Anglo-Chinese School

(Independent)

FINAL EXAMINATION 2017

YEAR 5 IB DIPLOMA PROGRAMME

MATHEMATICS

HIGHER LEVEL

PAPER 2

Monday 16th October 2017

2 hours

Candidate Session Number

0	0	2	3	2	9				

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the writing paper provided. Fill in your session number on each answer sheet, and attach them to this examination paper using the string provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- A clean copy of the **Mathematics HL and Further Mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**

Section A (50 Marks)		Section B (50 Marks)	
Question	**Marks**	**Question**	**Marks**
1		**9**	
2			
3		**10**	
4			
5			
6		**11**	
7			
8			
Subtotal		**Subtotal**	
TOTAL	**/ 100**		

This question paper consists of 12 printed pages including this cover page.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

Solve the following equations

$$2^{x+3y} = \frac{1}{e}$$
$$\log_2(3x + y) = e$$

2. [Maximum mark: 7]

At the start of the first year, a student puts in 1000 dollars into a new savings account. At the end of that year, he earned a fixed interest of 1.2% after which he withdraws a sum of p dollars. The student continues with this practice of withdrawing p dollars annually for a total of 7 years.

If the total balance in the account at the end of 7 years is \$439.52, find p .

3. [Maximum mark: 6]

Given that $z_1 = 3 + 4i$ and $z_3 = 4 + \left(4 + \sqrt{3}\right)i$,

(a) Find z_2 if $z_1 + z_2 = z_3$. [2]

(b) **Hence** find the coordinates of a point B which is on a bearing of $330°$ and a distance of 4 km from point A, which has coordinates $(3, 4)$ from a given origin. Leave your final answer in surd form. [4]

4. [Maximum mark: 5]

How many four digit numbers can be formed using the digits 0, 1, 2, 3, 4 and 5 which are even, without repeating the digits? [5]

5. [Maximum mark: 5]

Given the function $f(x) = \ln\left(x^2 + 3\right) + \left(x - 1\right)^2 + ax$ is an even function, find a.

6. [Maximum mark: 7]

Given a function $f(x) = \frac{e^x + 2}{x^2 + 1}$, find the range of values of x for which

(a) $f'(x) \leq f(x)$. [4]

(b) $f(x)$ is concave downwards. [3]

7. [Maximum mark: 7]

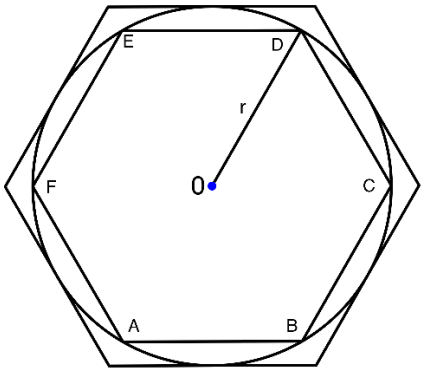
The plane π_1 is given by $\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

(a) Find a unit vector $\hat{\mathbf{n}}$ normal to the plane and the equation of plane π_1 in $\mathbf{r} \bullet \hat{\mathbf{n}} = \phi$ form. [4]

(b) **Hence** find the possible equations of plane π_2, given that it is parallel to π_1 and a distance of $3\sqrt{3}$ units from it. [3]

8. [Maximum mark: 7]

A regular hexagon $ABCDEF$ has a circumscribed circle of radius r and centre at O, and a second regular polygon is drawn such that its sides are tangential to the same circle. Adjacent vertices of both hexagons are concurrent with the centre of the circle. A drawing of the arrangement is shown below.



(a) Find the perimeters of both hexagons, leaving your answer in terms of r and in surd form. [4]

(b) **Hence** show that $3 \leq \pi \leq 2\sqrt{3}$. [3]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer sheets provided. Please start each question on a new page.

