ZHONGHUA SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2022
SECONDARY 4 EXPRESS/ 5 NORMAL (ACADEMIC)

Candidate's Name	Class	Register Number
MARKING SCHEME		

ADDITIONAL MATHEMATICS 4049/01

PAPER 1

12 September 2022
2 hour 15 minutes

Candidates answer on the Question Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **90**.

For Examiner's Use
90

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

[Turn over

1 Express $\dfrac{2x^3-4x^2+x-18}{(x-2)(x^2+4)}$ in partial fractions. [6]

$$\begin{array}{r l} & \quad\quad\quad\quad\quad\quad\quad\quad 2 \\ x^3-2x^2+4x-8 \,\big) & 2x^3-4x^2+x-18 \\ - & (2x^3-4x^2+8x-16) \\ \hline & \quad\quad\quad -7x-2 \end{array}$$

M1 attempt at long division

$$\frac{2x^3-4x^2+x-18}{(x-2)(x^2+4)}=2-\frac{7x+2}{(x-2)(x^2+2x+4)}$$

$$\frac{7x+2}{(x-2)(x^2+4)}=\frac{A}{(x-2)}+\frac{Bx+C}{(x^2+4)}$$ B1

$7x+2=A(x^2+4)+(Bx+C)(x-2)$

Let $x=2$, $7\times 2+2=A(8)$ M1 attempts to find A,B,C

$A=2$

Comparing the coefficients of x^2 and constant,

$0=A+B$, $B=-A$

$B=-2$

$2=4A-2C$

$2C=8-2$ A2 all correct A,B,C

$C=3$ (A1 any 2 correct values)

$$\frac{2x^3-4x^2+x-18}{(x-2)(x^2+4)}=2-\frac{2}{(x-2)}-\frac{-2x+3}{(x^2+4)}$$ A1

2 Without using a calculator,

(i) show that $\tan 285° = \dfrac{\sqrt{3}+1}{1-\sqrt{3}}$, [3]

$$\tan 285° = -\tan 75° \qquad \text{B1}$$
$$= -\tan(45°+30°)$$
$$= -\frac{\tan 45° + \tan 30°}{1-\tan 45° \tan 30°} \qquad \text{M1 using addition formula}$$
$$= -\frac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}} \times \frac{\sqrt{3}}{\sqrt{3}}$$
$$= -\frac{\sqrt{3}+1}{\sqrt{3}-1}$$
$$= \frac{\sqrt{3}+1}{1-\sqrt{3}}$$

B1

(ii) express $\sec^2 285°$ in the form $a+b\sqrt{3}$, where a and b are integers. [4]

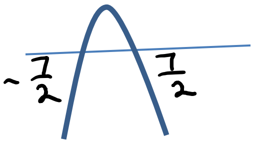
$$\sec^2 285° = 1+\tan^2 285° \qquad \text{B1}$$
$$= 1+\left(\frac{\sqrt{3}+1}{1-\sqrt{3}}\right)^2$$
$$= 1+\frac{3+2\sqrt{3}+1}{1-2\sqrt{3}+3}$$
$$= 1+\frac{4+2\sqrt{3}}{4-2\sqrt{3}} \qquad \text{B1 correct expansion of } (\ )^2$$
$$= 1+\frac{2(2+\sqrt{3})}{2(2-\sqrt{3})} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} \qquad \text{M1 multiplying by conjugate surds}$$
$$= 1+\frac{4+4\sqrt{3}+3}{1}$$
$$= 8+4\sqrt{3} \qquad \text{A1}$$

3 Find the range of values of k for which the curve $y = kx^2 + 8x$ lies entirely above the line $y = x - k$. [4]

For $kx^2 + 8x > x - k$, $kx^2 + 7x + k > 0$

$k > 0$ B1 and $7^2 - 4(k)(k) < 0$ M1

$k > 0$ and $(7-2k)(7+2k) < 0$



$k > 0$ and $k < -\frac{7}{2}$ or $k > \frac{7}{2}$ B1

Ans: $k > \frac{7}{2}$ A1

4 A curve is such that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -\frac{4}{x^3} - \frac{15}{(3x-2)^2}$ and the point $P(1, -6)$ lies on the curve.

