Raffles Institution
Year 5-6 Physics Department

2021 Year 5 H2 Physics Term 3 Common Test Solution

Section B

1 (a)

$$\begin{aligned}\text{base unit of pressure} &= \frac{\text{base unit of force}}{\text{base unit of area}}\\ &= \frac{\text{kg m s}^{-2}}{\text{m}^2}\\ &= \text{kg m}^{-1}\ \text{s}^{-2}\ \text{(shown)}\end{aligned}$$

(b)

$$\begin{aligned}\text{base unit of } Z &= \frac{(1)(\text{m})^3\left(\text{kg m}^{-1}\ \text{s}^{-2}\right)}{\text{m}}\left(\frac{\text{kg mol}^{-1}}{\text{kg m}^2\ \text{s}^{-2}\ \text{K}^{-1}\ \text{mol}^{-1}\ \text{K}}\right)^{\frac{1}{2}}\\ &= \text{kg m s}^{-2}\left(\frac{\text{s}^2}{\text{m}^2}\right)^{\frac{1}{2}}\\ &= \text{kg m s}^{-2}\left(\text{s m}^{-1}\right)\\ &= \text{kg s}^{-1}\end{aligned}$$

(c) (i)

$$\frac{\Delta(p_1-p_2)}{p_1-p_2}=\frac{2.5}{100}$$

$$\begin{aligned}\Delta(p_1-p_2) &= \frac{2.5}{100}\times 800101\\ &= 20003\\ &= 2\times10^4\ \text{Pa (1 s.f.)}\end{aligned}$$

(ii)

$$\begin{aligned}(p_1-p_2) &= (0.80\pm0.02)\times10^6\ \text{Pa}\\ &= (0.80\pm0.02)\ \text{MPa}\end{aligned}$$

(d) (i) Systematic errors result in all the measurements being either above or below the true value by a fixed amount.

Random errors result in the measurements being scattered about a mean value. These measurements have equal probability of being above or below the mean value.

(ii)

$$\begin{aligned}\frac{\Delta Z}{Z} &= 3\left(\frac{\Delta r}{r}\right)+\frac{\Delta(p_1-p_2)}{(p_1-p_2)}+\frac{\Delta L}{L}+\frac{1}{2}\left(\frac{\Delta m}{m}\right)+\frac{1}{2}\left(\frac{\Delta T}{T}\right)\\ &= 3\left(\frac{0.03}{3.34}\right)+\frac{2.5}{100}+\frac{0.001}{0.153}+\frac{1}{2}\left(\frac{1.7}{100}\right)+\frac{1}{2}\left(\frac{2.8}{100}\right) \qquad \text{since } r=\frac{1}{2}d \Rightarrow \frac{\Delta r}{r}=\frac{\Delta d}{d}\\ &= 0.08098\\ &= 8.1\%\ \text{(2 s.f.)}\end{aligned}$$

Comments

(a) Students should not use square brackets [ ] as they represent dimensions and not units. Students must write "units of" of a physical quantity.

(b) The most common mistake is missing the power of ½ for the term in square root. Students who are weak in manipulating algebraic expressions tend to get this wrong.

(c) **(i)** The pressure gauge measures the pressure difference directly, and the percentage uncertainty of 2.5% refers to the pressure difference. Weaker students erroneously double the percentage uncertainty to 5%.

(ii) Those who got this wrong tend to miss out the "mega" prefix in "MPa", or they presented inconsistent decimal places for the numerical values.

(d) **(i)** Many were careless in their use of terminologies in their statements. Marks were not awarded for inaccurate descriptions.

(ii) Common mistakes include: using r = d/2, but not Δr = Δd/2; missing out factor of 3 for percentage error in r; missing out factor of ½ for percentage error in *m* and *T*; not using the given 2.5% for percentage error in $\Delta(p_1 - p_2)$.

2 **(a)** **(i)** Take direction upwards and to the right as positive.
Consider the trajectory of the arrow from the archer to the cauldron.

