1 (i) $32^y \times 5^{4y} = 2^{4y+4} \times 5^{3y-1}$

$\Rightarrow 32^y \times 5^{4y} = 2^{4y} \times 2^4 \times \frac{5^{3y}}{5}$

$\Rightarrow \frac{32^y \times 5^{4y}}{2^{4y} \times 5^{3y}} = \frac{2^4}{5}$

$\Rightarrow \left(\frac{32 \times 5^4}{2^4 \times 5^3}\right)^y = \frac{16}{5}$

$\Rightarrow \underline{10^y = \frac{16}{5}}$ [3]

(ii) $\therefore y = \lg \frac{16}{5}$ [1]

$= \underline{0.505}$ (3 s.f.)

2) $y = \frac{x^2 - 2ax + 2}{x - a}$

(i) $\frac{dy}{dx} = \frac{(2x-2a)(x-a) - (x^2 - 2ax + 2)}{(x-a)^2}$

$= \frac{2(x-a)^2 - (x^2 - 2ax + 2)}{(x-a)^2}$

$= \frac{2(x^2 - 2ax + a^2) - (x^2 - 2ax + 2)}{(x-a)^2}$

$= \underline{\frac{x^2 - 2ax + (2a^2 - 2)}{(x-a)^2}}$ [2]

(ii) For increasing y, $\frac{dy}{dx} > 0$

$\therefore \frac{x^2 - 2ax + (2a^2-2)}{(x-a)^2} > 0$

$\Rightarrow x^2 - 2ax + (2a^2 - 2) > 0$ since $(x-a)^2 > 0$.

This implies that $D < 0$

ie. $(-2a)^2 - 4(2a^2 - 2) < 0$

$4a^2 - 8a^2 + 8 < 0$

$\therefore 4a^2 - 8 > 0$

$a^2 - 2 > 0$

$(a + \sqrt{2})(a - \sqrt{2}) > 0$ $\quad -\sqrt{2} \quad \sqrt{2} \quad a$

$\therefore \underline{a < -\sqrt{2} \text{ or } a > \sqrt{2}}$ [3]

3) $P(x) = (2x-1)(x^3 + k) - 12$

$P(x)/x$, $R = -4$

(i) By Remainder Theorem, $P(0) = -4$

$\therefore (-1)(k) - 12 = -4$

$\underline{k = -8}$. [2]

(ii) $P(x) + 12 = 0$

$\Rightarrow (2x-1)(x^3 - 8) - 12 + 12 = 0$

$(2x-1)(x^3 - 8) = 0$

$\therefore (2x-1)(x-2)(x^2 + 2x + 4) = 0$

$\Rightarrow x = \frac{1}{2}$ or $x = 2$ or $x^2 + 2x + 4 = 0$

$D = 2^2 - 4 \times 4$

$= -12$

< 0

$\therefore x^2 + 2x + 4 = 0$ has no real solution.

$\therefore$ $\underline{P(x) + 12 = 0 \text{ has only 2 real solutions.}}$ [4]

4) (i) $A = \left(\frac{6}{2}, 1\right) = \underline{(3, 1)}$

For B, $y = 0 \Rightarrow |2x - 6| = 1$

$2x - 6 = 1$ or $2x - 6 = -1$

$x = \frac{7}{2}$ or $x = \frac{5}{2}$

$\therefore$ $\underline{B = \left(\frac{7}{2}, 0\right)}$. [3]

(ii) $y = 3x$ ... (1)

$y = 1 - |2x - 6|$ ... (2)

$\Rightarrow 1 - |2x - 6| = 3x$

$\Rightarrow |2x - 6| = 1 - 3x$, $3x \leq 1$

$2x - 6 = 1 - 3x$ or $2x - 6 = 3x - 1$

$x = \frac{7}{5}$ or $x = -5$

Since $x \leq \frac{1}{3}$, $\therefore$ $\underline{x = -5}$ [2]

(iii) Let l_1 be the line parallel to the right arm of the graph.