The gradient of the curve at P is 3. Find the equation of the curve. [6]

$\frac{\mathrm{d}y}{\mathrm{d}x} = \int -4x^{-3} - 15(3x-2)^{-2}\,\mathrm{d}x$ M1 attempt to integrate

$= -\frac{4x^{-2}}{-2} - \frac{15(3x-2)^{-1}}{(-1)(3)} + C$ A1 (or equivalent)

$= \frac{2}{x^2} + \frac{5}{(3x-2)} + C$

At P, $x = 1$, $\frac{\mathrm{d}y}{\mathrm{d}x} = 3$

$3 = 2 + 5 + C$ M1 substituting to find C

$-4 = C$

$\frac{\mathrm{d}y}{\mathrm{d}x} = 2x^{-2} + 5(3x-2)^{-1} - 4$

$y = \frac{2x^{-1}}{-1} + \frac{5\ln(3x-2)}{3} - 4x + D$ $\int (3x-2)^{-1}\mathrm{d}x = \frac{\ln(3x-2)}{3} + C$ B1

Substituting $x = 1, y = -6$

$-6 = -2 + \frac{5\ln 1}{3} - 4 + D$

$D = 0$ A1

$y = -\frac{2}{x} + \frac{5\ln(3x-2)}{3} - 4x$ A1

5 The equation of a polynomial is given by $p(x) = 3x^3 - 5x^2 - 3x + 2$.

(a) Show that $(x-2)$ is a factor of $p(x)$. [1]

$$\left.\begin{aligned} p(2) &= 3(2)^3 - 5(2)^2 - 3(2) + 2 \\ &= 24 - 20 - 6 + 2 \\ &= 0 \\ &\text{By factor theorem, } (x-2) \text{ is a factor of } p(x). \end{aligned}\right\} \text{B1}$$

Alternatively by long division, need to state remainder = 0

(b) Hence, solve the equation $p(x) = 0$, expressing non-integer roots in surd form. [3]

$p(x) = (x-2)(3x^2 + bx - 1)$ M1

comparing coefficient of x^2

$-6 + b = -5$

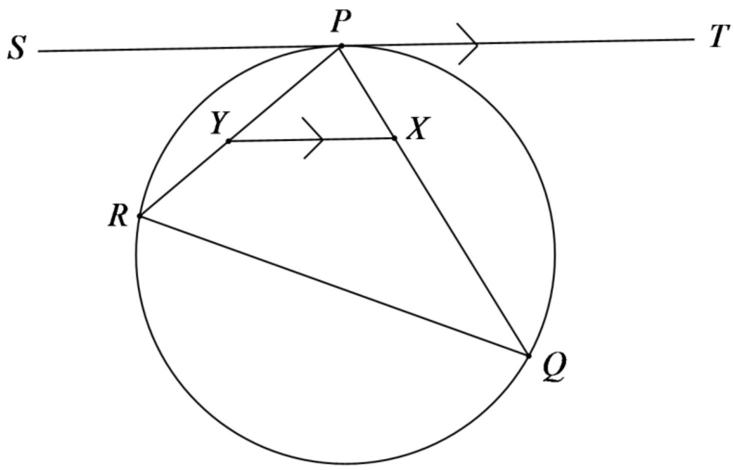
$b = 1$

$(x-2)(3x^2 + x - 1) = 0$ B1 correct quadratic factor

$x = 2$ or $x = \dfrac{-1 \pm \sqrt{1 - 4(3)(-1)}}{2(3)}$

$x = 2$ or $x = \dfrac{-1 \pm \sqrt{13}}{6}$ A1

6



The diagram shows a circle passing through the points P, Q and R. The points X and Y lie on QP and RP respectively. The tangent ST to the circle at P is parallel to YX.

(a) Prove that Q, R, Y and X lie on a circle.