Vertically:

$$v_y^2 = u_y^2 + 2a_y s_y$$
$$0 = u_y^2 + 2(-9.81)(24)$$
$$u_y = \sqrt{470.88}$$
$$= 21.70 \text{ m s}^{-1}$$

$$v_y = u_y + a_y t$$
$$0 = 21.70 + (-9.81)\,t$$
$$t = 2.21$$
$$= 2.2 \text{ s (shown)}$$

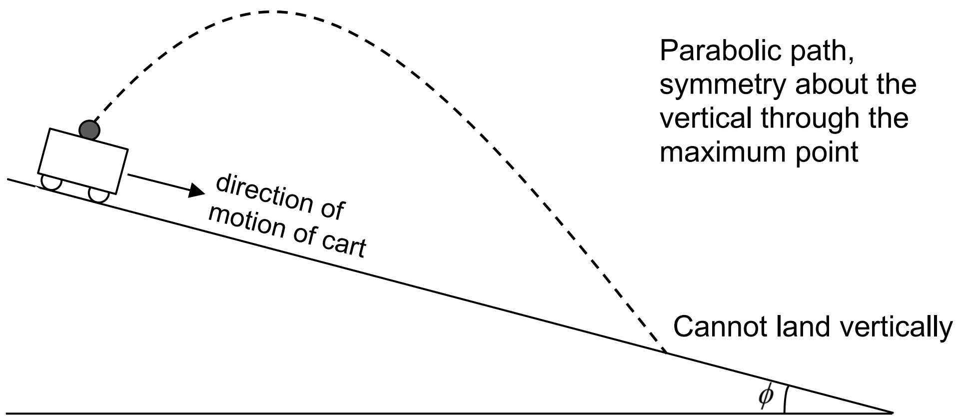
(ii) Since there is no air resistance, horizontal component of the velocity of the arrow remains constant throughout and acceleration in the horizontal direction is zero.

$$s_x = u_x t$$
$$u_x = \frac{s_x}{t}$$
$$= \frac{60}{2.2}$$
$$= 27.27 = 27.3 \text{ m s}^{-1}$$

(iii)

$$\tan\theta = \frac{u_y}{u_x}$$
$$\theta = \tan^{-1}\left(\frac{21.70}{27.27}\right)$$
$$= 38.51 = 38.5°$$

(b) (i)



(ii) cart's acceleration along plane $= g\sin\phi$

(iii) Yes, the ball will land back on the cart.

Assuming no air resistance, the ball undergoes free-fall acceleration g in air. The component of this acceleration parallel to the plane, $a_{//}$ is $g\sin\phi$, the same as the cart's acceleration.
Furthermore, the component of the initial velocity that is parallel to the plane, $u_{//}$, is also the same for the cart and the ball.
Hence, at any point in time t, their displacements parallel to the plane will be the same as shown by $s_{//} = u_{//}t + \frac{1}{2}a_{//}t^2$.
So the ball will land on the cart when it falls back to the plane.

Comments

(a) (i) Students who used symbols with subscripts y (e.g. u_y) had no problem substituting u_y into second kinematics equation. In comparison, for some who used $u\sin\theta$, it was less obvious to them they had to substitute a value for $u\sin\theta$ in the second equation. Some went to find θ value which was unnecessary.

(ii)-(iii) Generally well-done.

(b) (i) Some students drew curves that were obviously not parabolic. Many also erroneously drew paths with symmetry about a *line perpendicular to the slope*. This is wrong as the symmetry should be about the *vertical through the peak* (as is typical of projectile motion with ball projected at an angle to the horizontal).

The final part of trajectory should not be vertical as there is a non-zero horizontal component of velocity throughout the flight and thus ball should not land vertically.

Note the initial part of trajectory should not be perpendicular to the motion of the cart, the question states the ball is launched such that its initial velocity *with respect to the cart* is *perpendicular* to the motion of the cart. This means the initial velocity is perpendicular when viewed from *within* the moving cart, but this is not true for the observer outside. The initial velocity must factor in the velocity of the cart (and ball before launch) that is along the plane.

(ii) Most answered correctly.

(iii) Some students loosely said a_{ball} is $g\sin\phi$ (not true as a_{ball} is g), without clarifying this is only true for the // component of a_{ball}. There is a serious misconception that the acceleration of the ball remains the same as the cart after launch (just because the ball is one system with the accelerating cart at first). Some erroneously reasoned that since the launching angle is 90°, the // component of a_{ball} is unaffected by the launch (not true as a_{ball} in air is due to free-fall g, regardless of launching force/angle). These conceptual errors had to be penalised. One needs to know right after launch, there is only gravitational force at work and hence a_{ball} is g, which we then resolve into // and perpendicular components as required by the question.

Very few students recognised the importance of mentioning the component of the initial velocity parallel to the plane, $u_{//}$ is also the same for the cart and ball. If this condition is not met, the $s_{//}$ will not be the same for both and the ball will not land back in the cart. One can justify since the launching angle is 90°, $u_{//}$ is unaffected by the launch.

3 **(a)** Since the balls are moving at the same speed, they will meet at the midpoint.

Distance travelled by each ball $= \dfrac{0.950}{2} - 0.030 = 0.475 - 0.030 = 0.445$ m

$$\text{time taken} = \frac{\text{distance}}{\text{speed}} = \frac{0.445}{0.20} = 2.225 = 2.23 \text{ s}$$

(b) **(i)** The collision is inelastic and since the system loses kinetic energy, total kinetic energy is not conserved.

