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南 华 中 学

NAN HUA HIGH SCHOOL

PRELIMINARY EXAMINATION 2023

Subject : Mathematics
Paper : 4052/02
Level : Secondary Four Express
Date : 21 August 2023
Duration : 2 hours 15 minutes

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
 Write in dark blue or black pen.
 You may use an HB pencil for any diagrams or graphs.
 Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions. The number of marks is given in brackets [] at the end of each question or part question.
 If working is needed for any question, it must be shown with the answer.
 Omission of essential working will result in loss of marks.
 The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
 If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
 For π , use either your calculator value or 3.142.

For Examiner's Use

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	U		TOTAL
									P		
									A		

This paper consists of **22** printed pages and **2** blank pages.

Compound interest

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

- 1 (a) (i) Solve the inequalities $\frac{4x-1}{2} \leq \frac{3x}{4} \leq \frac{5x+9}{5}$.

Answer [2]

- (ii) Represent the solutions of the inequalities on the number line below.



[1]

- (iii) State the smallest integer which satisfies the inequalities.

Answer [1]

(b) Simplify $\frac{1}{ab^{-2}} \div \left(\frac{2a}{\sqrt[3]{b^5}} \right)^{-3} \times \frac{ab^0}{10}$, leaving your answer in positive index form.

Answer [3]

(c) Express $\frac{x+10}{3x^2-21x+30} + \frac{1}{2-x}$ as a single fraction in its simplest form.

Answer [3]

- (d) (i) y is inversely proportional to the square of x .

Which of these diagrams represents the graph of y against x ?

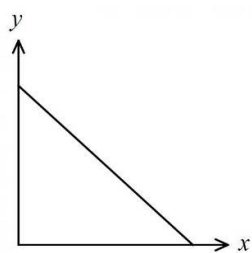


Diagram 1

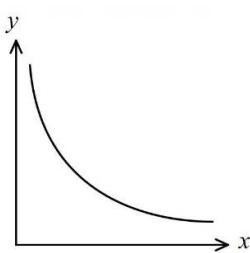


Diagram 2

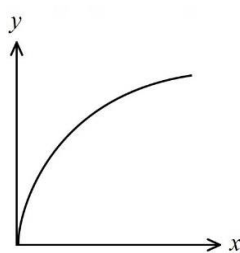


Diagram 3

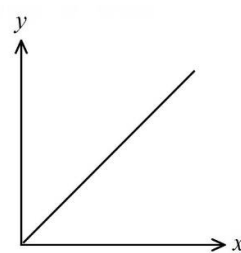


Diagram 4

Answer Diagram [1]

- (ii) Find the percentage change in y when x is increased by 150%.

Answer% [3]

- 2 At a store, 20 red apples cost \$ x .

The same number of green apples in the store cost \$4 more.

- (a) Write an expression, in terms of x , for the number of red apples that can be bought with \$2.40.

Answer [1]

- (b) Write an expression, in terms of x , for the number of green apples that can be bought with \$2.40.

Answer [1]

- (c) \$4.80 can buy a total of 10 apples at the store.

Write down an equation in x to represent this information and show that it reduces to

$$5x^2 - 28x - 96 = 0.$$

Answer

[3]