$\therefore$ gradient of $l_1 = -2$ [2]

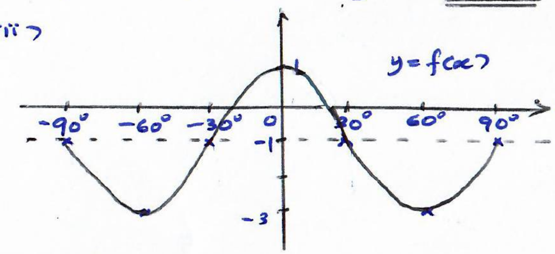
$\therefore$ $\underline{-2 < m < 0}$

5) $f(x) = 2\cos(3x) - 1$

(i) amplitude of $f = \underline{2}$

period of $f = \frac{360°}{3} = \underline{120°}$ [2]

(ii)



[3]

(iii) $f(x) = k$ has

(a) 4 solutions if $\underline{-3 < k \leq -1}$ [1]

(b) 2 solutions if $\underline{k = -3 \text{ or } -1 < k < 1}$ [1]

7 (a) (i) $(2 + bx)^8$

$= 2^8 + \binom{8}{1}2^7(bx) + \binom{8}{2}2^6(bx)^2 + \ldots$

$= 256 + 8 \times 128\, bx + 28 \times 64\, b^2x^2 + \ldots$

Comparing,

constant term: $a = 256$

coeff of x: $8 \times 128\, b = 256$

$\therefore b = \frac{1}{4}$

coeff of x^2: $c = 28 \times 64\, b^2$

$= 28 \times 64 \left(\frac{1}{4}\right)^2$

$= 112$

$\therefore$ $\underline{a = 256,\ b = \frac{1}{4},\ c = 112}$ [3]

(ii) $(2 + bx)^8 \left(2x - \frac{3}{x}\right)^2$

$= (256 + 256x + 112x^2)\left[4x^2 - 12 + \frac{9}{x^2}\right]$

$\therefore$ term independent of x

$= 256(-12) + 112(9)$

$= \underline{-2\ 064}$ [2]

7(b) $\left(4x^2 - \frac{1}{2x}\right)^{10}$

$T_{r+1} = \binom{10}{r}(4x^2)^{10-r}\left(-\frac{1}{2x}\right)^r$

power of $x = 2(10-r) + (-r)$

$= 20 - 3r$

For constant term, power of $x = 0$

$\therefore \; 20 - 3r = 0$

$r = \frac{20}{3}$.

But r must be a non-negative integer,

$\therefore$ The expansion does not contain a constant term. (shown). [3]

8(a) Area of square: $l^2 = 49 + 20\sqrt{6}$.

and $l = \sqrt{a} + \sqrt{b}$, $a < b$.

$\therefore (\sqrt{a} + \sqrt{b})^2 = 49 + 20\sqrt{6}$

$a + 2\sqrt{ab} + b = 49 + 20\sqrt{6}$

comparing,

rational part: $a + b = 49$ ...(1)

irrational part: $2\sqrt{ab} = +20\sqrt{6}$

$\sqrt{ab} = 10\sqrt{6}$

$\Rightarrow ab = 600$ ...(2)

From (1): $b = 49 - a$

Sub into (2): $a(49 - a) = 600$

$a^2 - 49a + 600 = 0$

$(a - 24)(a - 25) = 0$

$\therefore a = 24$ or $a = 25$

From (1): $b = 25$ or $b = 24$

Since $a < b$, $\underline{a = 24, \; b = 25}$. [4]

(b) $(\sqrt{5} - 3)x^2 + 3x + (\sqrt{5} + 3) = 0$

$\therefore x = \dfrac{-3 \pm \sqrt{3^2 - 4(\sqrt{5}-3)(\sqrt{5}+3)}}{2(\sqrt{5}-3)}$

$\therefore x = \dfrac{-3 \pm \sqrt{9 - 4(5-9)}}{2(\sqrt{5}-3)}$

$= \dfrac{-3 \pm 5}{2(\sqrt{5}-3)}$

$= -\dfrac{8}{2(\sqrt{5}-3)}$ or $\dfrac{2}{2(\sqrt{5}-3)}$

$= -\dfrac{4}{(\sqrt{5}-3)}$ or $\dfrac{1}{\sqrt{5}-3}$

$\dfrac{1}{\sqrt{5}-3} = \dfrac{\sqrt{5}+3}{(\sqrt{5}-3)(\sqrt{5}+3)}$

$= \dfrac{\sqrt{5}+3}{5-9}$

$= \dfrac{\sqrt{5}+3}{-4}$

$\therefore x = -4\left(\dfrac{\sqrt{5}+3}{-4}\right)$ or $\dfrac{\sqrt{5}+3}{-4}$