Let $\angle SPY = x°$

$\angle PQR = \angle SPY$ (angles in alternate segment) B1

$= x°$

$\angle PYX = \angle SPY$ (alternate angles, ST parallel to YX) B1

$= x°$

[5]

$\angle XYR = 180° - \angle PYX$ (adjacent angles on a straight line) B1

$= 180° - x°$

$\angle XQR = \angle PQR = x°$

since $\angle XYR + \angle XQR = 180° - x° + x°$

$= 180°$ B1

$\angle XYR$ and $\angle XQR$ are angles in opposite segments B1

hence Q, R, Y and X lie on a circle.

[Turn over

6 **(b)** The line RP is extended to Z such that angle $PZT = 90°$.
Explain why a circle passing through the points P, T and Z has its centre at the midpoint of PT. [2]

Since $\angle PZT = 90°$, it is an angle in semicircle, B1
PT is the diameter of the circle passing through P, T and Z B1
with centre at the midpoint of PT.

7 **(a)** The equation of a curve is $y = x(2x-5)^3$. Find the range of values of x for which y is decreasing. [4]

$$\frac{dy}{dx} = 1 \times (2x-5)^3 + x \times 3(2x-5)^2 \times 2$$ M1 product rule or chain rule

$$= (2x-5)^2[2x-5+6x]$$

$$= (2x-5)^2(8x-5)$$

For y to be decreasing, $\frac{dy}{dx} < 0$ M1

$(2x-5)^2(8x-5) < 0$ B1 $(2x-5)^2(8x-5)$

Since $(2x-5)^2 \geq 0$

$(8x-5) < 0$

$x < \frac{5}{8}$ A1

(b) The equation of a curve is $y = \frac{2x}{x-1} - 2x^2$ where $x > 1$. Find the value of the maximum gradient on the curve. [4]

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(x-1)2-(2x)(1)}{(x-1)^2} - 4x$$ Quotient Rule M1

$$= -\frac{2}{(x-1)^2} - 4x$$

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{4}{(x-1)^3} - 4$$

For maximum gradient, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 0$

$$\frac{4}{(x-1)^3} - 4 = 0$$ M1

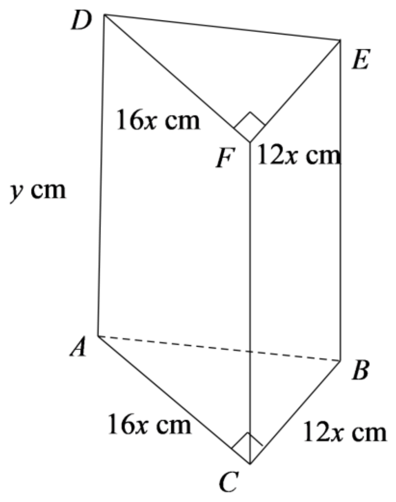
$$\frac{4}{(x-1)^3} = 4$$

$$(x-1)^3 = 1$$

$x = 2$ A1

Maximum gradient = $-2-8=-10$ A1

8



The diagram shows a solid prism with triangular base and height y cm. The triangles have right angles at C and F. The edges AC and DF are each of length $16x$ cm and the edges BC and EF are each of length $12x$ cm. The volume of the prism is 12 000 cm^3.

(a) Express y in terms of x . [2]

Volume of pyramid = $\frac{1}{2}(12x)(16x)\times y = 12000$ M1

$96x^2 y = 12000$

$y = \frac{125}{x^2}$ A1

(b) Show that the total surface area, S cm^2, is given by

$$S = 192x^2 + \frac{6000}{x}.$$ [3]

$AB = \sqrt{(12x)^2 + (16x)^2} = 20x$ B1

$S = 2\times\frac{1}{2}\times16x\times12x + (12x+16x+20x)y$ M1

$= 192x^2 + 48xy$

$= 192x^2 + 48x\times\frac{125}{x^2}$ All correct working B1

$= 192x^2 + \frac{6000}{x}$

[Turn over

Continuation of working space for question 8(b).