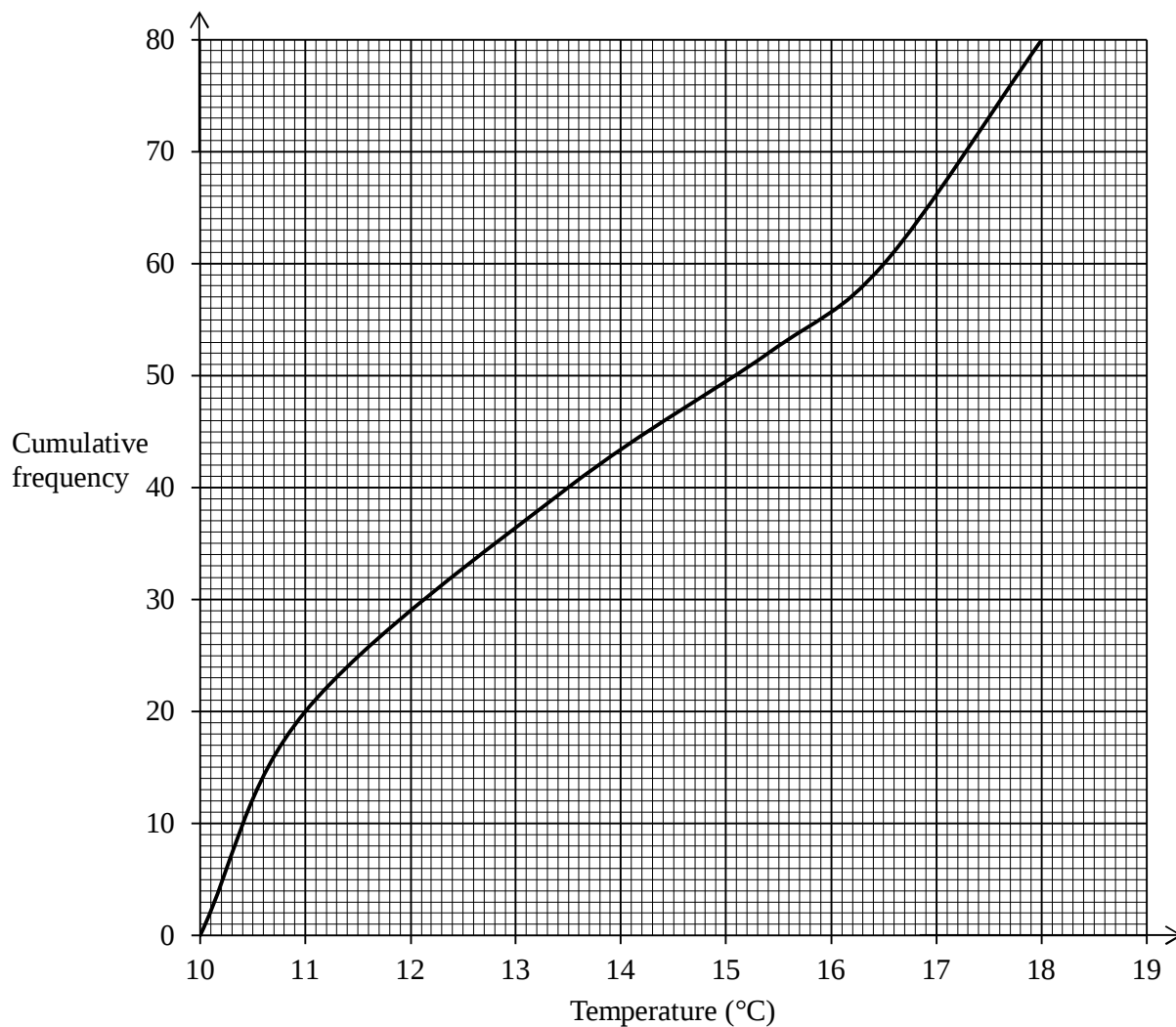
- (d) Solve the equation $5x^2 - 28x - 96 = 0$.

Answer $x = \dots\dots\dots$ or $\dots\dots\dots$ [2]

- (e) Find the cost of 30 green apples in store A.

Answer \$..... [2]

- 3 The cumulative frequency curve shows the distribution of the daily temperature of City X in the first 80 days of the year 2021.



