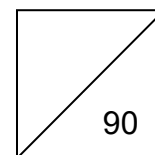




NORTH VISTA SECONDARY SCHOOL
End-of-Year Examination 2023
Secondary 3 Express



CANDIDATE
NAME

CLASS

INDEX
NUMBER

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ADDITIONAL MATHEMATICS

4049

4 OCTOBER 2023

2 hour 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your centre number, index number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use	
Category	Question No.
Accuracy	
Brackets	
Fractions	
Units	
Others	
Marks Deducted	

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$\text{Area of } \Delta = \frac{1}{2} bc \sin A.$$

Answer **all** questions.

1 Do not use a calculator in this question.

The curved surface area of a cylinder is $10\pi\sqrt{5} - 18\pi \text{ cm}^2$. The radius of the base of the cylinder is $3 + 2\sqrt{5} \text{ cm}$.

Find an expression for the height of the cylinder in the form $a + b\sqrt{5}$, where a and b are integers.

[4]

- 2 (a) The angles A and B are such that $\sin A = \frac{5}{13}$, where $90^\circ < A < 270^\circ$ and $\cos B = \frac{4}{5}$, where B is acute. Calculate
- (i) $\tan(-B)$, [1]
- (ii) $\tan(90 - A)$, [1]
- (iii) $\cos A \sin B$. [2]
- (b) **Without using a calculator**, find the exact value of $\left(\sin \frac{\pi}{6} + \cos \frac{\pi}{4}\right) \tan \frac{\pi}{3}$.
 Leave your answer in a single fraction. [2]

- 3 **(a)** Express $y = -2x^2 + 8x + 14$ in the form of $y = a(x+b)^2 + c$, where a , b and c are constants. [2]
- (b)** Sketch the graph of $y = -2x^2 + 8x + 14$ indicating the coordinates of the turning point and of the y -intercept. [2]

- 4 Express $\frac{x^3 + 7x^2 - 5x + 12}{(x-1)(x^2 + 2)}$ in partial fractions. [6]

- 5** The expression $4x^3 + ax^2 + bx - 12$, where a and b are constants, has a factor of $x - 2$ and leaves a remainder of -18 when divided by $2x - 1$.

(a) Show that $a = 2$ and find the value of b . [4]

(b) Using these values of a and b , solve the equation $4x^3 + ax^2 + bx - 12 = 0$. [4]

- 6** **(a)** Find the coordinates of the points of intersection between the straight line $3x + 2y = 5$ and the curve $\frac{3}{x} + \frac{2}{y} = 10$. [4]

- 6 (b) Show that the line $y = ax + 2a - 1$ cuts the curve $y = 8 + 2x - x^2$ at two distinct points for all values of a . [3]

- (c) Find the range of values of p for which the graph $y = px^2 + 4x + p$ lies entirely below the line $y = -3$. [4]

- 7 **(a)** Find the first 3 terms in the expansion, in descending powers of x , of $\left(3x - \frac{1}{2}\right)^5$.
Give the terms in their simplest form. [2]

- 7 **(b)** **(i)** By considering the general term in the the binomial expansion of $\left(x^2 - \frac{3}{x}\right)^7$,
explain why every term is dependent on x . [3]

- (ii)** Hence find the term independent of x in the expansion of $\left(x^2 - \frac{3}{x}\right)^7 (4 - 3x)$. [3]

8 (a) Solve the equation $\frac{125^{2+x}}{25} = 5^x \times 625^{2x-1}$. [4]