(ii) $\text{KE}_{\text{final}} = 0.80\ \text{KE}_{\text{inital}}$

$$\frac{1}{2}m_\text{A}v_\text{A}^2 + \frac{1}{2}m_\text{B}v_\text{B}^2 = 0.80\left(\frac{1}{2}m_\text{A}u_\text{A}^2 + \frac{1}{2}m_\text{B}u_\text{B}^2\right)$$

Since $m_\text{A} = m_\text{B}$, $u_\text{A} = u_\text{B} = 0.20 \text{ m s}^{-1}$ and $v_\text{A} = v_\text{B} = v$

$$\frac{1}{2}v^2 + \frac{1}{2}v^2 = 0.80\left(\frac{1}{2}(0.20)^2 + \frac{1}{2}(0.20)^2\right)$$

speed after collision, $v = \sqrt{0.80(0.20)^2} = 0.1789 = 0.179 \text{ m s}^{-1}$

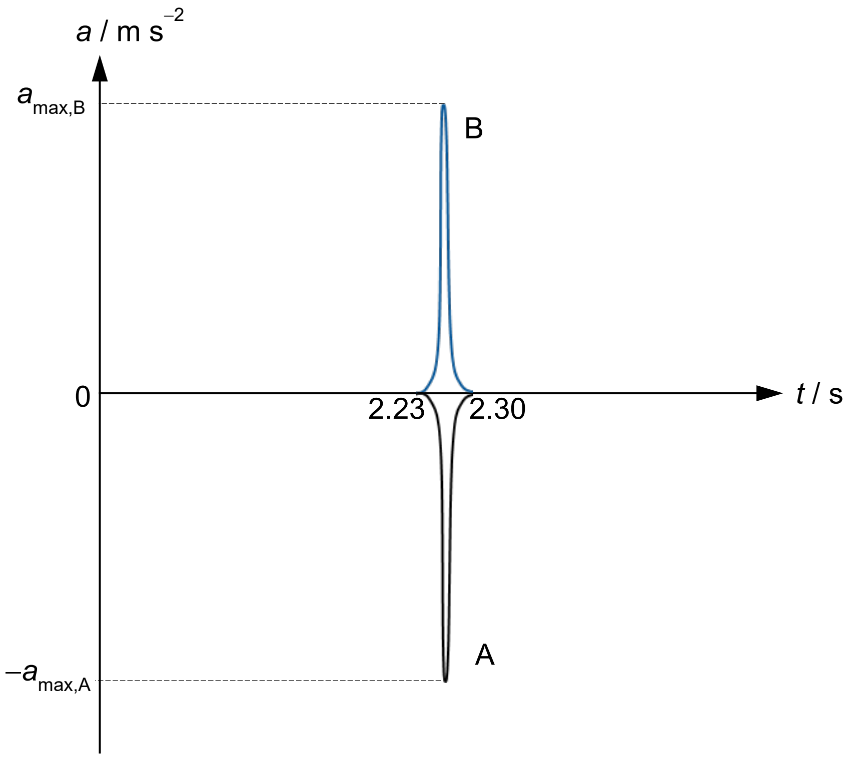
(iii) Taking direction to the right as positive,

$$\langle F \rangle \Delta t = \Delta p_A$$

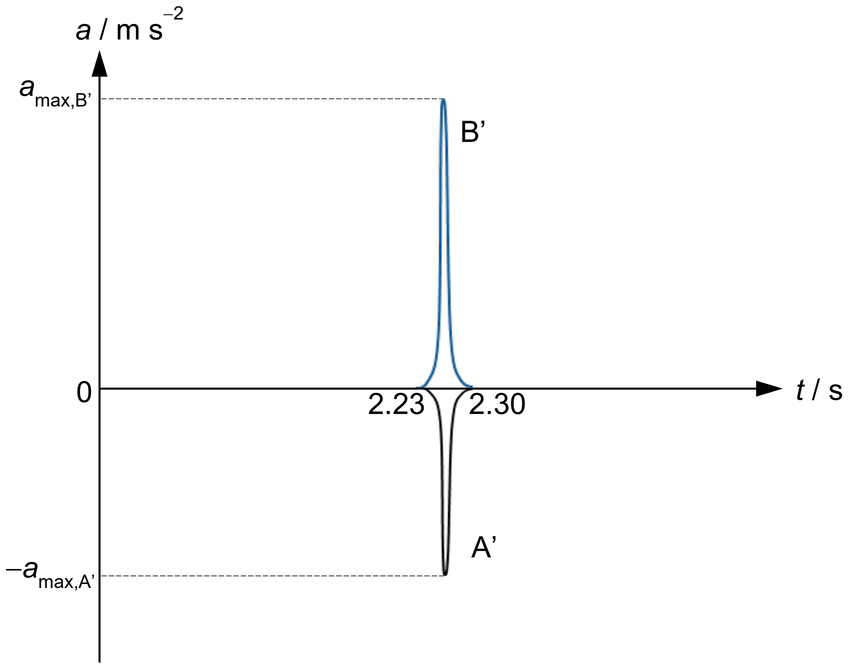
$$\langle F \rangle = \frac{\Delta p_A}{\Delta t} = \frac{m(v-u)}{\Delta t} = \frac{0.22(-0.179 - 0.20)}{0.070} = -1.191 = -1.19 \text{ N} \quad \text{(force on A by B is to the left)}$$