- (i) Use the curve to estimate
(a) the median temperature,

Answer °C [1]

- (b) the interquartile range of the temperatures,

Answer °C [2]

- (c) the 65th percentile,

Answer °C [1]

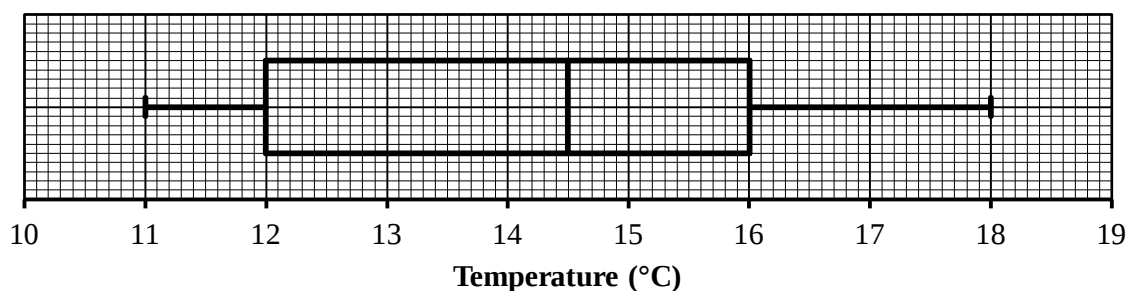
- (d) the percentage of days where the temperature was at least 17°C .

Answer % [1]

- (ii) The temperatures of the first 80 days of City X for the year 2022 were also recorded.
The median temperature for the year 2022 is smaller than that of the year 2021.
The interquartile range of the temperatures for the year 2022 is the same as that of the year 2021.
Describe how the cumulative frequency curve for the year 2022 may differ from the curve in the year 2021.

.....
..... [1]

- (iii) The temperatures of the first 80 days of City Y in the year 2021 were also recorded.
The box-and-whisker plot shows the distribution of the temperatures.



Make two comments comparing the temperatures of both cities.

.....
.....
.....
.....
..... [2]

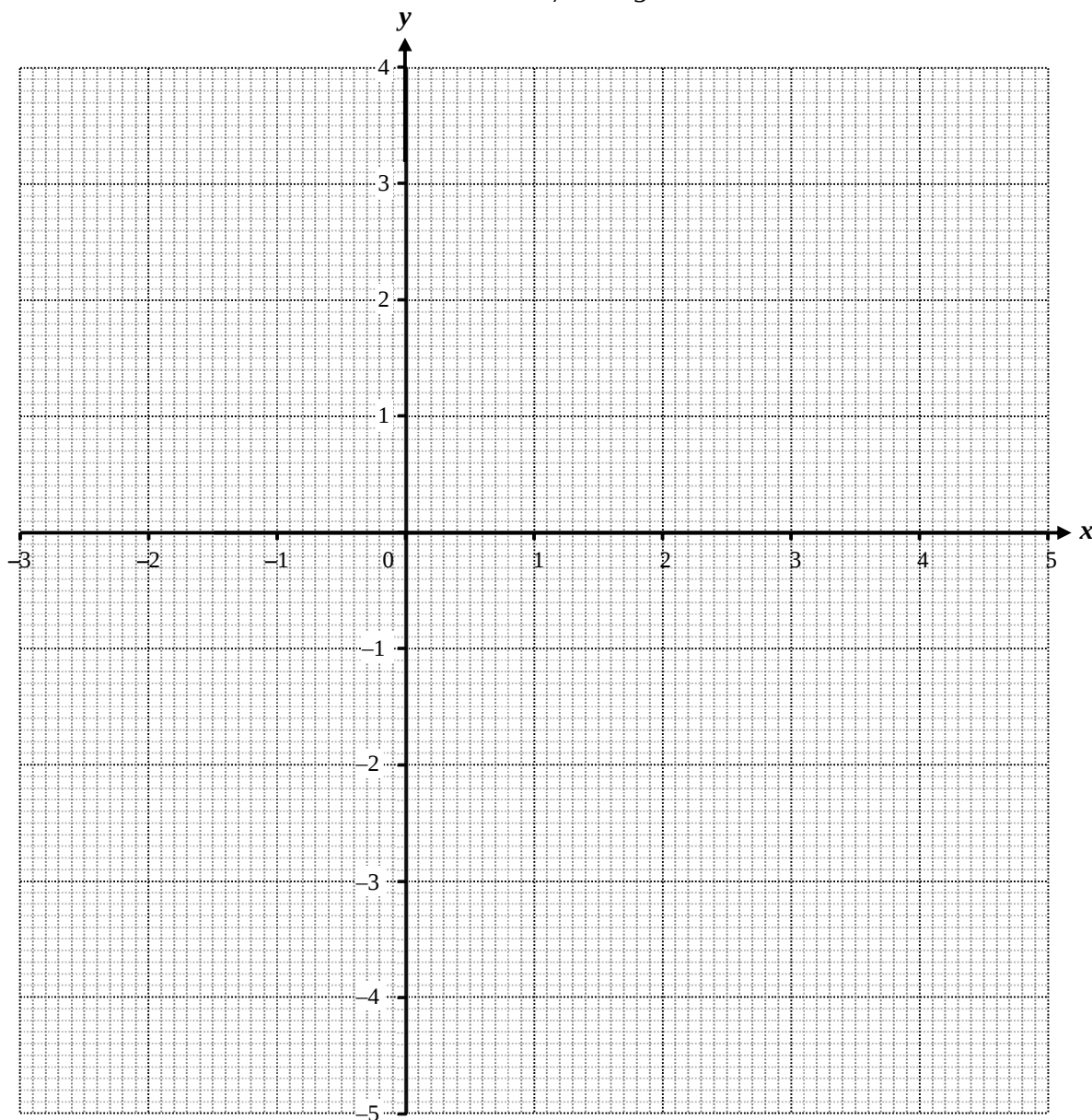
- 4 (a) Complete the table of values for $y = \frac{(x-1)^3}{7} - \frac{4x}{3} + 1$.