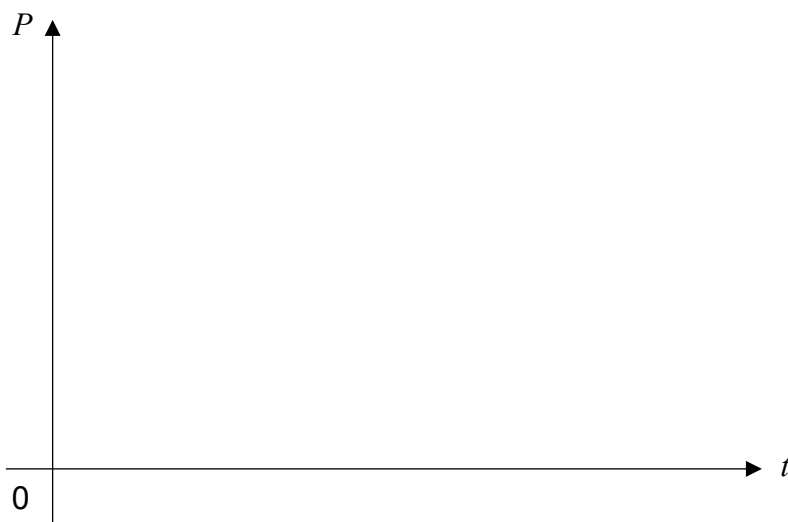
(b) Solve the equation $\log_4 x + 3\log_x 4 = 4$. [5]

- 9 The population, P , of a certain bacterium t days after the start of an experiment is modelled by $P = 5600e^{kt}$, where k is a constant.

(a) State what the value 5600 represents in this experiment. [1]

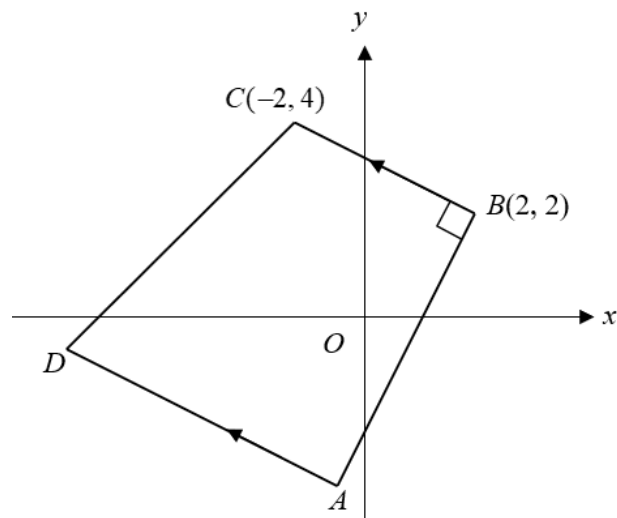
(b) Given that the population four days after the start of the experiment is 28 000, calculate the population of the bacterium 2 weeks after the start of the experiment. [4]

(c) Sketch the graph of $P = 5600e^{kt}$ for $t \geq 0$. [2]



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10



In the trapezium $ABCD$, AD and BC are parallel and angle ABC is a right angle. The coordinates of the points B and C are $(2, 2)$ and $(-2, 4)$ respectively.

(a) Find the equation of the line AB .

[3]

- 10 (b)** The y -intercept of the line DA produced is $-4\frac{1}{2}$, show that the coordinates of A is $(-1, -4)$. [2]

- (c)** It is given that the midpoint of the line segment BD is $\left(-2\frac{1}{2}, \frac{1}{2}\right)$.
Find the coordinates of D . [2]

- (d)** Find the area of the trapezium $ABCD$. [2]

- 11** The points P and Q both lie on a circle and have coordinates $(-2, -4)$ and $Q(5, -3)$ respectively. The equation of the tangent at Q is $4y + 3x = 3$.

(a) Find the equation of the perpendicular bisector of PQ .

[3]