magnitude of the average force on A by B = 1.19 N

(iv) **1.**



2.



By Newton's third law, the force on ball A by ball B and the force on ball B by ball A are <u>equal in magnitude</u> and opposite in direction.
By Newton's second law, the <u>accelerations</u> of balls A and B are <u>inversely proportional to their masses</u> for the same force.
Since the mass of ball A is greater than the mass of ball B, the acceleration of ball A will be smaller than the acceleration of ball B.

Comments

(a) Many students make the mistake of neglecting the radius of the ball when calculating the distance travelled by each ball.

(b) (i) No credit is given to students who identified the type of colision with no reason.

Students who stated that kinetic energy was lost, different, not a constant, does not remain the same will not be given credit. It is important to differentiate between kinetic energy and total kinetic energy when you have a system of multiple bodies. No marks will be awarded if the phrasing was vague. The keyword 'not conserved' is strictly marked for.

A small handful of students wrongly identified the collision as completely inelastic. Do note that the balls did not stick together and move off with the same velocity after collision. The balls only possess the same speed after collision.

(ii) Generally well done. Careless mistakes include missing out the v^2 for the equation for kinetic energy or forgetting to take square root in the final calculation of the answer to determine v from v^2.

(iii) Students generally made the following mistake which resulted in marks lost:

1. Sign convention was not consistently applied.
2. Substituting change in velocity, Δv as $v_{initial} - v_{final}$ or lack of clarity in the substitution for Δv .
3. Including the negative sign when asked for magnitude.

(iv) 1. The questions was generally not well done despite the fact that students may have scored full credit. Many details were ommitted which could have been penalised if marking was stricter. They include:

1. Graphs were drawn way to wide giving the impression that the collision time was long. We are not expecting precise scale drawing but some attempts to differentiate $\Delta t = 0.07$ s and $\Delta t = 2.23$ s on the x-axis should have been made since you are given that the graph started from $t = 0$.
2. Students should be consistent in their sign convention or articulate any change in a remark/statement which some conscientious student did when graphing quantities that are vectors.
3. More effort to draw symmetrical graphs.

Students who drew graphs with unconventional curvature will not be given full credit.

A significant handful of students drew the $v - t$ graph instead of the $a - t$ graph. No credit will be given.

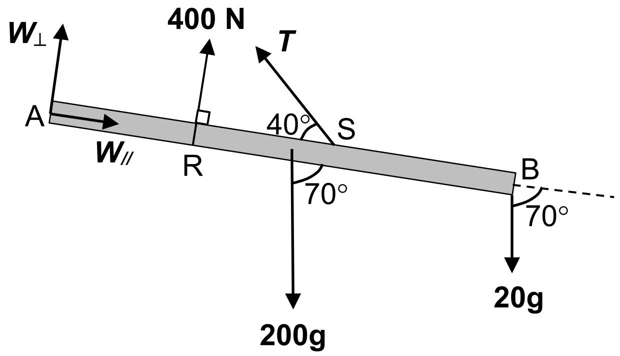
2. No credit is given if the graph was correctly drawn with the wrong explanation.

A significant handful of students possess the misconception that since mass of ball A is larger than mass of ball B, the force that ball A exerts on ball B is larger than the force that ball B exerts on ball A and hence ball B will experience a larger acceleration.

Many students do not understand the demand of an explain question. All equations or symbols used must be clearly defined and students need to aim to explain using prose instead of simply working out the analysis mathematically. Marks will be lost if the demand of question was not met. Students who explained with no reference to Newton's laws will also not be awarded full credit.

4 **(a)** The resultant force on the body is zero.
The resultant moment on the body about any axis is zero.

(b) **(i)**



Resolve forces along the awning and perpendicular to it.