Give your answer correct to 1 decimal place.

x	-3	-2	-1	0	1	2	3	4	5
y	-4.1	-0.2	1.2	0.9	-0.3	-1.5	-1.9		3.5

[1]

- (b) On the grid, draw the graph of $y = \frac{(x-1)^3}{7} - \frac{4x}{3} + 1$ for $-3 \leq x \leq 5$.



[3]

- (c) The equation $\frac{(x-1)^3}{7} - \frac{4x}{3} = 1$ has only one solution.

Explain how this can be seen from your graph.

.....

 [2]

- (d) By drawing a tangent, find the gradient of the curve at $(-2, -0.2)$.

Answer [2]

- (e) (i) On the same axes, draw the line $2y + x = 5$ for $-3 \leq x \leq 5$. [1]

- (ii) Write down the x -coordinate of the point where this line intersects the curve.

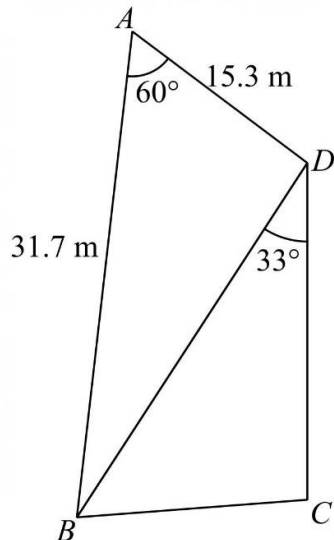
Answer $x =$ [1]

- (iii) This value of x is the solution of the equation $6(x-1)^3 - Ax - B = 0$.
 Find the value of A and the value of B .

Answer $A =$

$B =$ [2]

- 5 $ABCD$ represents a plot of land.



D is due north of C .

The bearing of B from C is 257° .

$AB = 31.7$ m and $AD = 15.3$ m.

Angle $BAD = 60^\circ$ and angle $BDC = 33^\circ$.

- (a) Find the length of BD .

