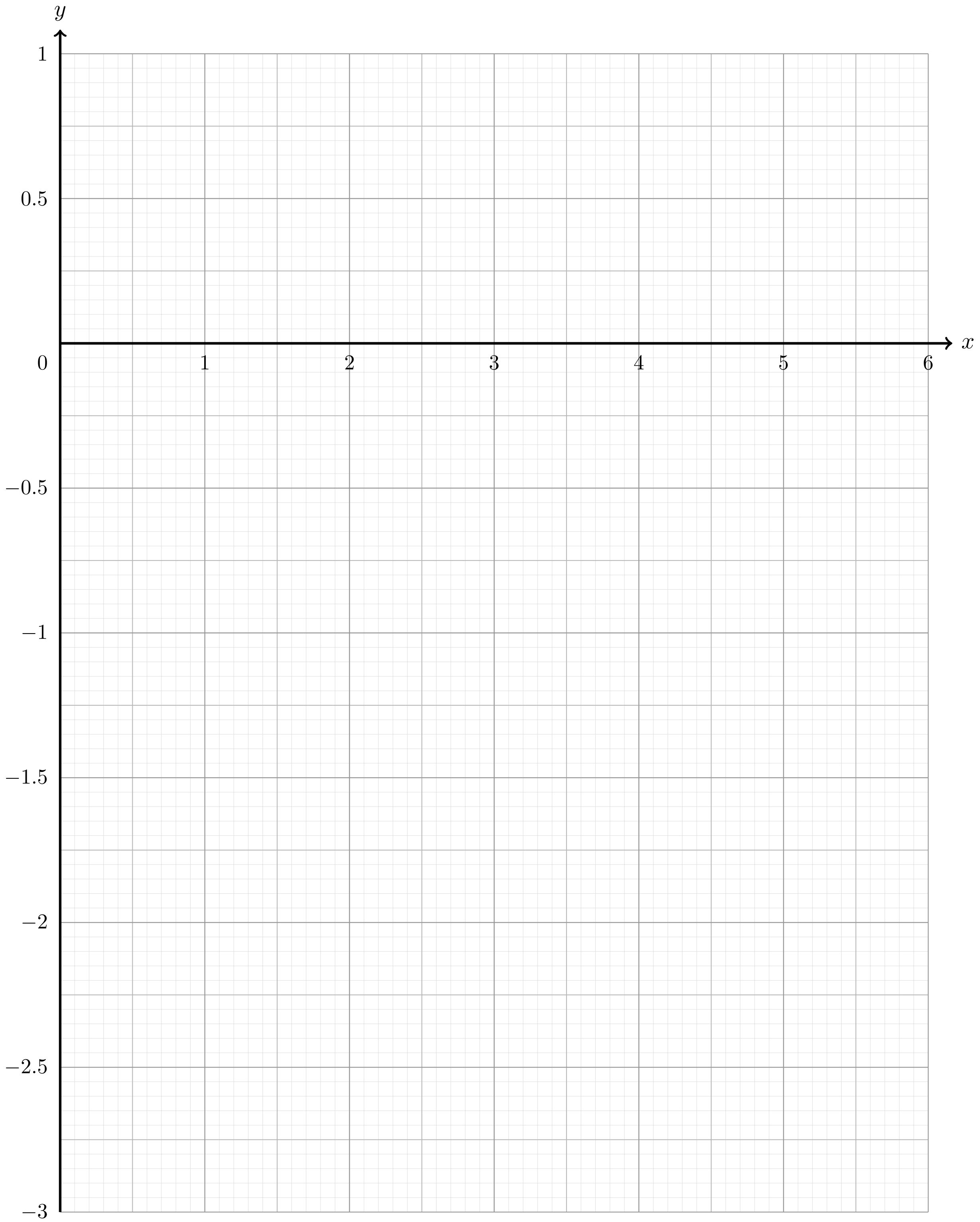
The table below shows some corresponding values of x and y, correct to 2 decimal places.

x	0.5	1	1.5	2	3	4	5	6
y	-0.98	-2.40	-2.55	-2.48	-2.04	p	-0.48	0.61

(a) Calculate the value of p. *[1]*

Answer $p =$ ....

(b) On the grid on the next page, draw the graph of $y = \dfrac{1}{2x^2} + \dfrac{1}{10}x^2 - 3$ for $0.5 \le x \le 6$. *[3]*



y
x
1
0.5
0
−0.5
−1
−1.5
−2
−2.5
−3
1
2
3
4
5
6

(c) By drawing a tangent, find the gradient of the curve at $x = 2$. *[2]*

Answer ....

