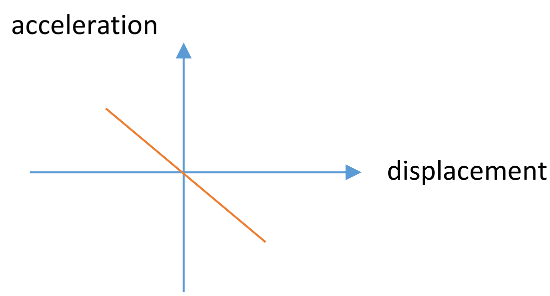
09 Oscillation Problem Solutions

P1	(a)	Only for half the cycle (see notes)
	(b)	Only for half the cycle (see notes)
	(c)	No (always in opposite direction $a = -\omega^2 x$)
P2	D	A – Amplitude is half of 70 cm B – Kinetic energy is max at equilibrium which occurs at T/4 C – Restoring force is maximum at amplitude and zero at equilibrium so restoring force decreases from t = 0 to t = T/4 D – Correct. Kinetic energy and speed highest at equilibrium.
P3	A	$E = \frac{1}{2}m\omega^2 {x_0}^2$ Hence $E \propto {x_0}^2$ New energy is ¾ E, hence new $x_0 = \frac{\sqrt{3}}{2}x$ So change in amplitude = $\left(1 - \frac{\sqrt{3}}{2}\right)x = 0.134x$
P4		**a) (i)** The radian is the unit of measurement for angles and is the ratio of the arc length subtended to the radius of the circle. **(a) (ii)** For simple harmonic motion, angular frequency is 2π times the frequency of oscillations. **(b) (i)** Total energy = maximum gravitational potential energy from equilibrium position = $mgh = 0.120 \times 9.81 \times 0.04 = 4.7 \times 10^{-3} J$ **(b) (ii)** Total energy = $\frac{1}{2}m\omega^2 {x_0}^2 = 4.7 \times 10^{-3}$ $\frac{1}{2}m4\pi^2 f^2 {x_0}^2 = 4.7 \times 10^{-3}$ $f = \sqrt{\frac{2\times4.7\times10^{-3}}{0.120\times4\times\pi^2\times0.08^2}} = 0.56\ Hz$ ($x_0 = 0.080\ m$)
P5	(i)	At t_1 and t_2, x = 1.3m from mid tide level, ie. equilibrium $1.3 = 3.7 \sin \omega t$ By sub $\omega = 2\pi/T$ ωt_1 = 0.35901 rad and ωt_2= 2.7826 rad t_1 = 2606 s and t_2 = 20195 s Hence Δt = $t_2 - t_1$= 17589 s = 18 000 s (2 s.f.) OR 290 min (2 s.f.)
	(ii)	Rate of rising tide and rate of falling tide is the gradient of tangent drawn at t_1 and t_2 respectively $= 3.7\omega \cos \omega t_1$