By the principle of moments, taking moments about A,
sum of anticlockwise moments = sum of clockwise moments

$$(T\sin 40^\circ)(1.20)+400(0.80)=(200\times 9.81)(1.00\sin 70^\circ)+(20\times 9.81)(2.00\sin 70^\circ)$$

$$T=2453.4=2450\text{ N}$$

(ii) resultant force along the awning = 0

$$W_{//}-2453.4\cos 40^\circ+(200\times 9.81)\cos 70^\circ+(20\times 9.81)\cos 70^\circ=0$$

$$W_{//}=1141.3\text{ N}$$

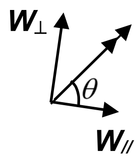
resultant force perpendicular to the awning = 0

$$W_{\perp}+400+2453.4\sin 40^\circ-(200\times 9.81)\sin 70^\circ-(20\times 9.81)\sin 70^\circ=0$$

$$W_{\perp}=51.029\text{ N}$$

resultant force on the awning by the wall

$$=\sqrt{W_{//}^{\,2}+W_{\perp}^{\,2}}=\sqrt{1141.3^2+51.029^2}=1142.4=1140\text{ N}$$



$$\tan\theta=\frac{W_{\perp}}{W_{//}}=\frac{51.03}{1141.27}$$

$$\theta=2.56^\circ$$

The resultant force on the awning at A has a magnitude of 1140 N and acts at an angle of 2.56° anti-clockwise above the beam.

(iii) S should be shifted towards B.

Considering the moments about A, the moments due to the rest of the forces remain unchanged. This means the moment due to the tension should also be unchanged to maintain rotational equilibrium. To reduce the tension, the perpendicular distance from A to the cable needs to increase. So S needs to be shifted towards point B.

Comments

(a) For questions asking for definitions or stating specific conditions, it is best to recall facts from the notes and write ad verbatim. Many students failed to do this and ended up being penalised. So, start memorising definitions!

The following answers are common but they are erroneous or irrelevant. They are:

- "sum of clockwise moments equal to sum of anti-clockwise moments about an axis" – this is stating the "definition" of the Principle of Moments and is not a condition for equilibrium.
- "translational and rotational equilibrium must be satisfied" are the effects of the conditions for equilibrium and hence not accepted.
- "total upward force = total downward force" is not accepted as forces may act in many other directions.
- "lines of actions of forces pass through the same point" applies to the case of equilibrium caused by only three forces acting on a system.

For rotational equilibrium, many students did not mention "resultant moment about any axis …." ,hence did not receive credit.

(b) **(i)** The most common mistake here is getting the angles wrong and mixing up the sine and cosine. Hence more practice needed in resolving forces or perpendicular distances!

Quite a number of students did this part by considering vertical equilibrium of forces, forgetting that there is another unknown force at point A (hinge) and did not take it into consideration, hence committing an error.

(ii) Again, wrong angles and sine-cosine mix-up are commonplace for this part. If forces were resolved parallel to the awning, some students carelessly omitted resolving the weights parallel to the awning.

In some answers, there were confusion in the resolving axes. For example, some students resolve forces vertically and parallel to the awning or resolve forces perpendicular to the awning and horizontally. These are obvious mistakes. The resolving axes should be perpendicular to each other.

In stating the direction of the force that the wall exerts on the awning at A, many students did not state any reference to the direction of the force. For some students who mentioned the reference, they failed to state if the direction of the force is clockwise or anti-clockwise from that reference.

(iii) Many students did not mention that the perpendicular distance from A must increase so that tension could be lower to keep the anti-clockwise moment it produces, constant in order to maintain rotational equilibrium, thus did not receive credit

5 **(a)** **(i)** Since the brick is at rest,

$$W - T = 0$$
$$T = W$$
$$kx = mg$$
$$x = \frac{mg}{k}$$
$$= \frac{(1.50)(9.81)}{(75.0)}$$
$$= 0.1962 = 0.196 \text{ m (shown)}$$