Answer m [2]

- (b) Find the length of CD .

Answer m [3]

- (c) Find the bearing of A from B .

Answer [2]

- (d) Alex walks from point B to point D .
(i) Calculate his distance from D when Alex is due west of C .

Answer m [2]

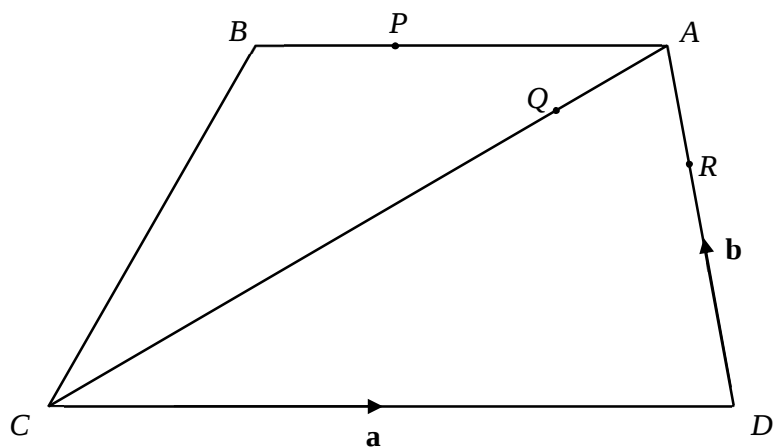
- (ii) A lamp post of height 11 m is at point C .
Calculate the greatest possible angle of elevation of the top of the lamp post from Alex during his walk.

Answer [2]

6 $ABCD$ is a trapezium where BA is parallel to CD and $3AB = 2CD$.

$AP = 3BP$, $2DA = 3DR$, and $QA = \frac{1}{4}CQ$.

$\overrightarrow{CD} = \mathbf{a}$ and $\overrightarrow{DA} = \mathbf{b}$.



(a) (i) Express $\overrightarrow{QA}$ in terms of $\mathbf{a}$ and $\mathbf{b}$, as simply as possible.

Answer $\overrightarrow{QA} = \dots\dots\dots$ [1]

(ii) Express $\overrightarrow{QR}$ in terms of $\mathbf{a}$ and $\mathbf{b}$, as simply as possible.

Answer $\overrightarrow{QR} = \dots\dots\dots$ [1]

- (iii) Express $\overline{PR}$ in terms of $\mathbf{a}$ and $\mathbf{b}$, as simply as possible.

Answer $\overline{PR} = \dots\dots\dots$ [2]

- (b) Explain why P , Q and R are collinear.

.....
.....
.....
..... [2]

- (c) Given that the area of triangle $ABC = 13 \text{ cm}^2$, find the area of trapezium $ABCD$.

Answer cm^2 [2]

- (d) The point S is the midpoint of CD . Determine if CA is parallel to SR .

.....
..... [2]

- 7 A café offers three set meals, A, B and C, during lunch and dinner on weekdays.

The matrix $\mathbf{M}$ shows the number of orders for set meals on a particular day.

$$\mathbf{M} = \begin{matrix} & \begin{matrix} \text{Set A} & \text{Set B} & \text{Set C} \end{matrix} \\ \begin{pmatrix} 19 & 23 & 32 \\ 25 & 12 & 29 \end{pmatrix} & \begin{matrix} \text{lunch} \\ \text{dinner} \end{matrix} \end{matrix}$$

- (a) The café charges \$20 for each set lunch, and \$30 for each set dinner.
Represent the prices in a 1×2 matrix $\mathbf{P}$.

Answer $\mathbf{P} =$ [1]

- (b) Evaluate the matrix $\mathbf{N} = 5\mathbf{M}$.

Answer $\mathbf{N} =$ [1]

- (c) Evaluate the matrix $\mathbf{T} = \mathbf{PN}$.

Answer $\mathbf{T} =$ [2]

- (d) State what each of the elements of $\mathbf{T}$ represents.

