Hwa Chong Institution 华侨中学
Mathematics (SMTP)

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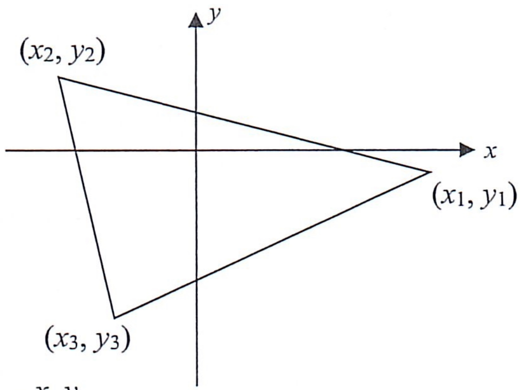
MA303.1.3 – Area of a Triangle

At the end of this unit, you should be able to

- **find the area of a triangle using the shoe-lace method**
- **use the shoe-lace method to find area of convex polygon**

We now explore an interesting method of finding the area of the triangle, given the coordinates of the three vertices. It is also known as the **shoe-lace method.**

Given the coordinates of the vertices of a triangle such that they are in an <u>anti clock-wise</u> order



The area of the triangle is $\dfrac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix}$

where $\begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix} = x_1y_2 + x_2y_3 + x_3y_1 - x_1y_3 - x_3y_2 - x_2y_1$

<u>E1</u>. Find the area of a triangle with vertices (2, 4), $(-1,-2)$ and $(-2,3)$.

$$\text{Area of triangle} = \frac{1}{2}\begin{vmatrix} 2 & -2 & -1 & 2 \\ 4 & 3 & -2 & 4 \end{vmatrix}$$

$$= \frac{1}{2}\left(6+4-4-(-8-3-4)\right).$$

$$= 10.5 \text{ units}^2$$

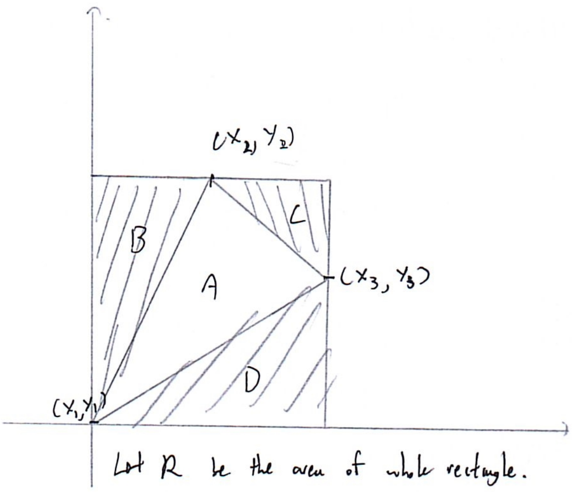
P1. Find the area of a triangle with vertices $(-2,-1)$, $(-1,-4)$ and $(3, 0)$.

$$\text{Area} = \frac{1}{2}\left|\begin{matrix} 3 & -2 & -1 & 3 \\ 0 & -1 & -4 & 0 \end{matrix}\right|$$

$$= \frac{1}{2}(-3 + 8 - 0 - 0 - 1 + 12)$$

$$= 8 \text{ units}^2$$

E2. Prove the shoe-lace formula.



Let R be the area of whole rectangle.

$A = R - B - C - D$

$$C = \frac{1}{2}(x_3 - x_2)(y_2 - y_3)$$

$$= \frac{1}{2}(x_3 y_2 + x_2 y_3) + \frac{1}{2}(x_2 y_2 - x_3 y_3)$$

$$D = \frac{1}{2}(x_3 - x_1)(y_1 - y_3)$$

$$= \frac{1}{2}(x_3 y_1 + x_1 y_2) + \frac{1}{2}(-x_1 y_1 - x_3 y_2)$$

$$A = (x_2 y_2 + x_1 y_3) - (x_3 y_2 + x_1 y_2)$$

$$= \frac{1}{2}\left|-x_1 y_2 - x_2 y_3 - x_3 y_1 + x_1 y_2 + x_2 y_3 + x_3 y_1\right|$$

$$= \frac{1}{2}\left|\begin{matrix} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \end{matrix}\right| \quad \text{(shown)}$$

E3. The shoe-lace method also works for any n-sided polygon with vertices (x_1, y_1), (x_2, y_2), (x_3, y_3), ... and (x_n, y_n) arranged in an anti-clockwise manner such that the edges do not intersect one another (simple polygon). What adjustments must you make when the coordinates are arranged in a clockwise manner instead?

The method works when the polygon is simple, whether it is a concave or convex polygon.
You have to take the negative of the answer when it is clockwise.

P2. Find the area of the polygon with vertices $(-5,-4)$, $(0,3)$, $(1,-2)$ and $(4, 0)$.

$$\text{Area of polygon} = \frac{1}{2}\left|\begin{matrix} 0 & -5 & 1 & 4 & 0 \\ 3 & -4 & -2 & 0 & 3 \end{matrix}\right|$$

$$= \frac{1}{2}(0 + 10 + 0 + 12 - 0 + 8 + 4 + 15)$$

$$= \frac{49}{2} \text{ units}^2$$