$= \underline{\sqrt{5}+3}$ or $\underline{-\dfrac{\sqrt{5}+3}{4}}$ [4]

9(i) Vol of open cylinder: $x(4x)h = 800$

$h = \dfrac{200}{x^2}$

$\therefore$ Area, $S = 2xh + 2(4x)h + 4x^2$

$= 2x \times \dfrac{200}{x^2} + 8x\left(\dfrac{200}{x^2}\right) + 4x^2$

$= \dfrac{400}{x} + \dfrac{1600}{x} + 4x^2$

$= 4x^2 + \dfrac{2000}{x}$ (shown) [4]

(ii) $\dfrac{dS}{dx} = 8x - \dfrac{2000}{x^2}$

For stationary value of S, $\dfrac{dS}{dx} = 0$

$\therefore 8x = \dfrac{2000}{x^2}$

$x^3 = \dfrac{2000}{8}$

$= 250$

$x = \sqrt[3]{250}$

$\therefore$ Stationary value of $S = 4(\sqrt[3]{250})^2 + \dfrac{2000}{\sqrt[3]{250}}$

$\therefore$ Stationary value of $S = \underline{476}$ (3 s.f.)

$\dfrac{d^2S}{dx^2} = 8 + \dfrac{4000}{x^3}$

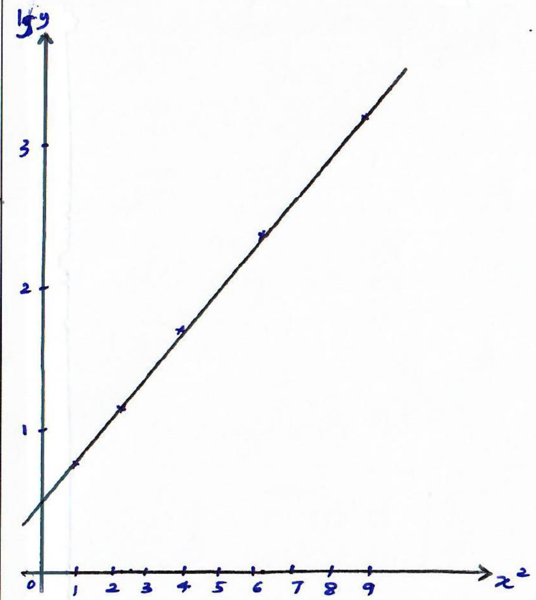
> 0 for $x = \sqrt[3]{250}$.

$\therefore$ The stationary value of S is a $\underline{\text{minimum}}$. [4]

10) $y = Ab^{x^2} \Rightarrow \lg y = \lg A + \lg b^{x^2}$

$= \lg A + x^2 \lg b$

x^2	1	2.25	4	6.25	9
$\lg y$	0.78	1.16	1.68	2.36	3.19



[3]

(ii) $\lg A = Y$-intercept

$= 0.5$

$\Rightarrow A = 10^{0.5} = \underline{3.16}$

$\lg b =$ gradient of line

$= \dfrac{3.2 - 0.5}{9}$

$= 0.30$

$\therefore b = 10^{0.30}$

$= \underline{2.00}$ [4]

(iii) When $y = 100$,

$\lg y = 2$

$\therefore x^2 = 5$

$x = \sqrt{5}$

$= \underline{2.24}$ [2]

11 (a) $\log_a x = p, \quad \log_a y = q$

(i) $\log_a \frac{a^2x^3}{y} = \log_a(a^2x^3) - \log_a y$

$= (2\log_a a + 3\log_a x) - \log_a y$

$= [2(1) + 3p] - q$

$= \underline{\underline{2 + 3p - q}}$ [2]

(ii) $\log_x a + \log_y a$

$= \frac{1}{\log_a x} + \frac{1}{\log_a y}$

$= \underline{\underline{\frac{1}{p} + \frac{1}{q}}}$ [1]

(iii) $xy = a^p \times a^q$

$= \underline{\underline{a^{p+q}}}$ [2]