.....
.....
..... [1]

- (e) Using matrix multiplication only, evaluate the total number of set meals sold on this particular day.

Answer [2]

8 Bag A contains 4 red discs, 5 blue discs and 8 yellow discs.

Bag B contains 5 red discs and 8 blue discs.

A disc is chosen at random from Bag A and placed into Bag B.

A disc is then chosen at random from Bag B.

(a) Draw a tree diagram to show the possible outcomes and their probabilities.

Answer

[2]

- (b) (i) Find, as a fraction in its simplest form, the probability that both discs chosen are the same colour.

Answer [2]

- (ii) Find, as a fraction in its simplest form, the probability that one disc is red, and the other disc is blue.

Answer [2]

- (iii) Find, as a fraction in its simplest form, the probability that the disc chosen from bag A is not blue.

Answer [1]

- (iv) Find, as a fraction in its simplest form, the probability that at least one disc chosen is blue.

Answer [2]

- 9 Yan is intending to travel from Singapore to Kuala Lumpur with his family.

He is considering between the following three different modes of transportation.

Option 1: Direct flight from Singapore to Kuala Lumpur

Option 2: Direct drive from Singapore to Kuala Lumpur

Option 3: Trains following this route from Singapore to Kuala Lumpur

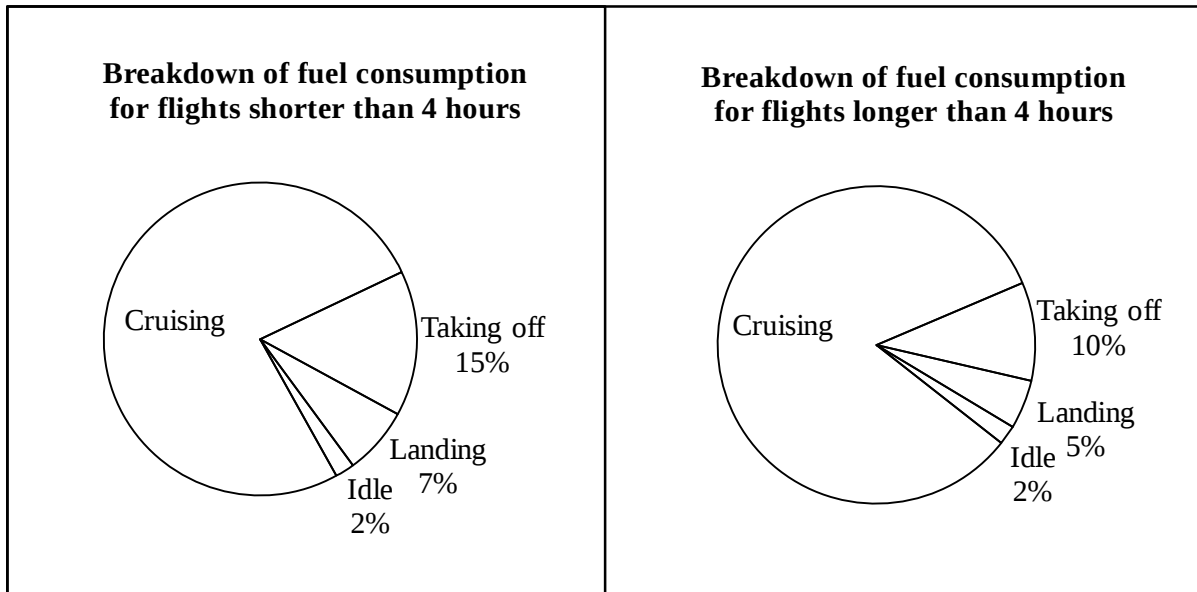
Yan finds the information below.

