RAFFLES INSTITUTION
H2 Mathematics (9758)
2025 Year 6

Tutorial S3: Normal Distribution

1 Given that $X \sim \mathrm{N}(1.5, 2.5)$, evaluate

(a) $\mathrm{P}(X < 0.7)$, **(b)** $\mathrm{P}(X \geq 3.7)$, **(c)** $\mathrm{P}(1.1 < X < 2.3)$,

(d) $\mathrm{P}(|X| \leq 0.5)$, **(e)** $\mathrm{P}(|X| > 0.5)$ **(f)** $\mathrm{P}(|X - 1.5| < \sqrt{2.5})$.

[(a) 0.306 **(b)** 0.0821 **(c)** 0.293 **(d)** 0.161 **(e)** 0.839 **(f)** 0.683**]**

2 Given that $Y \sim \mathrm{N}(22, 5)$ and $Z \sim \mathrm{N}(0,1)$, find the value(s) of k such that

(a) $\mathrm{P}(Y < k) = 0.35$,

(b) $\mathrm{P}(Y > k) = 0.8$,

(c) $\mathrm{P}(k < Y < 25) = 0.46$,

(d) $\mathrm{P}(|Z| > k) = 0.27$,

(e) $\mathrm{P}(22 - k < Y < 22 + k) = 0.67$,

(f) $\mathrm{P}(Y < k) > 0.3$.

[(a) 21.1 **(b)** 20.1 **(c)** 21.7 **(d)** 1.10 **(e)** 2.18 **(f)** $k > 20.8$**]**

3 The random variable X follows a normal distribution with mean μ and standard deviation σ. Given that $\mathrm{P}(X < 20) = 0.14$ and that $\mathrm{P}(X < 50) = 0.65$, calculate the values of μ and σ.

[$\mu = 42.1$, $\sigma = 20.5$**]**

4 The independent random variables X and Y are each normally distributed with means 6 and 8 respectively, and variances 9 and 16 respectively. Find

(a) **(i)** $\mathrm{P}(X < Y)$ **(ii)** $\mathrm{P}(|X - Y| < 1.5)$.

(b) X_1, X_2 and X_3 are three independent observations of X and Y_1 and Y_2 are two independent observations of Y. Find

(i) $\mathrm{P}(X_1 + X_2 + X_3 < 15)$,

(ii) $\mathrm{P}(X_1 + X_2 > Y_1 + Y_2)$,

(iii) $\mathrm{P}(2X > Y_1 + Y_2)$.

[(a)(i) 0.655 **(ii)** 0.218 **(b)(i)** 0.282 **(ii)** 0.286 **(iii)** 0.314**]**

5 The random variables X and Y are normally distributed with known means and variances. A student was asked to find the probability that sum of 3 independent observations of X differ from 3 times an observation of Y by at least 1. State an assumption that the student has to make to solve the question.

Part of the student's solution to this question is given as follows:

$$3X - 3Y \sim \mathrm{N}\left(3\mathrm{E}(X) - 3\mathrm{E}(Y), 9\mathrm{Var}(X) - 9\mathrm{Var}(Y)\right)$$

$$\mathrm{P}(3X - 3Y \geq 1) = \ldots.$$

Point out 3 mistakes that the student has made. **[3]**

6 The pineapples in a large warehouse have masses with a normal distribution. The mean of this distribution is 1.52 kg and the standard deviation is 0.070 kg. Six randomly chosen pineapples are packed in a box of mass 2.15 kg. Find the probability that the total mass of the filled box is less than 11.2 kg.

Find the probability that, out of 3 randomly chosen filled boxes, at least 1 has total mass less than 11.2 kg. **[4]**

[0.342 , 0.715**]**

7 A company sells bags of cement in two sizes. The mass of a large bag of cement can be modelled by a normal distribution with mean 50 kg and standard deviation 2 kg. The mass of a small bag of cement can be modelled by a normal distribution with mean 30 kg and standard deviation 1.5 kg.