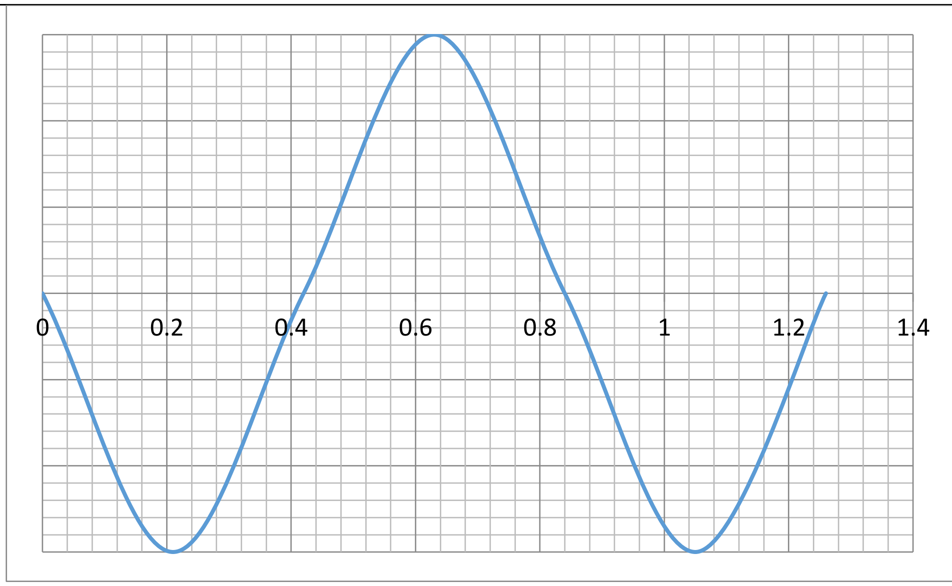
		$= \left(3.7 \times \frac{2\pi}{45600}\right) \cos\left[\left(\frac{2\pi}{45600}\right)(2606)\right]$ = 4.8 × 10^{-4} m s^{-1}	
P6	D	GPE should increase linearly with upwards displacement. The extension decreases with upwards displacement so, EPE should decrease. It is a curve as EPE = ½ kx^2 KE is maximum at equilibrium and zero at amplitude positions. TE is constant.	
P7	A	Total energy must be constant, also check value of KE is max at centre and equal to $\frac{1}{2}mv^2 = 1.8J$. EPE has negative value as it is set to be zero at the center. The spring has extension at the center so at the top where the extension, the EPE must be lower and hence a negative value.	
P8	(a)	(i)	t_3, t_7
		(ii)	t_4, t_8
	(b)	(i)	$f = \frac{4200}{60} = 70Hz$
		(ii)	$a_0 = \omega^2 x_0$ $= 4\pi^2 f^2 x_0 = 4\pi^2 \times 70^2 \times 0.025$ $= 4800\ ms^{-2}$
	(c)		 B1 straight line through the origin B1 negative gradient B1 correct values for amplitude (2.5 cm) and maximum acceleration
P9	(a)	(i)	Simple harmonic motion occurs when the acceleration of an object is directly proportional to its displacement and the acceleration is always opposite in direction to the displacement.
		(ii)1	The graph shows that displacement can be both positive and negative, hence the mass is oscillating.
		(ii)2	The graph is not a straight line (curved at one end), hence acceleration is not directly proportional to displacement and the oscillations are not simple harmonic.
	(b)	(i)1	At the upper amplitude position
		(i)2	When the sand first loses contact, the maximum acceleration is equal to the acceleration due to free fall $\omega^2 x_0 = g$ $(2\pi \times 13)^2 x_0 = 9.81$

			$x_0 = 1.47\ mm$
		(ii)	The minimum amplitude will not be different. The mass loses contact when the acceleration of the oscillations exceed the acceleration due to free fall. This is not dependent on mass and hence the size of the mass does not matter.
	(c)	(i)1	$E_T == \frac{1}{2} \times 1.2 \times (2\pi \times 2.5)^2 \times 0.034^2 = 0.171\ J$
		(i)2	When the potential energy is half the total energy, potential and kinetic energy are equal $E_P = \frac{0.171}{2} = \frac{1}{2} m\omega^2 x^2$ $x = 2.4\ cm$
		(i)3	Total energy – straight line at 0.171 J from -3.4 cm to +3.4 cm Kinetic energy – inverted U shape curve. Kinetic energy is zero at amplitude and 0.171 J at zero displacement. Potential energy – U shape curve. 0.171 J at amplitude and zero at zero displacement. At 2.4 cm, Kinetic and potential energy graphs intercept at 0.086 J
P10	(c)	(i)	Since *ρ*, *A*, *g* and *m* are all constants, the value of $\frac{\rho A g}{m}$ is a constant. Hence *a* is directly proportional to *x* but in the opposite direction to the displacement from the equilibrium position as indicated by the negative sign.
	(c)	(ii)	Since it is in SHM, acceleration a = - $\omega^2 x$ This implies $\omega^2 = \frac{\rho A g}{m}$ $$f = \frac{1}{2\pi}\sqrt{\frac{\rho A g}{m}}$$ $f = \frac{1}{2\pi}\sqrt{\frac{(1000)(4.2 \times 10^{-4})(9.81)}{0.032}}$ = 1.81 Hz
	(d)	(i)	Using Fig. 2 1. For 3 cycles, total time from graph = 1.50 s Period = 0.50 s f = 1/0.50 = 2.0 Hz 2. Since $\omega^2 = \frac{\rho A g}{m}$ only density changed. $$4\pi^2 f^2 = \frac{\rho A g}{m}$$ $\rho = \frac{4\pi^2 f^2 m}{Ag} = \frac{4\pi^2 (2.0)^2 (0.032)}{(4.2\times10^{-4})(9.81)} = 1226$ kg m^{-3}
		(ii)	1. The decrease in amplitude shows that the total energy is decreasing with time due to light damping. This comes about due to (1) the viscous force between the liquid and the tube and (2) the resistive force between the sand and the tube.

