$\frac{77}{80}$.

1\. (a) $8x^3-27y^3 = (2x)^3-(3y)^3$

$= (2x-3y)(4x^2+6xy+9y^2)$

(b) $\frac{2}{x-3} - \frac{x+1}{x^2-4x+3} - \frac{x}{1-x} = \frac{2}{x-3} - \frac{x+1}{(x-3)(x-1)} + \frac{x}{x-1}$

$= \frac{2(x-1)-(x+1)+x(x-3)}{(x-3)(x-1)}$

$= \frac{2x-2-x-1+x^2-3x}{(x-3)(x-1)}$

$= \frac{x^2-2x-3}{(x-3)(x-1)}$

$= \frac{\cancel{(x-3)}(x+1)}{\cancel{(x-3)}(x-1)}$

$= \frac{x+1}{x-1}$

6

2\. $u=\sqrt{x-5}$

$3u+\frac{8}{u}=14$

$3u^2-14u+8=0$

$(3u-2)(u-4)=0$

$u=\frac{2}{3}$ or $u=4$

$\sqrt{x-5}=\frac{2}{3}$ $\quad \sqrt{x-5}=4$

$x-5=\frac{4}{9}$ $\quad x-5=16$

$x=5\frac{4}{9}$ $\quad x=21$

4

3\. $x+3\sqrt{x+2}=2$

$3\sqrt{x+2}=2-x$

$9x+18=4-4x+x^2$

$x^2-13x-14=0$

$(x-14)(x+1)=0$

$x=14$ or $x=-1$

(rej.)

4

4\. (a) $(2-\sqrt{5})^2 - \frac{20}{3+\sqrt{5}} = 4-4\sqrt{5}+5-\frac{20(3-\sqrt{5})}{9-5}$

$= 9-4\sqrt{5}-\frac{60-20\sqrt{5}}{4}$

$= 9-4\sqrt{5}-15+5\sqrt{5}$

$= -6 + \sqrt{5}$

$\therefore p = -6, \quad q = 1$

(b) $4^{x+y} \times 2^{-3y} = 1$

$2^{2x+2y} \times 2^{-3y} = 2^0$

$2x - y = 0$

$\frac{81^x}{3^{3x+2y}} = \frac{1}{27}$

$\frac{3^{4x}}{3^{3x+2y}} = 3^{-3}$

$4x - (3x + 2y) = -3$

$x - 2y = -3$

$2x - y = 0$

$4x - 2y = 0$

$3x = 3$

$\therefore x = 1$

$\therefore y = 2$

(c) $y = x^2 + (w+6)x + \frac{3}{4}w + 18$

$D = 0$

$D = (w+6)^2 - 4(\frac{3}{4}w + 18) = 0$

$w^2 + 12w + 36 - 3w - 72 = 0$

$w^2 + 9w - 36 = 0$

$(w+12)(w-3) = 0$

$w = -12$ or $w = 3$

12

5. (a) $\log_4(2-x) - 1 = \log_{16} x + \log_4(\frac{1}{4})$

$\log_4(2-x) - \log_4 4 = \frac{\log_4 x}{\log_4 16} + \log_4(\frac{1}{4})$

$\log_4(\frac{2-x}{4}) = \frac{\log_4 x}{2} + \log_4 4^{-1}$

$\log_4(\frac{2-x}{4}) = \frac{\log_4 x}{2} - 1$

$2\log_4(\frac{2-x}{4}) = \log_4 x - 2$

$\log_4(\frac{2-x}{4})^2 = \log_4 x - \log_4 16$

$\log_4\left(\frac{4-4x+x^2}{16}\right) = \log_4\left(\frac{x}{16}\right)$

$x^2-4x+4=x$

$x^2-5x+4=0$

$(x-4)(x-1)=0$

$x=4$ or $x=1$

(rej.)

5

(b) $5^{2x-3} = 2e^{x+2}$

$\ln 5^{2x-3} = \ln 2e^{x+2}$

$(2x-3)\ln 5 = \ln 2 + \ln e^{x+2}$

$2x\ln 5 - 3\ln 5 = \ln 2 + x + 2$

$2x\ln 5 - x = \ln 2 + 2 + 3\ln 5$

$x(2\ln 5 - 1) = \ln 2 + 2 + 3\ln 5$

$x = \frac{\ln 2 + 2 + 3\ln 5}{2\ln 5 - 1}$

$= 3.39$ (3s.f.)