(b) $\ln x = 4 \log_x e$

$= 4\left[\frac{\ln e}{\ln x}\right]$

$= \frac{4}{\ln x}$

$\Rightarrow [\ln x]^2 = 4$

$\ln x = \pm 2$

$\therefore \underline{\underline{x = e^2 \text{ or } e^{-2}}}$ [3]

12 (i) $M = \left(\frac{-3+5}{2}, \frac{5+(-1)}{2}\right)$

$= (1, 2)$

gradient of AB $= \frac{5-(-1)}{-3-5}$

$= -\frac{6}{8}$

$= -\frac{3}{4}$

Since $PM \perp AB$,

$\therefore$ gradient of $PM = \frac{4}{3}$

$\therefore$ Equation of PM is

$y - 2 = \frac{4}{3}(x - 1)$

$y = \frac{4}{3}x - \frac{4}{3} + 2$

$\therefore \underline{\underline{y = \frac{4}{3}x + \frac{2}{3}}}$ [3]

(ii) Since P lies on the line PM

$\therefore P(r, s)$ where $\underline{\underline{s = \frac{4}{3}r + \frac{2}{3}}}$. [1]

(iii) Since $PM = 10$ and $P = (r, s)$

$\therefore \sqrt{(r-1)^2 + (s-2)^2} = 10$

$\Rightarrow (r-1)^2 + \left[\left(\frac{4}{3}r + \frac{2}{3}\right) - 2\right]^2 = 100$

$\therefore (r-1)^2 + \left(\frac{4}{3}r - \frac{4}{3}\right)^2 = 100$

$\Rightarrow (r-1)^2 + \frac{16}{9}(r-1)^2 = 100$

$\therefore \frac{25}{9}(r-1)^2 = 100$

$(r-1)^2 = \frac{100 \times 9}{25}$

$= 36$

$\therefore r - 1 = \pm 6$

$r = 6 + 1$ or $-6 + 1$

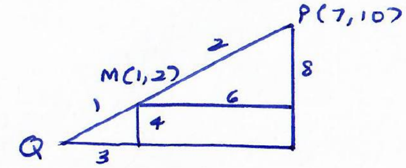
$= 7$ or -5

But $r > -3$, $\therefore r = 7$

When $r = 7$, $s = \frac{4}{3}(7) + \frac{2}{3}$

$= \underline{\underline{10}}$ [4]

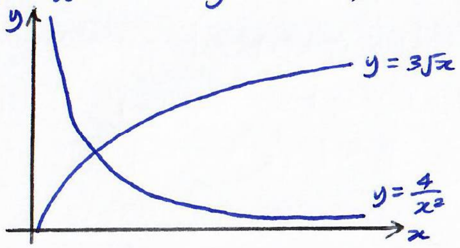
(iv) $PQ : MQ = 3 : 1$



$\therefore Q = (1 - 3, 2 - 4)$

$= \underline{\underline{(-2, -2)}}$ [2]

1(i) $y = \frac{4}{x^2}$ and $y = 3\sqrt{x}$, $x > 0$



[2]

(ii) $y = \frac{4}{x^2}$ ... (1)

$y = 3\sqrt{x}$ ... (2)

(1) = (2): $\frac{4}{x^2} = 3\sqrt{x}$

Squaring, $\frac{16}{x^4} = 9x$

$\Rightarrow x^5 = \frac{16}{9}$

$\therefore \underline{k = \frac{16}{9}}$ [2]

2) $f''(x) = 3\sin x + \cos(3x)$

(i) $f'(x) = \int 3\sin x + \cos(3x)\, dx$

$= 3(-\cos x) + \frac{\sin(3x)}{3} + c_1$

Since $(0, 0)$ is a stationary point, then when $x = 0$, $f'(x) = 0$

$\therefore 0 = 3(-1) + 0 + c$

$\Rightarrow c = 3$

$\therefore \underline{f'(x) = -3\cos x + \frac{\sin 3x}{3} + 3}$ [3]

(ii) $f(x) = \int -3\cos x + \frac{\sin 3x}{3} + 3\, dx$

$= -3(\sin x) + \frac{(-\cos 3x)}{3 \times 3} + 3x + c_2$

$= -3\sin x - \frac{\cos 3x}{9} + 3x + c_2$

When $x = 0$, $f(x) = 0$.