			2. Decrease in energy = (Total energy at t = 0) – (Total energy at t =1.0s) Since total energy = ½ m$\omega^2 x_0^2$ Decrease in energy = ½ (0.032)(2π × 2)^2[(1.5 × 10^{-2})^2 – (0.85× 10^{-2})^2] = 3.86 × 10^{-4} J
P11		ai(1) Displacement is the distance moved from the equilibrium position. ai(2) Amplitude is the maximum displacement. aii) Simple harmonic motion occurs when the acceleration of an object is directly proportional to its displacement and the acceleration is always opposite in direction to the displacement. b) simple pendulum – the restoring force is the component of the weight tangential to the tension in the string towards the equilibrium position Floating block – the restoring force is provided by the resultant force due to upthrust and weight. Below the equilibrium position, the upthrust is greater than weight and results in an upwards net force which is also the restoring force. Above the equilibrium position, the weight is greater than upthrust resulting in a downwards net force which is also the restoring force. c) At equilibrium when the steel strip is depressed, the weight of mass M causes a clockwise moment about the clamp. A counterclockwise moment needs to be provided to ensure that the net moment about the clamp is zero. The upper part of part of the steel strip is stretched and in tension. The bottom part of the steel strip is compressed and provides a pushing force on the mass. Together, these forces provide a counterclockwise moment which is needed to result in the net moment for the block mass M to be zero. d) As C, E, L and M are constants, the equation can be expressed in the form a = -kx, showing that acceleration is directly proportional to displacement and opposite in direction to the displacement. Hence, the strip is undergoing simple harmonic motion. ei) for 4 oscillations, time is 0.84s, period T = 0.21s $\omega = \frac{2\pi}{T} = 29.9\ rad\ s^{-1}$ eii) $\frac{CE}{L^3 M} = \omega^2$ $C = \frac{\omega^2 L^3 M}{E} = 3.44 \times 10^{-10}$ 7f) Since the frequency is the same, ω is the same L, C are the same, E is then proportional to M	

		$M = \frac{7.1\times10^{10}}{2.0\times10^{11}} \times 0.15 = 0.0533\ kg$

P12	(a)	(i)	2 cycles = 1.2 s T = 0.6 s
		(ii)	ω = 2π/T = 10.5 rad s^{-1}
	(b)	(i)	Damping is the phenomenon where the energy of the oscillating system decreases with time.
		(ii)	1. Light damping can be achieved by attaching a cardboard of large surface area to the base of the mass. This results in an increase in air resistance. OR Having the mass oscillate in a viscous fluid such as oil or honey. This results in an increase in viscous force between the fluid and the mass. 2. Increase the surface area of the cardboard OR viscosity of the fluid.
P13		(a) Extension of spring is given by x =(*l* - 12cm). From the graph, when *l* = 12.0 cm, extension is zero and after *l* = 12.0 cm (which is the extension), the gradient of the graph is positive and constant showing that F is directly proportional to the extension (*l* – 12 cm). Hence the spring obeys Hooke's law. (b) Work done = Area under the Force- extension graph = 0.5 × 1.4 × (18 × 10^{-2}) = 0.126 J (c) (i) Max speed = ωx_0 = (2π/0.84)(0.04) = 0.300 ms^{-1} (period is 0.84 s and amplitude 0.04 m) Or gradient of fig 1.2. (ii)	