(c) Given that x varies, find the stationary value of S and determine its nature. [5]

$$\frac{dS}{dx} = 384x - \frac{6000}{x^2} \qquad \text{B1}$$

For stationary value of S, $\frac{dS}{dx} = 0$

$$384x - \frac{6000}{x^2} = 0 \qquad \text{M1}$$

$$384x = \frac{6000}{x^2}$$

$$x^3 = \frac{6000}{384} = \frac{125}{8}$$

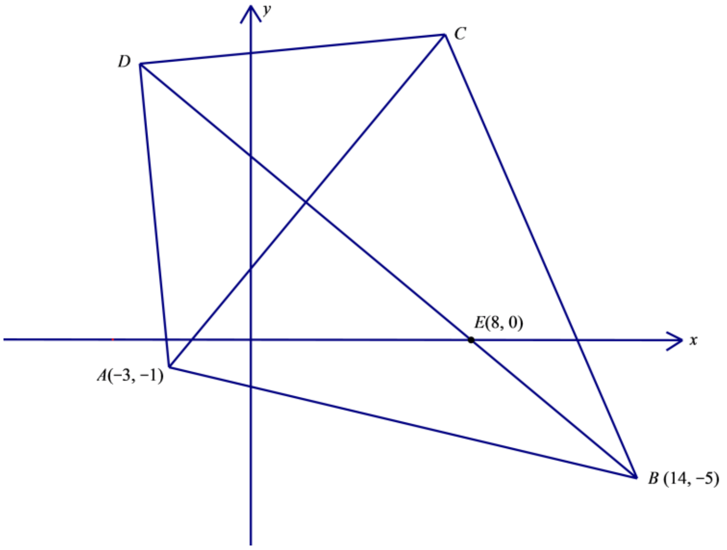
$$x = \frac{5}{2} \qquad \text{A1}$$

when $x = \frac{5}{2}$, stationary value of $S = 3600 \text{ cm}^2$ A1

$$\frac{d^2S}{dx^2} = 384 + \frac{12000}{x^3}$$

when $x = \frac{5}{2}$, $\frac{d^2S}{dx^2} = 1152 > 0$ S is a minimum A1

9



ABCD is a kite. The coordinates of *A* and *B* are $(-3, -1)$ and $(14, -5)$. The point $E(8, 0)$ lies on the diagonal of the kite.

(a) Find the equation of *AC*. [3]

gradient of $BE = \dfrac{0-(-5)}{8-14} = -\dfrac{5}{6}$ A1

gradient of $AC = -\dfrac{1}{\left(-\dfrac{5}{6}\right)}$

$= \dfrac{6}{5}$ A1

Equation of *AC* is $y+1 = \dfrac{6}{5}(x+3)$

$y = \dfrac{6}{5}x + \dfrac{13}{5}$ A1

(b) Find the coordinates of *C*. [5]

Equation of *BD* is $y-0 = -\dfrac{5}{6}(x-8)$

$y = -\dfrac{5}{6}x + \dfrac{20}{3}$ B1

For points of intersection of the 2 diagonals

$\dfrac{6}{5}x + \dfrac{13}{5} = -\dfrac{5}{6}x + \dfrac{20}{3}$ M1

$\dfrac{36x+25x}{30} = \dfrac{100-39}{15}$

$x = 2$ A1

$y = 5$

Continuation of working space for question 9(b).

Let $C = (x, y)$

midpoint of diagonals $= (2,\ 5)$

$\left(\dfrac{x-3}{2},\ \dfrac{y-1}{2}\right) = (2,\ 5)$ M1

$C = (7,\ 11)$ A1

(c) Given that $BD = 3BE$. Find the coordinates of D. [2]