$\therefore 0 = 0 - \frac{1}{9} + 0 + c_2$

$\therefore c_2 = \frac{1}{9}$

$\therefore \underline{f(x) = -3\sin x - \frac{\cos 3x}{9} + 3x + \frac{1}{9}}$ [2]

3) $B = 200e^{2t} + 800e^{-2t}$

(i) When $t = 0$, $B = 200(1) + 800(1)$

$= 1000$

$\therefore$ initial number of bacteria = $\underline{1\,000}$. [1]

(ii) $\frac{dB}{dt} = 200(2e^{2t}) + 800(-2e^{-2t})$

$= 400e^{2t} - 1600e^{-2t}$

$\therefore \frac{dB}{dt} = 1200 \Rightarrow 400e^{2t} - 1600e^{-2t} = 1200$

$\div 400$: $e^{2t} - 4e^{-2t} = 3$

$\times e^{2t}$: $e^{4t} - 4 = 3e^{2t}$

$\therefore e^{4t} - 3e^{2t} - 4 = 0$ [2]

(iii) Let $u = e^{2t}$, then the equation becomes

$u^2 - 3u - 4 = 0$

$(u - 4)(u + 1) = 0$

$\therefore u = 4$ or $u = -1$

$\therefore e^{2t} = 4$ or $e^{2t} = -1$ (no solution)

$\therefore 2t = \ln 4$

$= 2\ln 2$

$\therefore \underline{t = \ln 2 \text{ or } 0.693}$ [3]

4(a) $\sec\left(2x - \frac{\pi}{6}\right) = \sqrt{2}$ $\quad -\frac{\pi}{2} \le x \le \frac{\pi}{2}$

$\therefore \cos\left(2x - \frac{\pi}{6}\right) = \frac{1}{\sqrt{2}}$ $\quad -\pi - \frac{\pi}{6} \le 2x - \frac{\pi}{6} \le \pi - \frac{\pi}{6}$

basic angle $= \frac{\pi}{4}$

$\therefore 2x - \frac{\pi}{6} = -\frac{\pi}{4}, \frac{\pi}{4}$

$2x = -\frac{\pi}{12}, \frac{5\pi}{12}$

$\underline{x = -\frac{\pi}{24}, \frac{5\pi}{24}}$ [3]

(b) $f(x) = 3\sin^2 x + 2 - \cos^2 x$

$= 3\left(\frac{1 - \cos 2x}{2}\right) + 2 - \left(\frac{1 + \cos 2x}{2}\right)$

$= \frac{3}{2} - \frac{3}{2}\cos 2x + 2 - \frac{1}{2} - \frac{1}{2}\cos 2x$

$\therefore f(x) = \underline{3 - 2\cos 2x}$ [2]

(ii) $\max f(x) = 3 - 2(-1)$

$= \underline{5}$ [1]

5) $y = \frac{x + 5}{1 - 2x}$

(i) $\frac{dy}{dx} = \frac{(1 - 2x) - (x + 5)(-2)}{(1 - 2x)^2}$

$= \frac{1 - 2x + 2x + 10}{(1 - 2x)^2}$

$= \frac{11}{(1 - 2x)^2}$

At A, $x = 1$, $y = \frac{1 + 5}{1 - 2} = -6$

$\therefore A = (1, -6)$

$\frac{dy}{dx} = \frac{11}{(1 - 2)^2} = 11$

$\therefore$ gradient of normal $= -\frac{1}{11}$

$\therefore$ Equation of normal at A is

$y - (-6) = -\frac{1}{11}(x - 1)$.

$\therefore y = -\frac{1}{11}x + \frac{1}{11} - 6$.

$\underline{y = -\frac{1}{11}x - \frac{65}{11}}$ [4]

(ii) gradient of tangent $= \tan\theta$

$\therefore \tan\theta = 11$

$\theta = \tan^{-1} 11$

$= \underline{84.8^\circ}$

$\therefore$ Tangent makes an angle of 84.8° with the positive x-axis. [2]

(iii) $\frac{dy}{dx} = \frac{11}{(1 - 2x)^2}$

> 0 since $(1 - 2x)^2 > 0$ for all real values of x.

$\therefore \frac{dy}{dx} \ne 0$.

