NORTHBROOKS SECONDARY SCHOOL
MATHEMATICS DEPARTMENT
4EXP ADDITIONAL MATHEMATICS
Prelim Paper 1

MARKING SCHEME

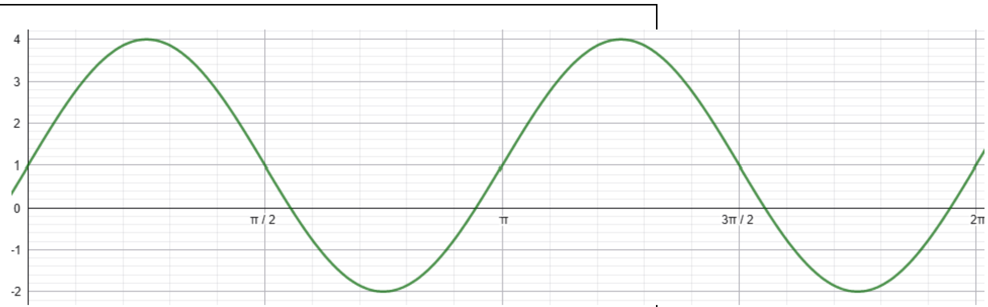
Qns	Answer
1(a)	Volume = $\pi r^2 h$ $\left(3\sqrt{5}-4\right)\pi=\pi r^2\left(3+\sqrt{5}\right)$ $r^2=\dfrac{\left(3\sqrt{5}-4\right)\pi}{\left(3+\sqrt{5}\right)\pi}$ $r^2=\dfrac{\left(3\sqrt{5}-4\right)}{\left(3+\sqrt{5}\right)}\times\dfrac{\left(3-\sqrt{5}\right)}{\left(3-\sqrt{5}\right)}$ $r^2=\dfrac{9\sqrt{5}-15-12+4\sqrt{5}}{9-5}$ $r^2=\dfrac{-27+13\sqrt{5}}{4}$
1(b)	$x-y=3$ --(1) and $x^2-3xy+y^2+19=0$ --(2) From (1) $y=x-3$ Sub $y=x-3$ into (2) $x^2-3x(x-3)+(x-3)^2+19=0$ $x^2-3x^2+9x+x^2-6x+9+19=0$ $-x^2+3x+28=0$ $x^2-3x-28=0$ $(x+4)(x-7)=0$ $x=-4$ or $x=7$ $y=-7$ or $y=4$ Coordinates are $\left(-4,-7\right)$ and $\left(7,4\right)$

2(a)	$y=\frac{3}{4}\left(\frac{x}{12}-1\right)^6$ $\frac{\mathrm{d}y}{\mathrm{d}t}=12\times\frac{\mathrm{d}x}{\mathrm{d}t}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=12$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3}{4}(6)\left(\frac{x}{12}-1\right)^5\left(\frac{1}{12}\right)$ $12=\frac{3}{8}\left(\frac{x}{12}-1\right)^5$ $32=\left(\frac{x}{12}-1\right)^5$ $2^5=\left(\frac{x}{12}-1\right)^5$ $x=36$ $y=\frac{3}{4}\left(\frac{36}{12}-1\right)^6$ $y=48$
2(b)	$\mathrm{f}(x)=\frac{e^{2x}}{1-x}$ $\mathrm{f}'(x)=\frac{(1-x)2e^{2x}-e^{2x}(-1)}{(1-x)^2}$ $\mathrm{f}'(x)=\frac{e^{2x}(3-2x)}{(1-x)^2}$ for $x>1$, **change condition to x > 1.5** $e^{2x}>0$, $(1-x)^2>0$, $(3-2x)<0$ $\therefore \frac{e^{2x}(3-2x)}{(1-x)^2}<0$ $\therefore \mathrm{f}'(x)$ is a decreasing function

3(a)	$\dfrac{7x^2-30x+3}{(x-5)(x^2+3)}=\dfrac{A}{x-5}+\dfrac{Bx+C}{x^2+3}$ $7x^2-30x+3=A(x^2+3)+(Bx+C)(x-5)$ Let $x=5$ $28=28A$ $A=1$ Let $x=0$ $3=1(3)+C(-5)$ $C=0$ Let $x=1$ $-20=1(4)+B(-4)$ $B=6$ $\dfrac{7x^2-30x+3}{(x-5)(x^2+3)}=\dfrac{1}{x-5}+\dfrac{6x}{x^2+3}$
3(b)	$\dfrac{\mathrm{d}}{\mathrm{d}x}\ln(x^2+3)=\dfrac{2x}{x^2+3}$
3(c)	$\int\dfrac{7x^2-30x+3}{(x-5)(x^2+3)}\ \mathrm{d}x$ $=\int\dfrac{1}{x-5}+\dfrac{6x}{x^2+3}\ \mathrm{d}x$ $=\ln(x-5)+3\ln(x^2+3)+c$
4(a)	103 represents the height of the roller coaster car at the start
4(b)	$3t^2-24t+103$ $=3(t^2-8t)+103$ $=3\left[(t-4)^2-16\right]+103$ $=3(t-4)^2+55$