9

4

6. (i) $T = 25 + T_0 e^{-at}$

$85 = 25 + T_0 e^{-a(0)}$

$85 = 25 + T_0$

$T_0 = 60$

$70 = 25 + 60e^{-10a}$

$45 = 60e^{-10a}$

$e^{-10a} = \frac{3}{4}$

$\ln e^{-10a} = \ln\frac{3}{4}$

$-10a = \ln\frac{3}{4}$

$a = \ln\frac{3}{4} \div (-10)$

$= 0.0288$ (3s.f.)

(ii) $T = 25 + 60e^{-0.0288(60)}$

$= 35.7°C$ (1d.p.)

(iii) The temperature will be 25°C after a very long time, as e^{-at} will become very near to 0 and 25 is a constant.

7

7. $y=-x^2-6x+3$

$y=mx+4$

$-x^2-6x+3=mx+4$

$x^2+(m+6)x+1=0$

$D=(m+6)^2-4>0$

$m^2+12m+36-4>0$

$m^2+12m+32>0$

$(m+8)(m+4)>0$

$m<-8$ or $m>-4$

4

8. (i) $f(x)=10x^3+x^2-5x+1$

$2x-1$ $f(\frac{1}{2})=10(\frac{1}{2})^3+(\frac{1}{2})^2-5(\frac{1}{2})+1$

$=0$

$\therefore$ $2x+1$ is a factor

(ii) $f(x)=0$

$10x^3+x^2-5x+1=0$

$(2x-1)(5x^2+3x-1)=0$

$x=\frac{1}{2}$ or $x=\frac{-3\pm\sqrt{9-4(5)(-1)}}{2(5)}$

$=\frac{-3\pm\sqrt{29}}{10}$

6

9. $x-3$ $P(3)=ax+b=4$

$3a+b=4$

$x+2$ $P(-2)=ax+b=-1$

$-2a+b=-1$

$5a=5$

$a=1$

8

$b=1$

remainder $= ax+b$

$= x+1$

4

10. (i) Maximum point $= \left(\frac{10+0}{2}, 7\right)$

$= (5, 7)$

$y = a(x-5)^2+7$

$0 = a(0-5)^2+7$

$25a = -7$

$a = -\frac{7}{25}$

$\therefore y = -\frac{7}{25}(x-5)^2+7$

(ii) $2.5 = -\frac{7}{25}(x-5)^2+7$

$-4.5 = -\frac{7}{25}(x-5)^2$

$16\frac{1}{14} = (x-5)^2$

$x-5 = 4.0089$

$x = 9.0089$

width $= 9.0089 - (10 - 9.0089)$

$= 8.0178$

$= 8.02$m (3 s.f.)

(iii) $\frac{4}{2}+5=7$

$y = -\frac{7}{25}(7-5)^2+7$

$= 5.88$m

(iv) It is not possible, as the height of the arch is 5.88m when its width is 4m, and 5.88m is less than 6m.