(ii) elastic potential energy $= \dfrac{1}{2}kx^2$

$$= \frac{1}{2}(75.0)(0.196)^2$$

$$= 1.4406 = 1.44 \text{ J}$$

(b) (i) 1. resultant force on brick $F_R = m_{brick}a$

$$= 1.50 \times 3.68$$

$$= 5.52 \text{ N}$$

2. work done on brick $= F_R s$

$$= 5.52 \times 1.20$$

$$= 6.624 = 6.62 \text{ J}$$

3. Work done on the brick by the resultant force ($mg - T$), increases the kinetic energy of the brick.

increase in KE $= 6.62$ J

4. instantaneous rate of increase in KE $= F_R v$

$$= 5.52 \times 2.80$$

$$= 15.456 = 15.5 \text{ J s}^{-1}$$

(ii) 1. When length of rope stops changing,

$$T_f = kx_f \quad \Rightarrow \quad x_f = \frac{T_f}{k}$$

decrease in EPE

= EPE when system at rest – EPE when length stops changing

$$= 1.4406 - \frac{1}{2}kx_f^2$$

$$= 1.4406 - \frac{1}{2}k\left(\frac{T_f}{k}\right)^2$$

$$= 1.4406 - \frac{1}{2}\left(\frac{9.20^2}{75.0}\right)$$

$$= 0.8763 = 0.876 \text{ J}$$

2. increase in KE of system = decrease in GPE + decrease in EPE

final KE − initial KE $= m_{brick}g(\Delta h)$ + decrease in EPE

$$\text{final KE} - 0 = 1.50(9.81)(1.70) + 0.8763$$

$$\text{final KE} = 25.8918 = 25.9 \text{ J}$$

Comments

(a) (i) Many candidates failed to recognise that for a *Show* question, their reasoning must be clearly explained, and did not get awarded the mark for their working of simply doing

$$x = \frac{mg}{k}$$
$$= \frac{(1.50)(9.81)}{(75.0)}$$
$$= 0.1962 = 0.196 \text{ m (shown)}$$

The key concept tested here is the brick is in translational equilibrium, hence this point must be clearly stated first.

Note the presenting of the 4 s.f. value before rounding off to the shown value is important.

(ii) Generally well done, but a very small handful of candidates did not convert the extension into SI units and hence got an order of magnitude that is much larger. This is strange as part **(a)(i)** has already guided them to finding the extension in metres.

(b) (i) 1. Generally well done. Some candidates applied Newton's Second Law on the trolley first, then used the tension to find the resultant force acting on the brick. This is correct, with more experience candidates should seek the most elegant method of solving questions.

A handful of candidates thought the resultant force = *mg*, but it is important to note that this only applies for free fall; the brick is clearly not experiencing free fall due to the tension.

2. Generally well done. A small handful of candidates used change in gravitational potential energy as the work done, but this is wrong as the weight is not the resultant force.

4. Many candidates found the kinetic energy instead, but the question is asking for the *rate of change* of kinetic energy.

Another big group of candidates performed differentiation on kinetic energy to get the rate of change as *mva*, but some careless candidates forgot the apply chain rule on the term *v* and hence got just *mv* instead, which cannot be the case as can be checked using unit analysis on rate of change of energy (kg $m^2 s^{-1}$) and momentum *mv* (kg m s^{-1}).

(ii) 1. Quite a number of students who found 'change' in the elastic potential energy, did not present their working properly, not knowing the meaning of 'change'. Take note that 'change' (denoted by the symbol 'Δ') in a quantity always mean 'final – initial'. In this part, there is a decrease in the elastic potential energy which means the final value is less than the initial value. This means the change in elastic potential energy is a negative value. However, the question asked for the 'decrease', hence a positive value should be stated as the final answer.

2. The conservation of energy equation was very poorly written and applied. In many cases, students simply equated the final kinetic energy to be the same as the increase in kinetic energy without stating or explaining that this is because the initial kinetic energy of the system is zero. The final kinetic energy will not be equal to the increase in kinetic energy if the initial kinetic energy is not zero.

It is very important to learn to write the conservation of energy equation correctly and accurately. Acceptable equations are:

increase in KE = decrease in GPE + decrease in EPE
final KE – initial KE = decrease in GPE + decrease in EPE
OR
initial KE + initial GPE + initial EPE = final KE + final GPE + final EPE

Note that for this part, the conservation of energy is applied to the whole system of trolley, brick and rope. Hence energies of each of these must be considered.

For all questions:
Students are expected to present their solutions sequentially and clearly, including the correct formulae / law / concept used and short statements. Full substitution of values to the same number of significant figures as given in the question should also be shown, as this will affect the number of significant figures of the final answer. Note that in this particular question, the values given are mostly to 3 significant figures, hence the final answer should be to 3 significant figures and not 2.

In general, all workings should be in this format:
Statement, formulae in accepted symbols, substitution, 4 s.f. / 5 s.f. value, final answer to the correct number of s.f.

Any of these missing components may warrant deducting of marks.

6 **(a)** **(i)** Angular velocity is the rate of change of angular displacement (with respect to time).