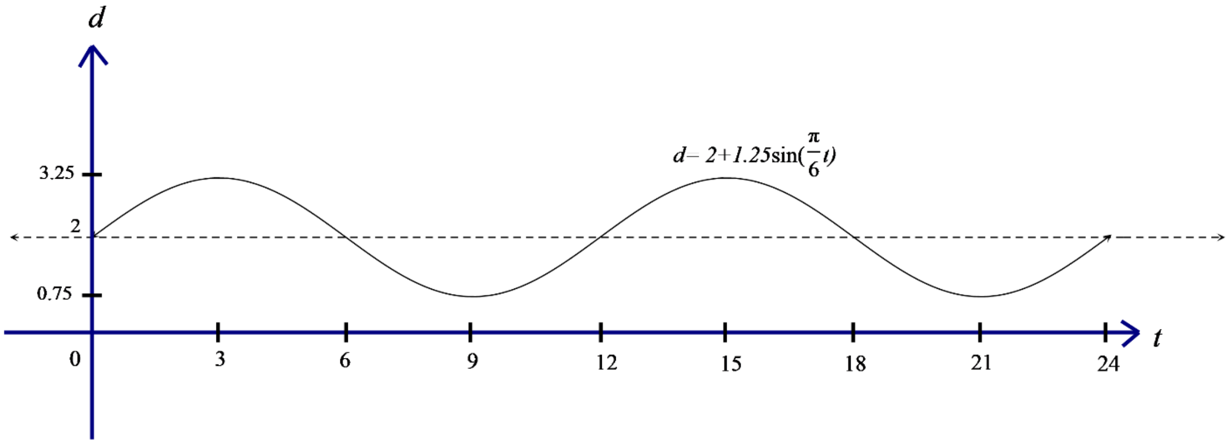
$D = (14 - 3(14-8),\ -5 + 3(5))$ M1

$= (14 - 18,\ -5 + 15)$

$= (-4,\ 10)$ A1

10 At a yacht club, the depth of water, d metres, at time t hours after 08 00 is modelled with a sinusoidal function $d = 2 + 1.25\sin\left(\frac{\pi}{6}t\right)$.

(a) Sketch the graph of d against t over a 24-hour period. [3]



10 Sailing from the yacht club is permitted only when the depth of water is at least 0.9 metres.

(b) Find the range of values of t between which sailing is **not** permitted over a 24-hour period. [4]

when $d = 0.9$

$$2 + 1.25\sin\left(\frac{\pi}{6}t\right) = 0.9$$

$$\sin\left(\frac{\pi}{6}t\right) = -\frac{1.1}{1.25} \qquad \text{M1}$$

$$\alpha = \sin^{-1}\frac{1.1}{1.25} = 1.0759$$

$$\frac{\pi}{6}t = \pi + 1.0759,\ 2\pi - 1.0759,\ 2\pi + (\pi + 1.0759),\ 2\pi + (2\pi - 1.0759) \qquad \text{M1}$$

$t = 8.05,\ 9.95,\ 20.1,\ 21.9$

For sailing to be permitted,

$8.05 < t < 9.95$ or $20.1 < t < 21.9$ A1, A1

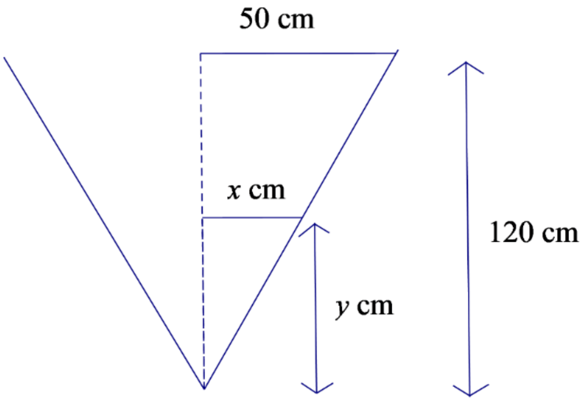
11 [The volume of a cone with base radius and vertical height h cm is given by $\frac{1}{3}\pi r^2 h$.]

In the industrial process, water flows at a constant rate of p cm^3/s into a conical funnel. The water flows out through a small hole of negligible radius in the vertex of the cone at a constant rate of q cm^3/s, where $q < p$. The axis of the cone is vertical. The volume of water in the cone at time t s is V cm^3. Initially the funnel is empty.

(a) Express $\frac{dV}{dt}$ in terms of p and q. [1]

$$\frac{dV}{dt} = p - q \qquad \text{A1}$$

The diagram shows the vertical cross-section of the funnel. The open top of the funnel is a horizontal circle of radius 50 cm which is 120 cm above the vertex.
At time t s, the water surface has radius x cm and is at a vertical height of y cm above the vertex.