(d) By drawing a suitable straight line on the same axes, solve the equation

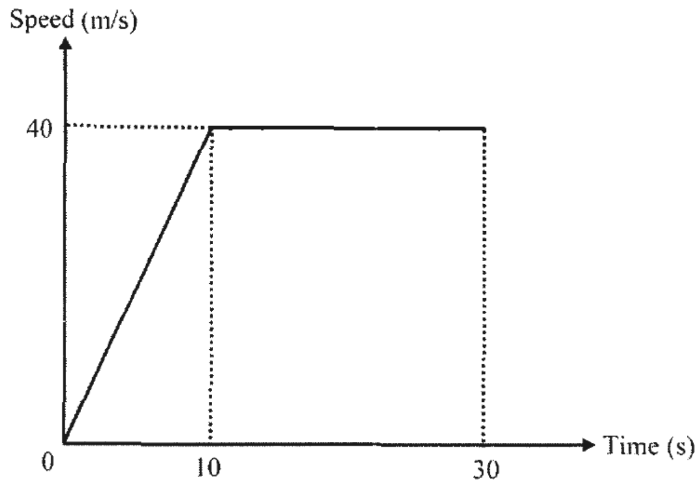
$$\frac{1}{2x^2} + \frac{1}{10}x^2 - x = 0.$$

[4]

Answer $x =$ ....

Section B: Graphs in Practical Situations [4 marks]

2. The diagram shows the speed-time graph for a motorist's journey.



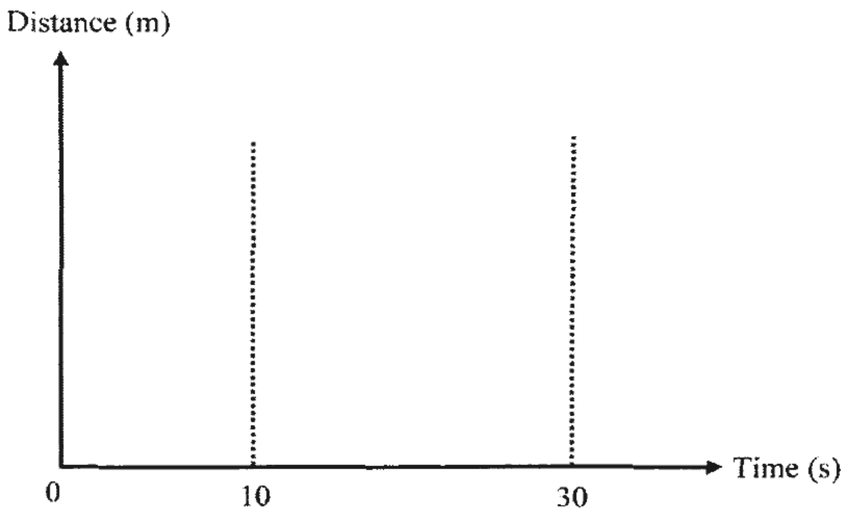
(a) Find the acceleration of the motorist during the first 10 seconds. *[1]*

Answer .... m/s^2

(b) Find the speed of the motorist at the 6th second. *[1]*

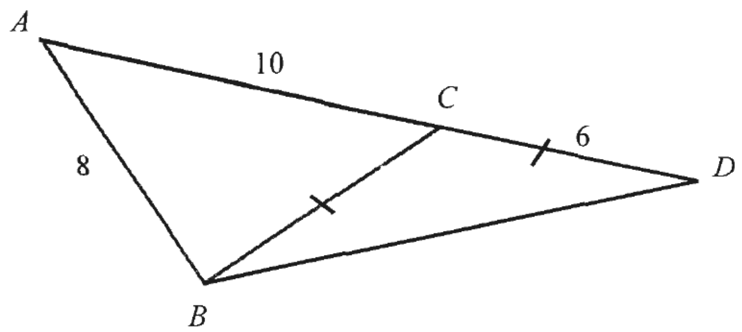
Answer .... m/s

(c) Sketch the distance-time graph of the motorist for the journey on the axes below. *[2]*



Section C: Further Trigonometry [7 marks]

3. In the diagram, $AB = 8$ cm, $CD = 6$ cm, $BC = CD$ and ACD is a straight line.



(a) Show that angle ABC is a right angle. *[2]*

Answer ....

....

....