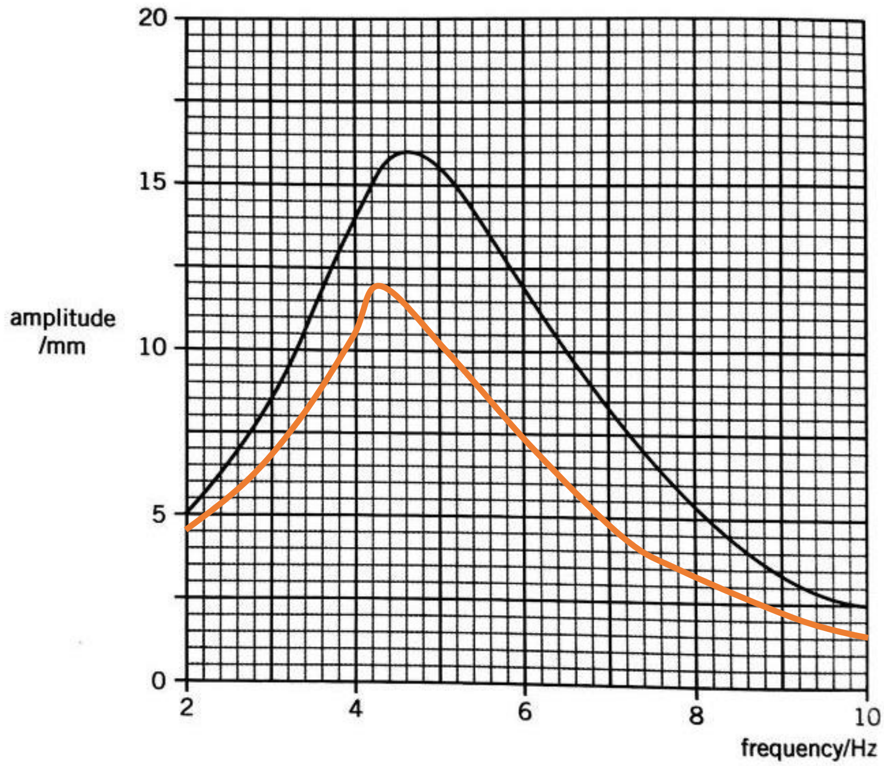
differentiate Fig. 1.2 (cos wt becomes – sin wt, period is 0.84 s, amplitude is 0.300 ms^{-1})

(d) (i) Weight of M = 1.40 N, Equilibrium length of spring = 0.30 m
Upthrust = ρVg = (1000)(2 × 10^{-5})(9.81) =0.1962 N
Since M is in equilibrium,
Tension = Weight – Upthrust = 1.4 – 0.1962 = 1.20 N
From Fig. 1.1, the new length is 0.275 m.

(ii) 1. Amplitude decreases over time: The viscous force acting on the lower part of M will result in the loss of energy of M overtime.
2. Period increases: Damping results in an increase in the period of the oscillation since the motion is slowed due to the presence of viscous force. (or velocity decreases.)

P14	(a)	(i)	Forced oscillations
		(ii)	At maximum amplitude, amplitude = 16 mm and frequency = 4.6 Hz
	(b)	(i)	Max acceleration = $\omega^2 x_0 = (2\pi \times 4.6)^2(16 \times 10^{-3}) = 13.4$ m s^{-2}
		(ii)	Max resultant force = ma = 0.150 × 13.365 = 2.0 N

(c)



Highest peak is lower and shifts to a lower frequency.
All amplitudes lower for all frequencies compared to previous.

P15 As the car goes over the bumps at regular interval at a particular speed, the normal contact force exerts a periodic driving force on the car suspension system. This driving force's frequency matches the natural frequency of the system resulting in an increased in the amplitude of the oscillation larger than before. Resonance is said to have occurred in this system.