Mode of transportation	Plane	Train (from Woodlands to JB Sentral)	Train (from JB Sentral to Gemas)	Train (from Gemas to KL Sentral)	Car
Passenger capacity	300	320	150	125	5
Travel duration	60 – 65 minutes	5 minutes	4 hours 45 minutes	2 hours 35 minutes	6 – 7 hours
Distance covered (km)	310	5	205	130	370
Amount of fuel		6 gallons	200 gallons	130 gallons	45 litres

Other useful information:

- Fuel efficiency is a measure of how much energy is produced by an engine in relation to the amount of fuel that it uses. In other words, it shows how far your vehicle can travel with a certain amount of fuel. It is normally measured in litres per kilometre.
- 1 gallon = 3.785 litres

- (a) The pie charts below show the breakdown of the total fuel consumption for a flight.



The amount of fuel needed for cruising during any flight is 5760 l / 1000 km.

In case of emergencies, it is mandatory for all flights to carry an additional 20% of the total fuel needed to complete the flight.

Find the amount of fuel, in litres, required for the plane to fly from Singapore to Kuala Lumpur.

Answer l [3]

- (b) An environmentalist, Yan is looking to travel using the mode that is the most fuel efficient per passenger.
Suggest a suitable mode of transport for Yan and his family to travel to Kuala Lumpur. Justify the decision you made and show your calculations clearly.

Answer

[6]

- (c) State one assumption you have made in the above calculations.

.....

..... [1]

Answer Key

1ai	$-7\frac{1}{5} \leq x \leq \frac{2}{5}$	4d	Gradient = 2.52 (accept 2.23 – 2.86)
1aiii	-7	4eii	$x = 4.3$
1b	$\frac{4a^3}{5b^3}$	4eiii	$A = 35, B = 63$
1c	$\frac{25 - 2x}{3(x - 2)(x + 5)}$	5a	27.5 m
		5b	19.6 m
1di	Diagram 2	5c	004.1°
1dii	-84%	5di	23.3 m
2a	$\frac{48}{x}$	5dii	45.9°
2b	$\frac{48}{x + 4}$	6ai	$\frac{1}{5}(a + b)$
2d	$x = -2.4$ or 8	6aai	$\frac{1}{15}(3a - 2b)$
2e	\$18	6aiii	$\frac{1}{6}(3a - 2b)$
3ia	13.5°C	6b	$\overrightarrow{QR} = \frac{1}{15}(3a - 2b)$ $= \frac{2}{5}\left(\frac{1}{6}\right)(3a - 2b)$ $= \frac{2}{5}\overrightarrow{PR}$ QR is parallel to PR. Since R is a common point, P, Q and R are collinear.
3ib	5.5°C		
3ic	15.4°C		
3id	17.5%		
3ii	The curve of the year 2022 will be to the left of the year 2021.		
3iii	The median temperature of City Y is 14.5°C which is higher than that of City X. The temperature in City Y is generally warmer/higher than in City X.	6c	32.5 cm ²
	The interquartile range of temperatures of City Y is 4°C which is lower than that of City X. There is a narrower/smaller spread of temperatures in City Y than in city X.	6d	CA is not parallel to SR .
		7a	$P = (20 \ 30)$
		7b	$\begin{pmatrix} 95 & 115 & 160 \\ 125 & 60 & 145 \end{pmatrix}$
4a	-0.5	7c	(5650 4100 7550)
4c	$\frac{(x-1)^3}{7} - \frac{4x}{3} = 1$ $\frac{(x-1)^3}{7} - \frac{4x}{3} + 1 = 2$ By adding in the line $y = 2$, from the graph, there is only one intersection. ∴ the equation has only one solution	7d	The estimated total amount collected from the sales of each type of set meal in a particular week.
		7e	140

8bi	$\frac{11}{34}$
8bii	$\frac{57}{238}$
8biii	$\frac{12}{17}$
8biv	$\frac{83}{119}$
9a	2820l
9b	Yan should choose the train as it is the most fuel efficient per passenger.
9c	The vehicles are at full capacity.