(b) Find the value of

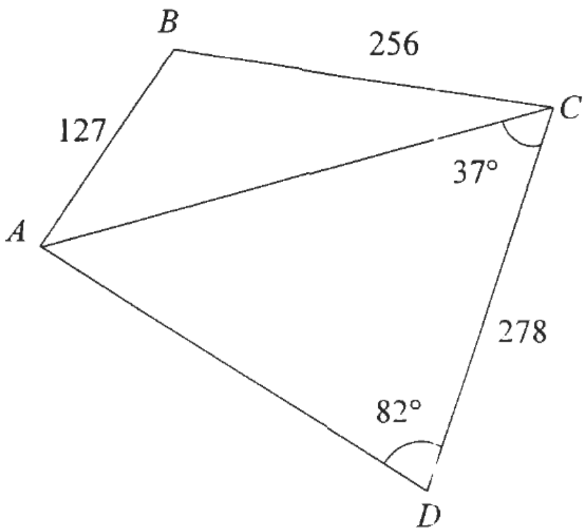
(i) sin $\angle BCD$, *[1]*

Answer ....

(ii) cos $\angle BCD$. *[1]*

Answer ....

4. In the diagram, $AB = 127$ km, $BC = 256$ km and $CD = 278$ km. Angle $ACD = 37°$ and angle $ADC = 82°$.

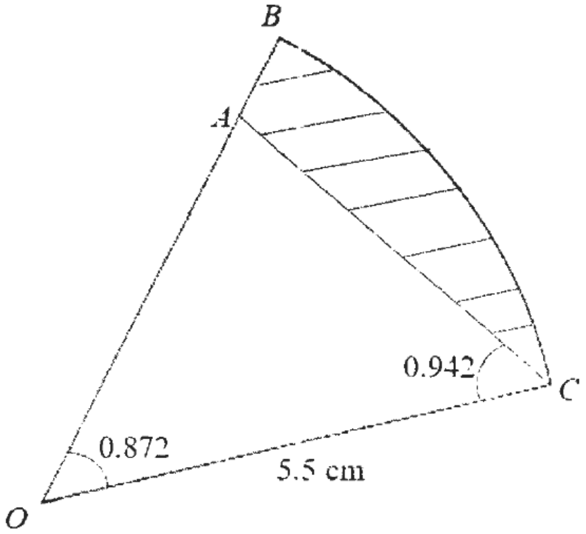


Find angle ABC. *[3]*

Answer *Angle* $ABC =$ .... °

Section D: Arc Length and Sector Area [11 marks]

5. OBC is a sector of a circle with centre O and radius 5.5 cm. A is a point on radius OB. Angle $AOC = 0.872$ radians and angle $ACO = 0.942$ radians.



Calculate

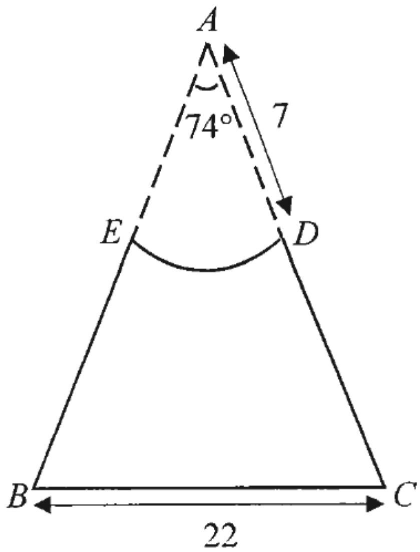
(a) OA, *[2]*

Answer $OA =$ .... cm

(b) the area of the shaded region ABC. *[3]*

Answer .... cm^2

6. A sector of a circle radius 7 cm is removed from an isosceles triangle ABC as shown in the diagram. Angle EAD is $74°$ and $BC = 22$ cm.



(a) Calculate the length of the arc ED. *[2]*

Answer .... cm

(b) (i) Show that the perpendicular height of the triangle ABC is 14.597 cm, correct to 5 significant figures. *[1]*

Answer ....

....

....