To find the spring constant:
F = kx
80 x9.81 = k (0.02)
k= 39240 N m^{-1}

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$T = 2\pi\sqrt{\frac{1000}{39240}}$$

$T = 1.003$ s

Speed = Distance (L) /Time (T)
15 = L/T, where T is also the period of the oscillation.
15 = L/1.003
L = 15.0 m

C1

Ans: D

(This is torsional oscillation, $\alpha = -\omega^2\theta$ where α is the angular acceleration and θ the angular displacement)

Angular displacement $= 0.050\ sin\ \frac{2\pi}{2.0}t$

$0.030 = 0.050\ sin\ \frac{2\pi}{2.0}t$

$t = 0.205s$

Angular velocity $= 0.050 \times \frac{2\pi}{2.0} cos \frac{2\pi}{2.0} t = 0.050 \times \frac{2\pi}{2.0} cos \frac{2\pi}{2.0} \times 0.205 = 0.125\ rads^{-1}$

C2

(a) When x=0, E_k is maximum.

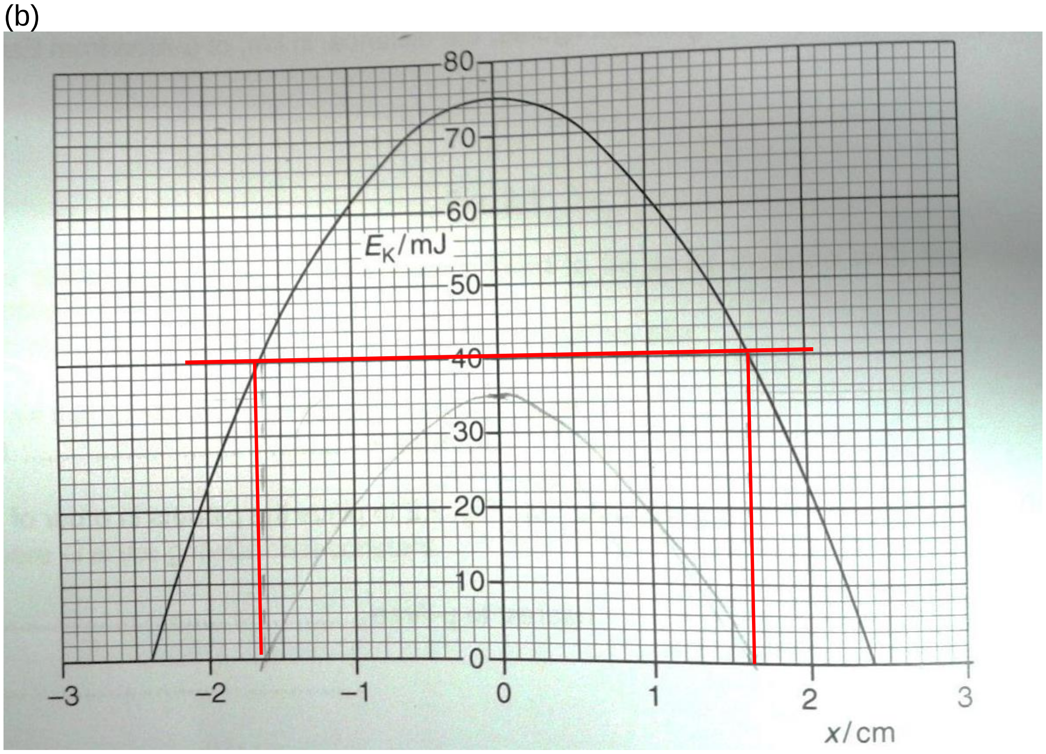
$E_k = 1/2\ mv^2_{max}$ where maximum speed of trolley at eqm position, $v_{max} = \omega x_o$ and x_o is amplitude

$75x10^{-3} = 1/2\ (0.590)\ \omega^2 {x_o}^2$

$75x10^{-3} = 1/2\ (0.590)\ (2\pi f)^2(0.024)^2$

f = 3.34 Hz

(b)



Construction: A horizontal line at E = 40 mJ. Intersection between the horizontal line and curve gives the new amplitude.

Imagine the horizontal line as the new x-axis. The graph above this new line represents the energy graph where total energy is reduced by 40 mJ. Where the graph meets the new x-axis must be the corresponding amplitude.

	New amplitude = 0.0165 m