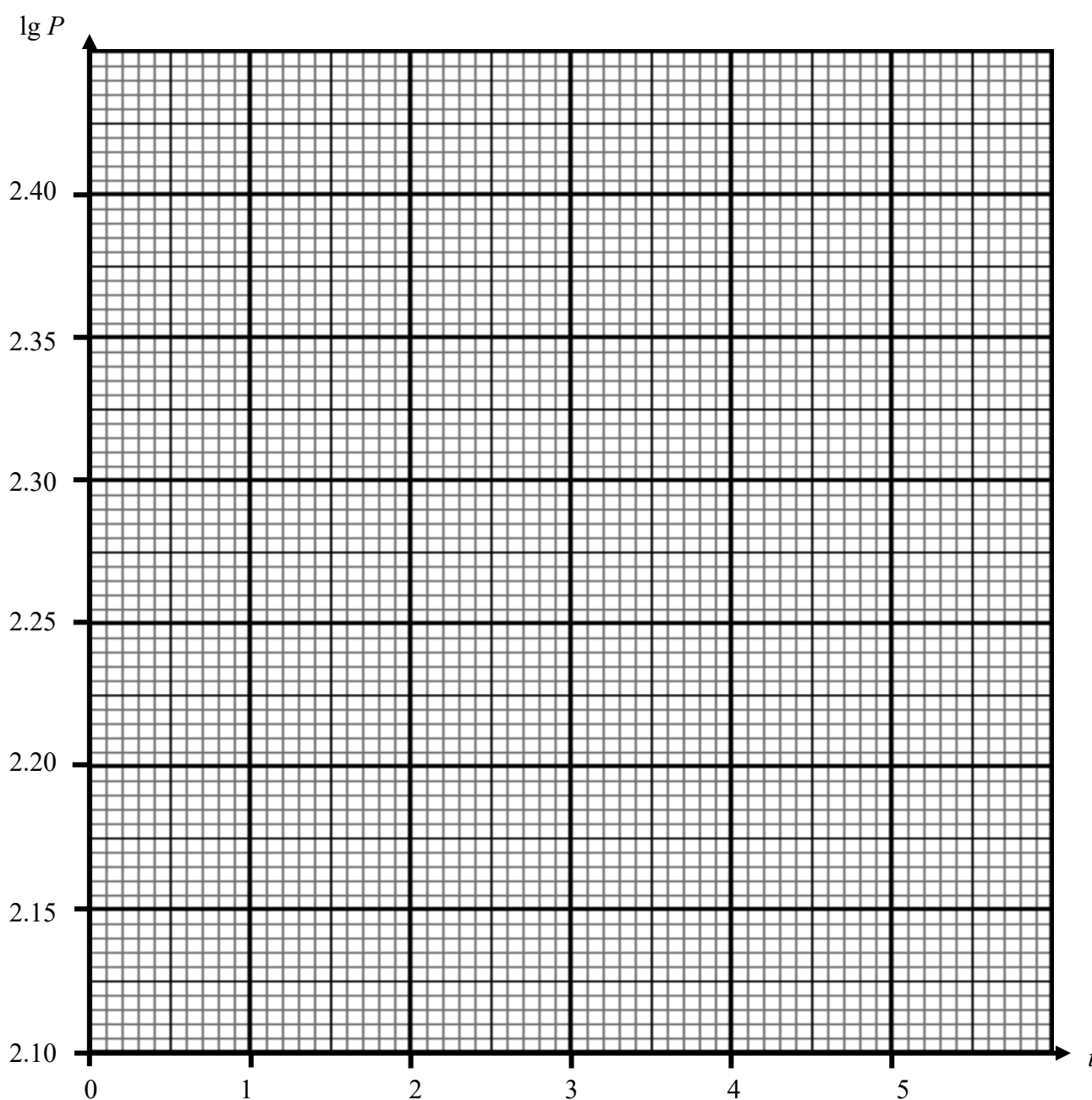
- 11 (b) Find the equation of the circle. [5]

- (c) Determine if the point (2, 1) lies within, outside or on the circle. [2]

- 12** The table shows the population, P , of a species, correct to 3 significant figures, measured daily for t days. It is known that t and P are related by the equation $P = ab^t$, where a and b are constants.

Number of days, t	1	2	3	4	5
Population, P	174	192	211	232	255

- (i) On the grid below, plot $\lg P$ against t and draw a straight line graph. [2]



- 12** **(ii)** Use your graph to estimate the value of a and of b . [4]

- (iii)** Assuming that the model is still appropriate, find the population when $t = 10$. [2]

END OF PAPER