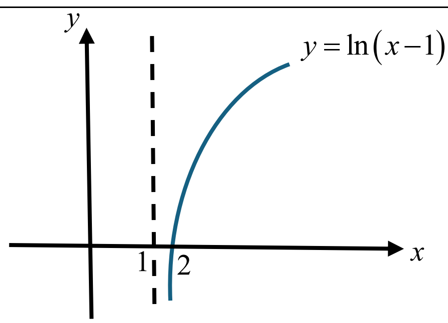
4(c)	The minimum height is 55 so the section of the roller coaster track has met the safety requirements
5(a)	$\left(2x-\frac{1}{x^2}\right)^8$ $T_{r+1}=\binom{8}{r}(2x)^{8-r}\left(-\frac{1}{x^2}\right)^r$ $T_{r+1}=\binom{8}{r}2^{8-r}x^{8-r}(-1)^r x^{-2r}$ Power of $x = 8-3r$ $8-3r=0$ $r=\frac{8}{3}$ Since r is not an integer, there is no independent term in the expansion.
5(b)	$\left(2x-\frac{1}{x^2}\right)^8$ $=(2x)^8-\binom{8}{1}(2x)^7\left(\frac{1}{x^2}\right)+\binom{8}{2}(2x)^6\left(\frac{1}{x^2}\right)^2+\ldots$ $=256x^8-1024x^5+1792x^2+\ldots$
5(c)	$\left(\frac{1}{x^3}+5\right)\left(2x-\frac{1}{x^2}\right)^8$ $=\left(\frac{1}{x^3}+5\right)\left(256x^8-1024x^5+1792x^2+\right)$ $=\ldots+\left(\frac{1}{x^3}\right)\left(-1024x^5\right)+(5)\left(1792x^2\right)+\ldots$ $=7936x^2$ Coefficient = 7936

6(a)	$y = x^2\sqrt{1-3x}$ $\frac{dy}{dx} = x^2 \frac{d}{dx}\left(\sqrt{1-3x}\right) + \left(\sqrt{1-3x}\right)\frac{d}{dx}x^2$ $\frac{dy}{dx} = x^2\left(\frac{1}{2}\right)(1-3x)^{-\frac{1}{2}}(-3) + \left(\sqrt{1-3x}\right)(2x)$ $\frac{dy}{dx} = \frac{-3x^2}{2\sqrt{1-3x}} + \left(\sqrt{1-3x}\right)(2x)$ $\frac{dy}{dx} = \frac{-3x^2 + (2x)(2)(1-3x)}{2\sqrt{1-3x}}$ $\frac{dy}{dx} = \frac{4x - 15x^2}{2\sqrt{1-3x}}$ $\frac{dy}{dx} = \frac{x(4-15x)}{2\sqrt{1-3x}}$
6(b)	$\int_{-1}^{0} \frac{2 + x(4-15x)}{2\sqrt{1-3x}}\,dx$ $= \int_{-1}^{0} \frac{2}{2\sqrt{1-3x}} + \frac{x(4-15x)}{2\sqrt{1-3x}}\,dx$ $= \int_{-1}^{0} \frac{1}{\sqrt{1-3x}} + \frac{x(4-15x)}{2\sqrt{1-3x}}\,dx$ $= \left[\frac{(1-3x)^{\frac{1}{2}}}{\frac{1}{2}(-3)}\right]_{-1}^{0} + \left[x^2\sqrt{1-3x}\right]_{-1}^{0}$ $= \frac{2}{3} + (-2)$ $= -\frac{4}{3}$
7(a)	Amplitude = 3 Period = 180° or π

7(b)	
7(c)	$k = 4$ or $k = -2$
8(a)	@ $y = 0 \quad x = 3$ $A = (3,0)$
8(b)	Grad of AD = $\dfrac{5-0}{-2-3} = -1$ Grad of DC = $\dfrac{-1}{-1} = 1$ $\dfrac{5k+3-5}{3k-(-2)} = 1$ $5k-2 = 3k+2$ $2k = 4$ $k = 2$
8(c)	C = (6, 13) Midpt of DC = $\left(\dfrac{-2+6}{2}, \dfrac{5+13}{2}\right)$ $= (2,9)$ Gradient of perpendicular = -1 Equation of perpendicular bisector: $y-9 = -1(x-2)$ $y = -x+11$