11

11. $y = f(x)$

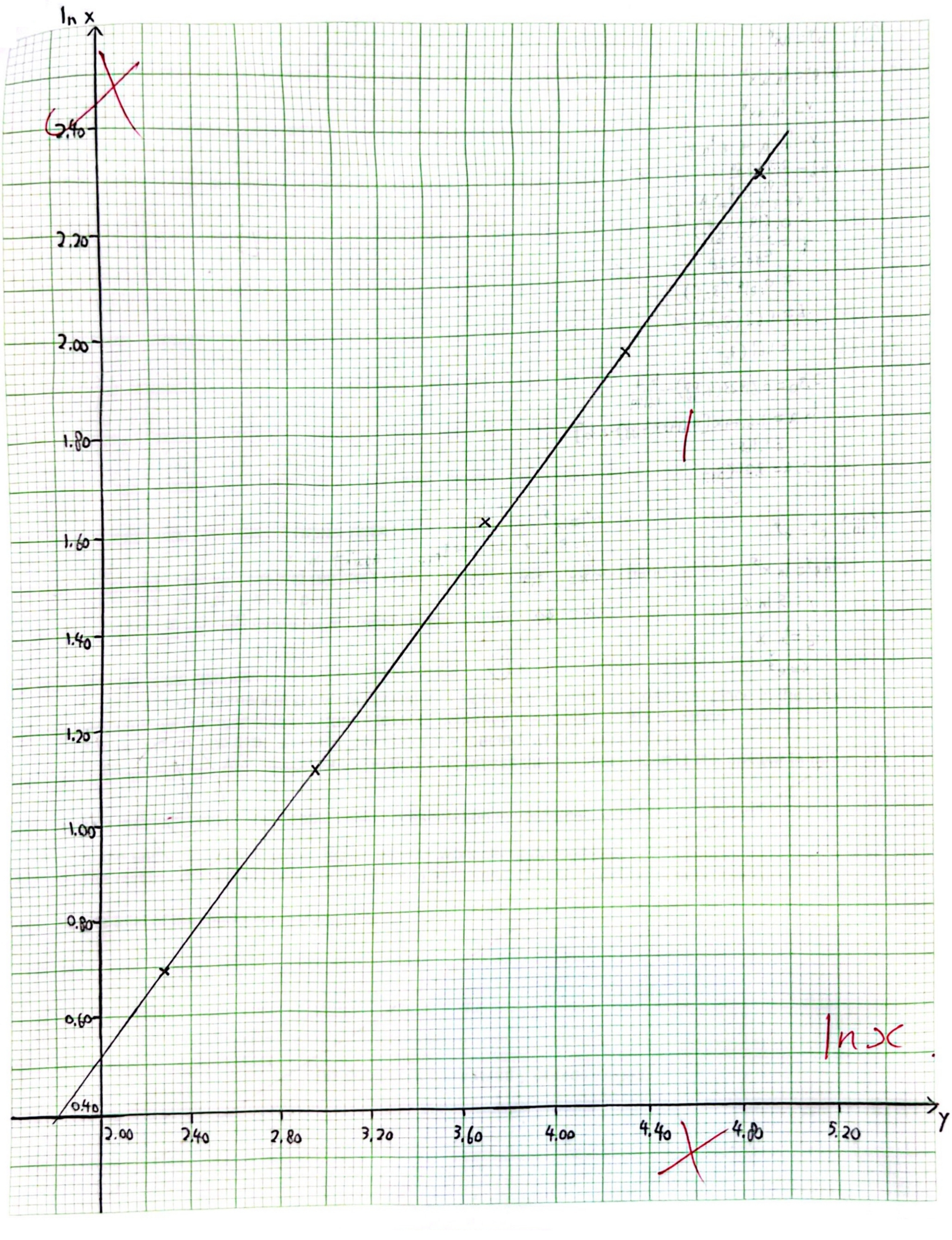
$= k(x-2)(x-\sqrt{3})(x+\sqrt{3})$

$= k(x-2)(x^2-3)$

$= k(x^3-2x^2-3x+6)$

$= 5x^3-10x^2-15x+30$

4


ln x
2.40
2.20
2.00
1.80
1.60
1.40
1.20
1.00
0.80
0.60
0.40
2.00
2.40
2.80
3.20
3.60
4.00
4.40
4.80
5.20
Y
ln x

12.

(i)

y	2.28	2.94	3.69	4.30	4.88
ln x	0.69	1.10	1.61	1.95	2.30

1.

(ii) $e^y = ax^b$

$\ln e^y = \ln ax^b$

$y = \ln ax^b$

$y = \ln a + b \ln x$

$b \ln x = y - \ln a$

$\ln x = \frac{1}{b}(y) - \frac{\ln a}{b}$

$\frac{1}{b} = \frac{2.30 - 0.69}{4.88 - 2.28}$

$= 0.61923$

$\therefore b = 1.6149$

$= 1.61$ (3 s.f.)

$2.30 = 0.61923(4.88) - \frac{\ln a}{1.6149}$

$\ln a = 1.6149 \times [0.61923(4.88) - 2.30]$

$= 1.1657$

$\therefore a = 3.21$ (3 s.f.)

4.

(iii) $e^y = x^5$

$\ln e^y = \ln x^5$

$y = 5 \ln x$

$\frac{y}{5} = \ln x$

$x = e^{\frac{y}{5}}$

$e^y = x^5$

$y = 5 \ln x \longrightarrow$ Draw y against ln x

$\ln x = 0.334$

$x = 1.40$

0

6.