Hwa Chong Institution 华侨中学
Mathematics (SMTP)

Name: ______________________ (   ) Class: _____ Date: _______

MA304.2.3 – Trigonometric Ratios of Any Angle

At the end of this unit, you should be able to
- **understand that trigonometric ratio of an angle is closely related to its basic angle**
- **understand and use ASTC to determine sign of trigonometric ratios of angles**
- **use ASTC to reduce trigonometric ratios of any angle in terms of its basic angle**
- **simplify trigonometric ratios of negative angles**

E1. Given $-360^\circ < A < 720^\circ$ and that A has a basic angle $\alpha = 80^\circ$, find all the values of A if A is in the

(a) 2^{nd} quadrant

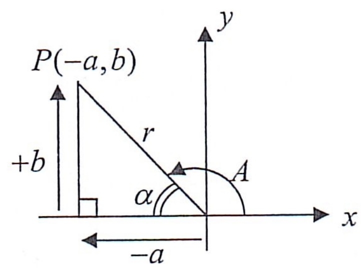
(b) 3^{rd} quadrant

(c) 1^{st} quadrant

So far, we are able to find trigonometric ratios of acute angles. What about angles which are not acute? That is when our basic angle comes into play – to evaluate the trigonometric ratio of an angle A which is not acute, it is *almost* the same as evaluating the trigonometric ratio of its basic angle, except for an important difference. Read on to find out.

Trigonometric ratio of *any* angle

Consider angle A which sits in the 2^{nd} quadrant. In this case, To move from the origin to a point on the line, $P(-a,b)$, where $a,b>0$, we have to go along the *negative* x-axis for a units before moving along the *positive* y-axis for b units.



r, the hypotenuse, is given by $r=\sqrt{a^2+b^2}$ and is always positive.
Hence for this quadrant, we define, with the 'help' of the basic angle α,

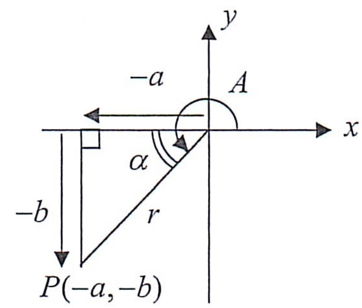
$$\sin A = \frac{b}{r} \text{ (since opposite side is positive)}$$

$$\cos A = \frac{-a}{r} = -\frac{a}{r} \text{ (since adjacent side is negative)}$$

and $$\tan A = \frac{b}{-a} = -\frac{b}{a}$$

So in this quadrant, **only the sine ratio is positive**. The cosine and tangent ratios are negative.

P1. Now consider an angle A which sits in the 3[rd] quadrant. Investigate and define the trigonometric ratios of angle A in that quadrant. The diagram is drawn for you here.

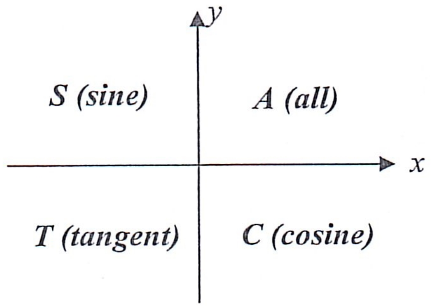


In this quadrant, **only the** __________ **ratio is positive.**

P2. Do the same for the above for an angle A which sits in the 4[th] quadrant.

In this quadrant, **only the** __________ **ratio is positive.**

Summarising, we can see that the trigonometric ratios take different signs over different quadrants, and the **positive** ones are illustrated as below:



How would you plan to memorise this order?

E2. Are the following trigonometric ratios positive or negative?

(a) $\tan\frac{5\pi}{3}$

(b) $\cos 160^\circ$

(c) $\sin(-100^\circ)$

(d) $\cos(-545^\circ)$

(e) $\sin\frac{7\pi}{3}$

(f) $\tan\left(-\frac{7\pi}{6}\right)$

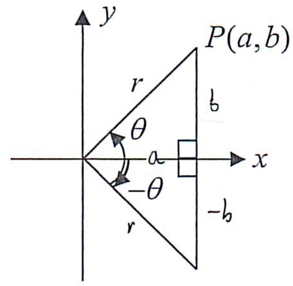
P3. Evaluate, to three significant figures $\sin 10\pi^\circ - \frac{\pi}{5}$.

E3. Given that A is obtuse and that $\tan A = -0.8$, find the value of $\cos A$ and of $\sin A$.

P4. If A is such that $\tan A = -1.4$ and $\cos A$ is positive, evaluate the exact values of $\sin A$ and $\cos A$. Hence show that $\sin^2 A + \cos^2 A = 1$.

Negative angles

E4. Consider $\theta > 0$ and acute. Write down, in terms of a, b and r, the expressions for $\sin\theta$, $\cos\theta$ and $\tan\theta$.



(a) Now express, using the triangle drawn above, $\sin(-\theta)$, $\cos(-\theta)$ and $\tan(-\theta)$ in terms of a, b and r.

(b) How are $\sin(-\theta)$ and $\sin\theta$ related?

(c) How are $\cos(-\theta)$ and $\cos\theta$ related?

(d) How are $\tan(-\theta)$ and $\tan\theta$ related?