$\therefore$ The curve has no stationary pt. [1]

6 (i) $\frac{\tan x}{1+\sec x} + \frac{1+\sec x}{\tan x}$

$= \frac{\tan^2 x + (1+\sec x)^2}{\tan x(1+\sec x)}$

$= \frac{\tan^2 x + 1 + 2\sec x + \sec^2 x}{\tan x(1+\sec x)}$

$= \frac{\sec^2 x + 2\sec x + \sec^2 x}{\tan x(1+\sec x)}$

$= \frac{2\sec x(1+\sec x)}{\tan x(1+\sec x)}$

$= \frac{2}{\cos x} \times \frac{\cos x}{\sin x}$

$= \frac{2}{\sin x}$ (shown) [4]

(ii) $\frac{\tan x}{1+\sec x} + \frac{1+\sec x}{\tan x} = 1 + 3\sin x$

$-90° \le x \le 90°$

$\Rightarrow \frac{2}{\sin x} = 1 + 3\sin x$

$\therefore 2 = \sin x + 3\sin^2 x$

$3\sin^2 x + \sin x - 2 = 0$

$(3\sin x - 2)(\sin x + 1) = 0$

$\therefore \sin x = \frac{2}{3}$ or $\sin x = -1$.

$\therefore x = 41.8°$ $x = -90°$.

$\therefore x = 41.8°$ or $-90°$ [3]

7) $8x^2 - 4x + 1 = 0$

Since the roots are $\frac{1}{\alpha^2\beta}$ and $\frac{1}{\alpha\beta^2}$, then

$\frac{1}{\alpha^2\beta} + \frac{1}{\alpha\beta^2} = -\frac{-4}{8}$

$\frac{\beta+\alpha}{\alpha^2\beta^2} = -\frac{1}{2}$ ... (1)

and $\frac{1}{\alpha^2\beta} \times \frac{1}{\alpha\beta^2} = \frac{1}{8}$

$\frac{1}{\alpha^3\beta^3} = \frac{1}{8}$

$\therefore \alpha^3\beta^3 = 8$

$\therefore \alpha\beta = 2$

Sub into (1): $\frac{\beta+\alpha}{2^2} = \frac{1}{2}$

$\Rightarrow \alpha+\beta = 2$

For the new equation,

Sum of roots $= \alpha^3 + \beta^3$

$= (\alpha+\beta)[(\alpha+\beta)^2 - 3\alpha\beta]$

$= 2[2^2 - 3(2)]$

$= -4$

product of roots $= \alpha^3\beta^3$

$= 2^3$

$= 8$

$\therefore$ The equation is $x^2 - (-4)x + 8 = 0$

ie $x^2 + 4x + 8 = 0$ [7]

8) $V = 40e^{-\frac{1}{3}t} - 15$

(i) $a = 40(-\frac{1}{3}e^{-\frac{1}{3}t})$

$= -\frac{40}{3}e^{-\frac{1}{3}t}$

When $t = 0$, $a = -\frac{40}{3}$

$\therefore$ Initial acceleration $= -\frac{40}{3}$ m/s^2. [2]

(ii) For the car to stop, $v = 0$

$\therefore 40e^{-\frac{1}{3}t} - 15 = 0$

$e^{-\frac{1}{3}t} = \frac{15}{40} = \frac{3}{8}$

$-\frac{1}{3}t = \ln\frac{3}{8}$

$t = -3\ln\frac{3}{8}$

$= 2.94$ (3s.f.) [2]

(iii) $s = \int 40e^{-\frac{1}{3}t} - 15 \, dt$

$= 40\left[\frac{e^{-\frac{1}{3}t}}{-\frac{1}{3}}\right] - 15t + c_1$

$= -120e^{-\frac{1}{3}t} - 15t + c_1$

When $t = 0$, $s = 0$

$\therefore 0 = -120 + c_1$

$\Rightarrow c_1 = 120$

$\therefore s = -120e^{-\frac{1}{3}t} - 15t + 120.$ [3]

(iv) When $t = 2.942$,

$s = -120e^{-\frac{1}{3}(2.942)} - 15(2.942) + 120$

$= 30.9$ m (3 s.f.) [1]

9) (i) Let $f(x) = (x+1)(2x^2 + kx - 3)$

By Remainder Theorem, $f(1) = -4$

$\therefore -4 = 2[2 + k - 3]$

$\therefore k - 1 = -2$

$k = -1$

$\therefore f(x) = (x+1)(2x^2 - x - 3)$

$= (x+1)(2x-3)(x+1)$

$= (x+1)^2(2x-3)$ [3]

(ii) $f(x) = 0 \Rightarrow x = -1$ or $x = \frac{3}{2}$

$f(x-1) = 0 \Rightarrow x-1 = -1$ or $x - 1 = \frac{3}{2}$

$x = 0$ or $x = \frac{5}{2}$.