(ii) Hence find the area of $BCDE$. *[3]*

Answer .... cm^2

— END OF PAPER —

Answer Key
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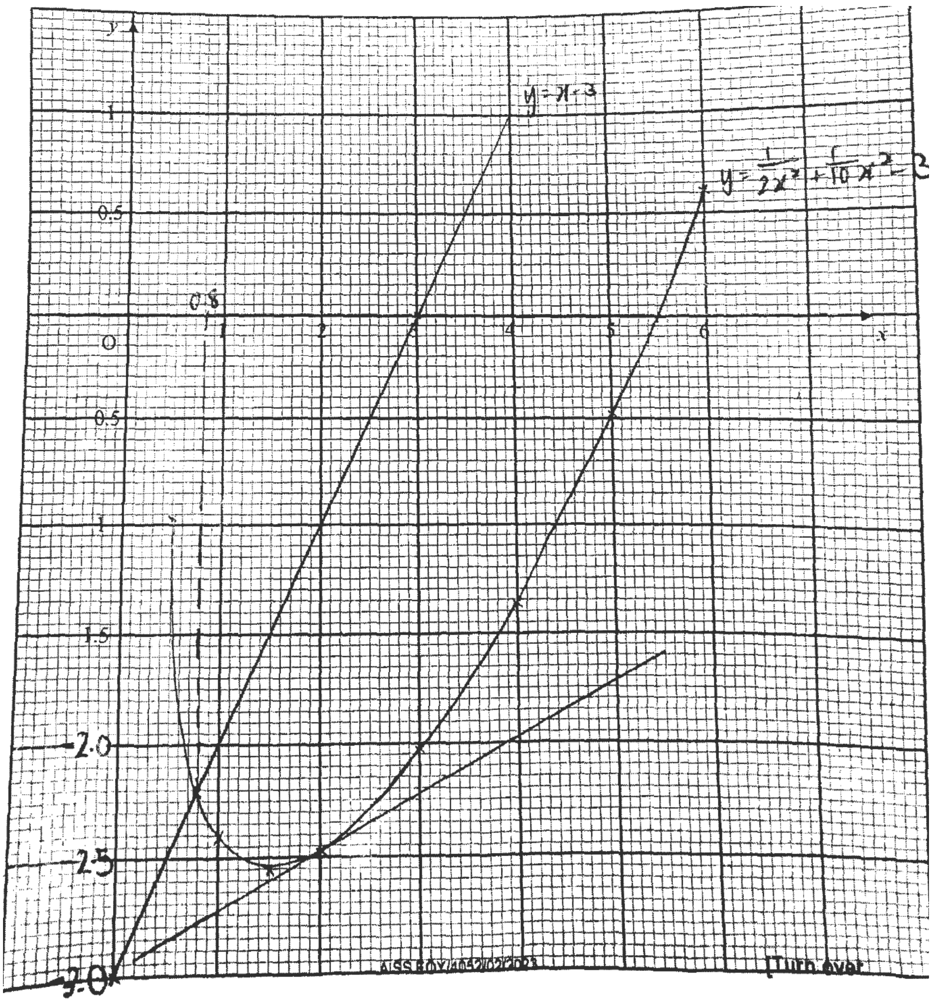
Section A: Graphs of Functions [10 marks]

1(a) At $x = 4$: $y = \dfrac{1}{2(4)^2} + \dfrac{1}{10}(4)^2 - 3 = 0.03125 + 1.6 - 3 = -1.36875$.
$\therefore p = -\mathbf{1.37}$ (2 d.p.). *[1]*

1(b) All eight tabulated points plotted and joined with a smooth curve: the curve falls from $(0.5,\ -0.98)$ to a minimum of about -2.55 near $x = 1.5$, then rises through $(6,\ 0.61)$. *[3]*



(Marking scheme graph shown, including the tangent for (c) and the line $y = x - 3$ for (d).)

1(c) Tangent drawn at $(2,\ -2.48)$.
Exact gradient: $\dfrac{dy}{dx} = -\dfrac{1}{x^3} + \dfrac{x}{5} = -0.125 + 0.4 = \mathbf{0.275}$ (accept 0.175 to 0.375). *[2]*

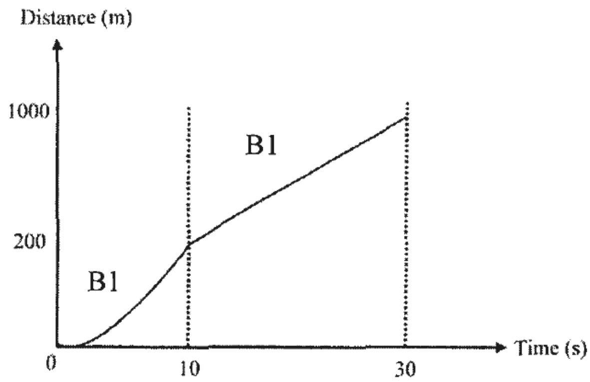
1(d) $\dfrac{1}{2x^2} + \dfrac{1}{10}x^2 - x = 0 \ \Leftrightarrow\ \dfrac{1}{2x^2} + \dfrac{1}{10}x^2 - 3 = x - 3.$
So draw the line $\mathbf{y} = \mathbf{x} - \mathbf{3}$ (e.g. through $(0.5,\ -2.5)$ and $(4,\ 1)$).
The line meets the curve once in range: $\mathbf{x} \approx \mathbf{0.8}$ (accept 0.7 to 0.9). *[4]*