(ii) Consider the motion of the ball from ground level to point Q.
By conservation of energy,

$$\frac{1}{2}mu^2 + mgd = \frac{1}{2}m{v_Q}^2$$

$$v_Q = \sqrt{2\left(\frac{1}{2}u^2 + gd\right)}$$

$$= \sqrt{2\left(\frac{1}{2}(4.0)^2 + (9.81)(0.50)\right)}$$

$$= 5.0804 \text{ m s}^{-1}$$

$$v_Q = r\omega_Q$$

$$\omega_Q = \frac{v_Q}{r}$$

$$= \frac{5.0804}{1.5}$$

$$= 3.387 = 3.39 \text{ rad s}^{-1}$$

(iii) The centripetal acceleration at point P is <u>lesser</u> than that at point Q.

The <u>speed v of the ball at P is lower</u> than its speed at Q as it has <u>greater gravitational potential energy</u> at P.

(Kinetic energy lost to gravitational potential energy at the top of the hill as compared to kinetic energy gained from gravitational potential energy at the bottom of the valley.)

Since centripetal acceleration is given by v^2/r, for the same radius of curvature, the centripetal acceleration at P will be lesser due to a lower v.

(b) (i) At point P,

$$mg - N = \frac{m{v_P}^2}{r}$$

Since the ball just loses contact at point P, $N = 0$.

$$mg - 0 = \frac{m{v_P}^2}{r}$$

$$v_P = \sqrt{rg} = \sqrt{(1.5)(9.81)} = 3.836 = 3.84 \text{ m s}^{-1}$$

*Newton's second law, which includes N is required, before equating N to zero.

(ii) The student's suggestion is incorrect.

The maximum speed a ball can have to remain in contact at point P is given by $\sqrt{rg}$ and this is independent of its mass. Hence, regardless of its mass, the third ball will lose contact since it has a speed greater than the speed of the second ball, which is already at the maximum speed possible to stay just in contact.

Comments

(a) (i) Students should learn to use proper terminology, especially when it comes to questions about definitions. The most common issue was "angular displacement" – students who did not get the mark used other weird and inadequate terms to replace an otherwise simple term such as "angular displacement".

Another common issue came from students who used "rate of change" and "per unit time" together. To clarify: the term "rate of change" already implies the idea of $\frac{d}{dt}$. The term "per unit time" also implies $\frac{d}{dt}$. Using them both together would be akin to a second derivative (e.g. rate of change of θ per unit time would be $\frac{d}{dt}\left(\frac{d\theta}{dt}\right) = \frac{d^2\theta}{dt^2}$). Hence, angular velocity should either be "rate of change of angular displacement" OR "change of angular displacement per unit time.

Lastly, units should never be used in defining quantities (e.g. angular velocity defined as radians per second is wrong.). Quantities should define quantities.

(ii) Rather poorly done, unfortunately.

The first part requires students to use conservation of energy to find the speed of the ball at Q. The most common mistake is for students to calculate the change in kinetic energy (KE) wrongly. Change in KE should be:

$$\Delta KE = \frac{1}{2}m\Delta\left(v^2\right) = \frac{1}{2}m\left({v_f}^2 - {v_i}^2\right)$$

Many students instead wrote

$$\Delta KE = \frac{1}{2}m(\Delta v)^2 = \frac{1}{2}m(v_f - v_i)^2 \text{ ------- WRONG}$$

The second part requires students to use $v = r\omega$. The most common mistake here is for students to use 0.50 m for r, instead of 1.5 m. 0.50 m is only the depth of the valley from ground level, but we should be using the radius of curvature for r.

(iii) To get full credit for the explanation, students must firstly explain <u>why</u> the velocity at Q is larger than at P by using relevant physics concepts (energy or gravitational acceleration), and not just state that it is larger. Secondly, students must explain that from $a = \frac{v^2}{r}$, for a <u>constant radius</u>, a larger velocity means a larger centripetal acceleration. Most students who did not get full credit would have missed out one of the underlined points. Explanations should be as clear as possible, invoking relevant physics concepts where relevant.

(b) **(i)** Most common problem lies in the first equation $mg - N = \frac{m{v_P}^2}{r}$. The centripetal force should be mg – N, and not mg + N or N – mg. Some of these wrong permutations might lead to the same answer (since N will be set to zero) but they will not be accepted as the fundaentals are incorrect.

Another common problem lies in students not presenting the logical flow of their working with enough detail. Jumping straight from N = 0 to $mg = \frac{m{v_P}^2}{r}$ does not show how those two statements link together. Worse offenders will be students who do not even state that N = 0. Worst of all will be students who just state $v = \sqrt{rg}$ without any prior steps in the working. None of these will be given full credit, even if the final answer is correct.

The best answer should always start with invoking Newton's Second Law, before stating N = 0 and moving on from there.