(b) Express V in terms of y. [2]

Using similar triangles

$$\frac{x}{50} = \frac{y}{120}$$

$$x = \frac{5}{12}y \qquad \text{M1}$$

$$V = \frac{1}{3}\pi x^2 y$$

$$= \frac{1}{3}\pi\left(\frac{5}{12}y\right)^2 y$$

$$= \frac{25\pi y^3}{432} \qquad \text{A1}$$

[Turn over

In the case where $p = 80$ and $q = 20$,

(c) find the vertical height of the water surface after one minute, [2]

$$\frac{dV}{dt} = 80 - 20 = 60 \text{ cm}^3 / s$$

After one minute, volume of water = $60 \times 60 = 3600 \text{ cm}^3$

$$\frac{25\pi y^3}{432} = 3600 \qquad \text{M1}$$

$$y^3 = \frac{3600 \times 432}{25\pi}$$

$$y = 27.054$$

$$= 27.1 \text{ cm} \qquad \text{A1}$$

(d) **(i)** Find the rate of change of the vertical height of the water surface after one minute. [2]

Using $\frac{dV}{dt} = \frac{dV}{dy} \times \frac{dy}{dt}$

$$60 = \frac{75\pi y^2}{432} \times \frac{dy}{dt} \qquad \text{M1}$$

when $y = 12.557$

$$\frac{dy}{dt} = \frac{60 \times 432}{75\pi (27.054)^2}$$

$$= 0.150 \text{ cm/s} \qquad \text{A1}$$

(ii) Determine, with working whether this rate will increase or decrease as t increases. [2]

$$\frac{dy}{dt} = \frac{1728}{5\pi y^2} \qquad \text{B1}$$

As t increases, y increases but the rate is inversely proportional to y^2, hence the rate will decrease with time t. B1

12 **(a)** State the set of values of x for which $\sin^{-1}\left(\frac{1}{5}x\right)$ is defined. [1]

$-5 \le x \le 5$

(b) Prove the identity $\dfrac{(\cos A - \sin A)(1 + \cos A \sin A)}{\sin^3 A} = \cot^3 A - 1$ [4]

$$\text{LHS} = \frac{\cos A - \sin A + \cos^2 A \sin A - \cos A \sin^2 A}{\sin^3 A}$$ M1 expansion

$$= \frac{\cos A - \sin A + (1 - \sin^2 A)\sin A - \cos A(1 - \cos^2 A)}{\sin^3 A}$$ B1 replacing $\cos^2 A$ and $\sin^2 A$

$$= \frac{\cos A - \sin A + \sin A - \sin^3 A - \cos A + \cos^3 A}{\sin^3 A}$$

$$= \frac{\cos^3 A - \sin^3 A}{\sin^3 A}$$

$$= \frac{\cos^3 A}{\sin^3 A} - \frac{\sin^3 A}{\sin^3 A}$$ using $\frac{\cos A}{\sin A} = \cot A$ B1

$$= \cot^3 A - 1$$ B1 All correct

13 (a) Without using a calculator, find the exact value of x in the form $k\lg\frac{4}{5}$ in [3]

$$80^x \times \frac{16}{25} = 20^{3x}$$

$\frac{16}{25} = \frac{20^{3x}}{80^x}$ M1 – all terms with power x on one side

$$= \left(\frac{20^3}{80}\right)^x$$

$$= \left(\frac{8000}{80}\right)^x$$

$$= 100^x$$

$\lg\frac{16}{25} = x\lg 100$ M1 taking log

$$x = \frac{\lg\frac{16}{25}}{\lg 100}$$

$$= \frac{2\lg\frac{4}{5}}{2}$$

$=\lg\frac{4}{5}$ A1

(b) Solve the equation $\left(\log_2 y\right)^3 - \dfrac{12}{\log_{16} y^3} = 0$ [4]

$$\left(\log_2 y\right)^3 = \frac{12}{\log_{16} y^3}$$

$$= \frac{12}{\dfrac{\log_2 y^3}{\log_2 16}} \qquad \text{M1 change of base law}$$

$$= \frac{12}{3\log_2 y} \times \log_2 2^4$$

$$= \frac{4 \times 4}{\log_2 y}$$

$\left(\log_2 y\right)^4 = 16$ B1 (or equivalent)

$\log_2 y = 2$ or $\log_2 y = -2$

$y = 4$ or $y = \dfrac{1}{4}$ A1, A1

End of Paper