8(d)	$\text{Area} = \frac{1}{2}\begin{vmatrix} 3 & 8 & 6 & -2 & 3 \\ 0 & 3 & 13 & 5 & 0 \end{vmatrix}$ $= \frac{1}{2}(9+104+30-18-(-26)-15)$ $= 68 \text{ units}^2$
9(a)	$\angle BFA = 90°$ (given) $\angle ABC = 90°$ (angle in semicircle) $\angle ABC = \angle BFA$ $\angle FAB = \angle BCA$ (angle in alt. seg) triangle *BFA* is similar to triangle *ABC*
9(b)	Since triangle *BFA* is similar to triangle *ABC* $\frac{AB}{AC} = \frac{BF}{AB}$ $\therefore AB^2 = AC \times BF$
9(c)	ΔTBA is a right angle triangle $TA^2 = TB^2 + AB^2$ ΔTAC is a right angle triangle $TA^2 = TC^2 - AC^2$ $\therefore TB^2 + AB^2 = TC^2 - AC^2$ $\therefore TB^2 + AB^2 = (TC + AC)(TC - AC)$
10(a)	$\pi x^2 y = 32$ $y = \frac{32}{\pi x^2}$ $A = 2\pi x^2 + 2\pi xy$ $A = 2\pi x^2 + 2\pi x\left(\frac{32}{\pi x^2}\right)$ $A = 2\pi x^2 + \frac{64}{x}$

10(b)	$\frac{dA}{dx}=4\pi x-\frac{64}{x^2}$ $0=4\pi x-\frac{64}{x^2}$ $4\pi x=\frac{64}{x^2}$ $x^3=\frac{64}{4\pi}$ $x=1.720508028$ $x=1.72$
10(c)	$\frac{d^2A}{dx^2}=4\pi+\frac{128}{x^3}$ $@\,x=1.720508028$ $\frac{d^2A}{dx^2}=37.69911184>0$ The stationary value of A is a minimum value. Hence with this dimension, the cost of the production will be minimum
11(a)	From B to FC $\sin\theta=\frac{d}{50}$ $d=50\sin\theta$ From B to AE $\cos\theta=\frac{d}{30}$ $d=30\cos\theta$ $W=50\sin\theta+30\cos\theta.$

11(b)	$50\sin\theta+30\cos\theta=R\sin(\theta+\alpha)$ $R=\sqrt{50^2+30^2}$ $R=\sqrt{3400}$ $R=10\sqrt{34}$ $\tan\alpha=\dfrac{30}{50}$ $\alpha=30.963756°$ $50\sin\theta+30\cos\theta=10\sqrt{34}\sin(\theta+31.0°)$
11(c)	$10\sqrt{34}\sin(\theta+31.0°)=40$ $\sin(\theta+30.963756°)=\dfrac{40}{10\sqrt{34}}$ $\theta=12.3°$
12(a)	
12(b)	$e^{x-1}=\sqrt{x-1}$ $\ln e^{x-1}=\ln\sqrt{x-1}$ $x-1=\dfrac{1}{2}\ln(x-1)$ $2(x-1)=\ln(x-1)$ $y=2(x-1)$

13	$\frac{dy}{dx}=-\frac{1}{2}e^{-\frac{x}{2}}$ $@\left(-4,e^{2}\right)\ \frac{dy}{dx}=-\frac{1}{2}e^{2}$ Equation of tangent: $y-e^{2}=-\frac{1}{2}e^{2}\left(x-(-4)\right)$ $y=-\frac{1}{2}e^{2}x-2e^{2}+e^{2}$ $y=-\frac{1}{2}e^{2}x-e^{2}$ $@\,y=0$ $0=-\frac{1}{2}e^{2}x-e^{2}$ $\frac{1}{2}x=-1$ $x=-2 \qquad \therefore B=(-2,0)$ Area = $\int_{-4}^{3}e^{-\frac{x}{2}}\ \mathrm{d}x-\frac{1}{2}\times(2)\times e^{2}$ $=\left[-2e^{-\frac{x}{2}}\right]_{-4}^{3}-e^{2}$ $=\left[-2e^{-\frac{3}{2}}-\left(-2e^{-\frac{-4}{2}}\right)\right]_{-4}^{3}-e^{2}$ $=-2e^{-\frac{3}{2}}+\left(2e^{2}\right)-e^{2}$ $=e^{2}-2e^{-\frac{3}{2}}$ $=6.942795779$ units $=6.94$ units