9. [Maximum mark: 18]

(a) Solve $z^3 - 3z^2 + (3-i)z - 2(1-i) = 0$ [4]

(b) Given that $z = \cos\theta + i\sin\theta$,

(i) Show that $\sin\theta + i\cos\theta = iz^*$, where z^* is the complex conjugate of z.

(ii) Expand $(\sin\theta + i\cos\theta)^3$ and **hence** show that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$. [5]

(c) A polynomial is given by the equation $f(z) = z^8 + 1$.

(i) Find the zeros of $f(z)$, leaving your answers in $re^{i\theta}$ form.

(ii) Sketch all the zeros of $f(z)$ in an Argand diagram.

(iii) **Hence** show that $\cos\frac{\pi}{8} + \cos\frac{3\pi}{8} + \cos\frac{5\pi}{8} + \cos\frac{7\pi}{8} = 0$. [9]

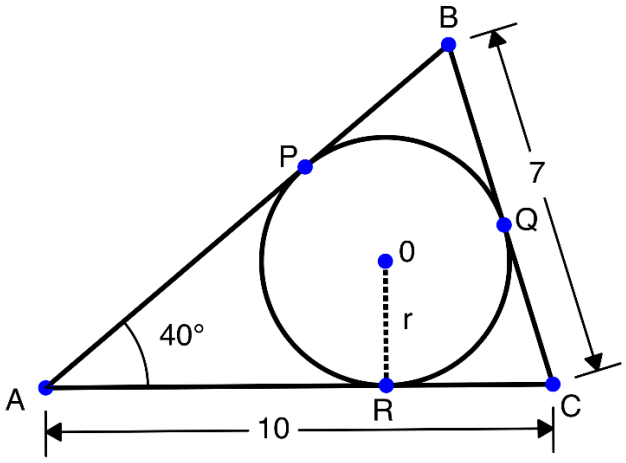
10. [Maximum mark: 17]

(a) A triangle ABC is such that $AC = 10$ m, $BC = 7$ m and $C\hat{A}B = 40°$.

(i) Find the possible length of the side AB and the area of the smaller triangle.

(ii) **Hence** calculate the ratio of the area of the smaller to the larger triangle. [8]

(b) A circle of radius r is inscribed in the larger triangle ABC such that the circumference is tangential to sides AB, BC and CA at P, Q and R respectively.

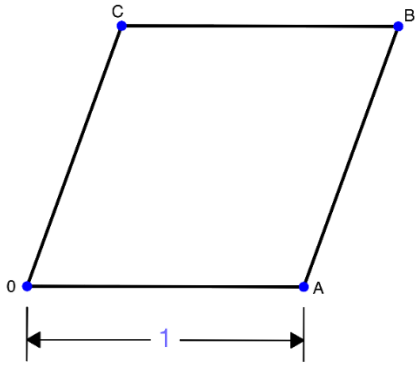


(i) Find the radius r

(ii) Calculate the area bounded by AP, AR and the minor arc PR [9]

11. [Maximum mark: 15]

(a) Consider a rhombus $OABC$ with sides $OA = OC = 1$ unit.



Given that $\overrightarrow{OA} =$ **a** and $\overrightarrow{OC} =$ **c,**

(i) Find $\overrightarrow{OB}$ and $\overrightarrow{AC}$ in terms of **a** and **c** and prove that diagonals of the rhombus are perpendicular.

(ii) Hence explain why OB is the angle bisector of $A\hat{O}C$. [4]

(b) Given that the two planes π_1 and π_2 have equations

$$\mathbf{r} \bullet \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 4 \text{ and } \mathbf{r} \bullet \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix} = 8 \text{ respectively.}$$

(i) Verify that the line of intersection between these two planes has equation

$$\mathbf{r} = \begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} -3 \\ 0 \\ 4 \end{pmatrix}$$

(ii) Find the equation of the plane through $A(2,4,0)$ and which is perpendicular to both planes π_1 and π_2.

(iii) By considering result in part (a), find the vector equation of the line through A which bisects the angle between the two planes π_1 and π_2. [11]

END OF PAPER