(iii) $\frac{3x^2 - 10x + 2}{f(x)}$

$= \frac{3x^2 - 10x + 2}{(x+1)^2(2x-3)}$

$= \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{2x-3}$

$\therefore 3x^2 - 10x + 2 = A(x+1)(2x-3) + B(2x-3) + C(x+1)^2$

$x = -1$: $3 + 10 + 2 = B(-2-3)$

$\therefore B = -3$

$x = \frac{3}{2}$: $3(\frac{9}{4}) - 10(\frac{3}{2}) + 2 = C(\frac{3}{2}+1)^2$

$-\frac{25}{4}C = \frac{25}{4}C$ $\Rightarrow C = -1$

comparing x^2: $2A + C = 3 \Rightarrow A = 2$

$\therefore \frac{3x^2 - 10x + 2}{f(x)} = \frac{2}{x+1} - \frac{3}{(x+1)^2} - \frac{1}{2x-3}$ [4]

10) $\triangle QRT$: $\sin\theta = \frac{QR}{24} \Rightarrow QR = 24\sin\theta$

$\triangle PQN$: $\cos\theta = \frac{PQ}{7} \Rightarrow PQ = 7\cos\theta$

$\therefore$ Perimeter, $P = 2[PQ + QR]$

$= 2[24\sin\theta + 7\cos\theta]$

$= \underline{\underline{14\cos\theta + 48\sin\theta}}$ [2]

(ii) $R = \sqrt{14^2 + 48^2} = 50$

$\tan\alpha = \frac{48}{14} \Rightarrow \alpha = 73.739°$

$\therefore \underline{\underline{P = 50\cos(\theta - 73.7°)}}$ [3]

(iii) $P = 48 \Rightarrow \theta = 90°$

But $0 < \theta < 90°$,

$\therefore$ it is not possible for P to be 48 m. [1]

(iv) $QR = 12 \Rightarrow \sin\theta = \frac{12}{17+7}$

$= \frac{1}{2}$

$\Rightarrow \theta = 30°$

$\therefore P = 14\cos 30° + 48\sin 30°$

$= 14(\frac{\sqrt{3}}{2}) + 48(\frac{1}{2})$

$= 7\sqrt{3} + 24$

$= \underline{\underline{36.1 \text{ cm}}}$.

11 (a) $y = (1 + e^{x^2})(x+5)$

(i) $\frac{dy}{dx} = (1 + e^{x^2}) + (x+5)(2xe^{x^2})$

$= 1 + e^{x^2}[1 + 2x(x+5)]$

$= \underline{\underline{1 + e^{x^2}(1 + 10x + 2x^2)}}$ [3]

(ii) When $x = 0.5$, $\frac{dy}{dx} = 1 + \frac{13}{2}e^{\frac{1}{4}}$

$= 9.34616$

Chain Rule : $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$2 = 9.34616 \times \frac{dx}{dt}$

$\therefore \frac{dx}{dt} = \frac{2}{9.34616}$

$= \underline{\underline{0.214}}$ (3 s.f.) [2]

(b) Area $A = \left| \int_0^4 x(\frac{1}{16}x^2 - 1)\,dx \right|$

$= \left| \int_0^4 (\frac{1}{16}x^3 - x)\,dx \right|$

$= \left| \left[\frac{1}{16}(\frac{x^4}{4}) - \frac{x^2}{2} \right]_0^4 \right|$

$= \left| (4 - \frac{4^2}{2}) - 0 \right|$

$= |-4|$

$= 4$ units^2.