(i) Find the mass that is exceeded by 99% of the large bags. **[2]**
(ii) Find the probability that total mass of three small bags is less than twice the mass of one large bag. **[3]**
(iii) State an assumption needed for your calculation in part **(ii)**. **[1]**

Three large bags are selected at random.
(iv) Find the probability that two weigh more than 53 kg and one weighs less than 53 kg. **[3]**

[(i) 45.3 **(ii)** 0.982 **(iv)** 0.0125**]**

8 In this question you should assume that T, W and D follow independent normal distributions.

James leaves home to go to work at T minutes past 8 am each day, where T follows the distribution $\mathrm{N}\left(5,\ 1.2^2\right)$.

(i) Sketch this distribution for the period from 8 am to 8.10 am. **[2]**

(ii) Find the probability that, on a randomly chosen day, James leaves for work later than 8.06 am. **[1]**

When the weather is fine, James walks to work. The time, W minutes, he takes to walk to work follows the distribution $\mathrm{N}\left(21,\ 3^2\right)$. James is supposed to start work at 8.30 am.

(iii) Find the probability that, on a randomly chosen day when James walks, he is late for work. **[2]**

When the weather is not fine, James drives to work. He still leaves at T minutes past 8 am each day; the time, D minutes, he takes to drive to work follows the distribution $\mathrm{N}\left(19,\ 6^2\right)$.

On average, the weather is fine on 70% of mornings.
(iv) One day, James is late for work. Find the probability that the weather is fine that day. **[5]**

[(ii) 0.202 **(iii)** 0.108 **(iv)** 0.606**]**

9 In this question you should state the parameters of any distributions that you use.
A manufacturer produces specialist light bulbs. The masses in grams of one type of light bulb have the normal distribution $\mathrm{N}\left(50, 1.5^2\right)$.

(i) Sketch the distribution for masses between 40 grams and 60 grams. **[2]**

(ii) Two light bulbs are randomly chosen. Find the probability that one of the light bulbs has mass less than 50.4 g and the other has mass more than 50.4 g. **[3]**

Each light bulb is packed into a randomly chosen box. The masses of the empty boxes have the distribution $\mathrm{N}\left(75, 2^2\right)$.

(iii) Find the probability that the total mass of 4 randomly chosen empty boxes is more than 297 grams. **[2]**

(iv) Find the probability that the total mass of a randomly chosen light bulb and a randomly chosen box is between 124.9 and 125.7 grams. **[3]**

In order to protect the bulbs in transit each bulb is surrounded by padding before being packed in a box. The mass of the padding is modelled as 30% of the mass of the bulb.

(v) The probability that the total mass of a box containing a bulb and padding is more than k grams is 0.9. Find k. **[4]**

(vi) Find the probability that the total mass of 4 randomly chosen boxes, each containing a bulb and padding, is more than 565 grams. **[3]**

[(ii) 0.478 **(iii)** 0.773 **(iv)** 0.126 **(v)** 137 **(vi)** 0.163**]**

10 Every weekday, the last train from Bedok Station leaves the station at 11.29 pm. Its travel time X (in minutes) from Bedok Station to Redbridge Station may be taken to follow a normal distribution with mean 30 and variance 9.

(i) Find the travel time exceeded by 90% of the trips made by the last train. **[2]**

At 11.40 pm every weekday, Abel walks from his workplace to Redbridge Station to catch the last train. The time Y (in minutes) he takes follows a normal distribution with mean 16 and variance 4.

(ii) Find the probability that the train arrives at Redbridge Station after 12 am and Abel arrives before 12 am. **[2]**

(iii) Find the probability that Abel will not miss the train. **[3]**

(iv) Explain why the answer to part **(iii)** is greater than the answer to part **(ii)**. **[1]**

The time W (in minutes) taken by Dina to walk from her workplace to Redbridge Station has a mean of 4 and a variance of 6.

Explain why W is unlikely to be normally distributed. **[1]**

[(i) 26.2 **(ii)** 0.361 **(iii)** 0.797**]**