(ii) The biggest and most common issue here is that many students misinterpret what the expression $v = \sqrt{rg}$ refers to. This speed refers to the MAXIMUM speed for the ball to move at P without losing contact, or the MINIMUM speed for the ball to move at P for it to lose contact. In other words, speeds above $\sqrt{rg}$ will cause the ball to lose contact, and speeds below $\sqrt{rg}$ will cause the ball to remain in contact.

Some students even misinterpret $\sqrt{rg}$ as "the speed of ball at P", which is not the case for the third ball. We should be comparing the speed of the third ball to the threshold speed – since the third ball is travelling faster than the threshold speed, it will lose contact despite the larger mass.

7 **(a)** Maximum depression is <u>inversely proportional to the product ab^3</u>.
Since the depth b of beam S is much <u>smaller</u> than beam T even though its width a is greater, <u>beam S</u> would sag more since b^3 is a more significant factor than a.

(b)

$$\text{maximum load } F = \frac{kab^3 x_{max}}{L^3} = \frac{(3.6\times10^{10})(0.050)(0.10^3)(0.010)}{3.0^3} = 666.7 = 667 \text{ N}$$

(c) **(i)**

$$\text{maximum depression} = \frac{L}{360} = \frac{4.20}{360} = 0.01167 = 0.0117 \text{ m}$$

(ii)

$$\text{bending moment } B = \frac{FL}{8} = \frac{33000\times4.20}{8} = 17325 = 17300 \text{ N m}$$

(iii)

$$\text{actual depression } x = \frac{BL^3}{c} = \frac{17325\times4.20^3}{3.35\times10^8} = 0.003831 = 3.83\times10^{-3} \text{ m}$$

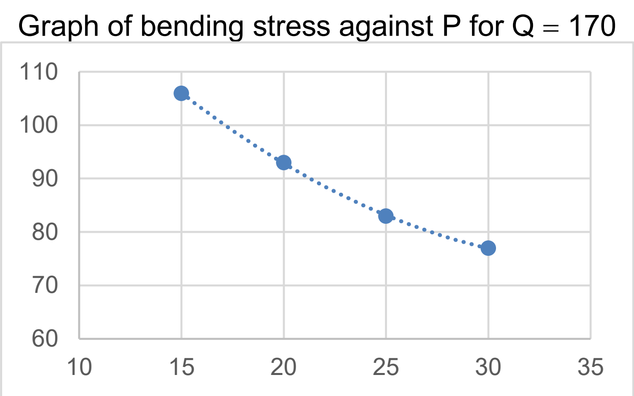
(d)

$$\text{total moment} = (10000\times0.50)+(6000\times1.3)+(22000\times2.3)+(7000\times3.1) = 85100 \text{ N m}$$

(e) **(i)** Assuming a linear variation between the values,

$$\text{for } Q = 170,\ \frac{s-93}{21-20} = \frac{83-93}{25-20}$$

$$\text{allowable bending stress } s = \left[\frac{83-93}{25-20}\times(21-20)\right]+93 = 91 \text{ MPa}$$



(ii) $\dfrac{\text{bending moment}}{\text{allowable bending stress}} = \dfrac{17325}{91\times10^{6}} = 1.90\times10^{-4}\ \text{m}^3$

Since the ratio is less than $2.0\times10^{-4}\ \text{m}^3$, the beam is safe.

Comments

(a) A number of students erroneously interpreted the width *a* of the beam as its span *L*.

Full credit is only awarded if student indicated the correct equation $x = \dfrac{FL^3}{kab^3}$ and also account for width a and depth b of the beam.

(b) This part was easily calculated as the equation was given in the question. Yet, there were too many students who made either one or a few of the following careless mistakes:

- Omitting the power 3 for X or b
- Writing the width of the beam to be 0.50 instead of 0.050
- Correct substitutions but incorrect calculation leading to the wrong numerical answer

There were also many who did not show any substitution of values before writing down the correct numerical answer. Such answers were penalized for presentation (P). Note that for 2 mark questions, 1 mark is typically awarded for correct substitution of values.

Students are reminded to read the question carefully and to be more meticulous and detailed in calculations. Questions like this are meant to help you score more marks.

There were a small number of students who quoted the answer as 666 N. Do note that we do not round down our values but we will take 667 N as the maximum load.

(c) **(i)** Generally well done with no issues.

(ii), (iii) Generally well done but several students left the answer in too many significant figures.

Again, there were some who did not show any substitution of values.

(d) Some students did not account for kN.

(e) **(i)** Most students were able to interpolate correctly.

(ii) A small number of students used total moment in **(d)** instead of the bending moment **(c)(ii)** to calculate the allowable bending stress.
Some students did not indicate the units of the calculated quantities.