Section B: Graphs in Practical Situations [4 marks]

2(a) Acceleration $=$ gradient $= \dfrac{40}{10} =$ **4** m/s^2. *[1]*

2(b) During the first 10 s the speed increases at 4 m/s^2, so at the 6$^{\text{th}}$ second:
$v = 4 \times 6 =$ **24** m/s. *[1]*

2(c) Distance at $t = 10$: $\dfrac{1}{2}(10)(40) = 200$ m (curved, increasing gradient).
Distance at $t = 30$: $200 + 40 \times 20 = 1000$ m (straight line, constant gradient). *[2]*



Section C: Further Trigonometry [7 marks]

3(a) $BC = CD = 6$ cm and $AC = 10$ cm (from the diagram).
$AB^2 + BC^2 = 8^2 + 6^2 = 100$ and $AC^2 = 10^2 = 100$.
Since $AB^2 + BC^2 = AC^2$, by the **Converse of Pythagoras' Theorem**, triangle ABC is right-angled at B, i.e. angle $ABC = 90°$. *[2]*

3(b)(i) $\sin\angle BCD = \sin(180° - \angle BCA) = \sin\angle BCA = \dfrac{AB}{AC} = \dfrac{8}{10} = \mathbf{\dfrac{4}{5}}$. *[1]*

3(b)(ii) $\cos\angle BCD = -\cos\angle BCA = -\dfrac{BC}{AC} = -\dfrac{6}{10} = -\mathbf{\dfrac{3}{5}}$. *[1]*

4. In triangle ACD: $\angle CAD = 180° - 37° - 82° = 61°$ ($\angle$ sum of $\triangle$).
By the Sine Rule: $\dfrac{AC}{\sin 82°} = \dfrac{278}{\sin 61°} \Rightarrow AC = \dfrac{278\sin 82°}{\sin 61°} = 314.759\ldots$ km.
In triangle ABC, by the Cosine Rule:
$\cos\angle ABC = \dfrac{127^2 + 256^2 - 314.759\ldots^2}{2(127)(256)} \Rightarrow \angle ABC =$ **105.5**° (1 d.p.). *[3]*

Section D: Arc Length and Sector Area [11 marks]

5(a) $\angle OAC = \pi - 0.872 - 0.942 = 1.3276$ rad ($\angle$ sum of $\triangle$).
By the Sine Rule: $\dfrac{OA}{\sin 0.942} = \dfrac{5.5}{\sin 1.3276}$

$OA = \dfrac{5.5\sin 0.942}{\sin 1.3276} = 4.5829\ldots =$ **4.58** cm (3 s.f.). *[2]*

5(b) Area of sector $OBC = \dfrac{1}{2}r^2\theta = \dfrac{1}{2}(5.5)^2(0.872) = 13.189$ cm^2.

Area of triangle $OAC = \dfrac{1}{2}(5.5)(4.5829\ldots)\sin 0.872 = 9.6491$ cm^2.

Shaded area $ABC = 13.189 - 9.6491 = 3.5399 =$ **3.54** cm^2 (3 s.f.). *[3]*

6(a) Arc $ED = \frac{74°}{360°} \times 2\pi(7) = 9.0408\ldots = \mathbf{9.04}$ cm (3 s.f.). *[2]*

6(b)(i) The perpendicular from A bisects BC (isosceles) and bisects the $74°$ apex angle.
$\tan 37° = \frac{11}{h} \Rightarrow h = \frac{11}{\tan 37°} = \mathbf{14.597}$ cm (5 s.f.). *[1]*

6(b)(ii) Area of triangle $ABC = \frac{1}{2}(22)(14.597) = 160.567$ cm^2.

Area of sector $EAD = \frac{74°}{360°} \times \pi(7)^2 = 31.6428$ cm^2.

Area of $BCDE = 160.567 - 31.6428 = 128.92\ldots = \mathbf{129}$ cm^2 (3 s.f.). *[3]*