Area $B = \int_4^6 (\frac{1}{16}x^3 - x)\,dx - \frac{1}{2} \times 2 \times 4$

$= \left[\frac{1}{16}(\frac{x^4}{4}) - \frac{x^2}{2} \right]_4^6 - 4$

$= \left[(\frac{81}{4} - 18) - (4 - 8) \right] - 4$

$= \frac{9}{4}$

$= 2\frac{1}{4}$ units

$\therefore$ Shaded area $= 4 + 2\frac{1}{4} = \underline{\underline{6\frac{1}{4} \text{ units}^2}}$ [5]

12) $C : x^2 + y^2 - 12x - 8y - 13 = 0$

(i) $x^2 - 12x + y^2 - 8y = 13$

$(x-6)^2 + (y-4)^2 = 13 + 36 + 16$

$= 65$

$\therefore$ centre $= (6, 4)$ and radius $= \sqrt{65}$ units. [3]

(ii) $4x + 7y = 117 \Rightarrow y = -\frac{4}{7}x + \frac{117}{7}$

gradient of required line $= \frac{7}{4}$

$\therefore$ Equation of line is

$y - 4 = \frac{7}{4}(x - 6)$

$y = \frac{7}{4}x - \frac{21}{2} + 4$

$\underline{\underline{y = \frac{7}{4}x - \frac{13}{2}}}$ [3]

(iii) $4x + 7y = 117$ ... (1)

$(x-6)^2 + (y-4)^2 = 65$ ... (2)

From (1) : $y = \frac{117 - 4x}{7}$

Sub into (2) : $(x-6)^2 + (\frac{117 - 4x}{7} - 4)^2 = 65$

$\therefore x^2 - 12x + 36 + (-\frac{4x}{7} + \frac{89}{7})^2 = 65$

$\therefore x^2 - 12x + 36 + \frac{16}{49}x^2 - \frac{712}{49}x + \frac{7921}{49} = 65$

$\Rightarrow 49x^2 - 588x + 1764 + 16x^2 - 712x + 7921 = 3185$

$\therefore 65x^2 - 1300x + 6500 = 0$

$x^2 - 20x + 100 = 0$

$\therefore D = (-20)^2 - 4(100)$

$= 400 - 400$

$= 0$

$\therefore$ The line is tangent to the circle.

$\therefore$ The point of contact is at $x = 10$.

$\therefore$ $\underline{\underline{\text{The point is } (10, 11)}}$ [5]

13) $y = x^2\sqrt{2x+1}$

$\Rightarrow \frac{dy}{dx} = 2x\sqrt{2x+1} + x^2 \cdot \frac{1}{2}(2x+1)^{-\frac{1}{2}} \times 2$

$= 2x\sqrt{2x+1} + \frac{x^2}{\sqrt{2x+1}}$

$= \frac{2x(2x+1) + x^2}{\sqrt{2x+1}}$

$= \frac{5x^2 + 2x}{\sqrt{2x+1}}$

$= \underline{\underline{\frac{x(5x+2)}{\sqrt{2x+1}}}}$ [3]

(i) For stationary pts, $\frac{dy}{dx} = 0$

$\frac{x(5x+2)}{\sqrt{2x+1}} = 0 \Rightarrow x = 0$ or $x = -\frac{2}{5}$

$y = 0$ or $y = \frac{4\sqrt{5}}{125}$

Using 1st Derivative Test,

x	0^-	0	0^+	$(-\frac{2}{5})^-$	$(-\frac{2}{5})$	$(-\frac{2}{5})^+$
$\frac{dy}{dx}$	$-$	0	$+$	$+$	0	$-$
	\	—	/	/	—	\

$\therefore (0, 0)$ is a minimum point.

$(-\frac{2}{5}, \frac{4\sqrt{5}}{125})$ is a maximum point. [4]

(iii) $\int_0^4 \frac{5x^2+2x-3}{\sqrt{2x+1}}\,dx$

$= \int_0^4 \frac{5x^2+2x}{\sqrt{2x+1}} - \frac{3}{\sqrt{2x+1}}\,dx$

$= \left[x^2\sqrt{2x+1}\right]_0^4 - 3\left[\frac{(2x+1)^{\frac{1}{2}}}{\frac{1}{2}(2)}\right]_0^4$

$= \left[16\sqrt{9} - 0\right] - 3\left[\sqrt{2x+1}\right]_0^4$

$= 16\times 3 - 3\left[\sqrt{9} - \sqrt{1}\right]$

$= 48 - 9 + 3$

$= \underline{\underline{42.}}$ [4]