Chapter 9: Data Representation

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Syllabus Learning Outcomes

3.1 Data Representation
Understand that values can be represented in different number bases: denary, binary and hexadecimal.
- 3.1.1 Represent data in binary and hexadecimal forms.
- 3.1.2 Write programs to perform the conversion of positive integers between different number bases: denary, binary and hexadecimal forms; and display results.

1 Number systems

In this chapter, we will learn about three number systems – denary, binary and hexadecimal. Learning about number systems will allow you to understand how computers represent data and how to interpret them.

1.1 Denary numbers

The number system that we are most familiar with and typically used is called the denary number system. It is also known as the decimal number system and is made up of 10 distinct digits – 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. This is therefore a base 10 number system.

Consider a denary number 346. The table below shows the representation of the denary number using place values.

Place value	$10^2 = 100$	$10^1 = 10$	$10^0 = 1$
Digit	3	4	6
Product of digit and place value	$3 \times 10^2 = 300$	$4 \times 10^1 = 40$	$6 \times 10^0 = 6$

This gives $(3 \times 10^2) + (4 \times 10^1) + (6 \times 10^0) = 346$

1.2 Binary numbers

The number system used by computers consists of two unique digits – 0 and 1. It is called the binary number system. A binary digit is referred to as a bit. A group of eight bits is called a byte.

Humans tend to use the denary number system. Hence, denary numbers must be converted into their binary equivalent before a computer can use them.

As with a denary number, the value of a binary number is defined by place values.

Consider the binary number 101110.

Place value	$2^5 = 32$	$2^4 = 16$	$2^3 = 8$	$2^2 = 4$	$2^1 = 2$	$2^0 = 1$
Digit	1	0	1	1	1	0
Product of digit and place value	$1 \times 2^5 = 32$	$0 \times 2^4 = 0$	$1 \times 2^3 = 8$	$1 \times 2^2 = 4$	$1 \times 2^1 = 2$	$0 \times 2^0 = 0$

This gives $(1 \times 2^5) + (0 \times 2^4) + (1 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (0 \times 2^0) = 46$

1.3 Hexadecimal numbers

There is another number system, called the hexadecimal number system. This number system consists of 16 unique digits – 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E and F. The symbols A through to F represent the denary values 10 to 15.

Hexadecimal digit	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
Denary equivalent	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

The value of a number is defined by place values. Consider the hexadecimal number 2A6.

Place value	$16^2 = 256$	$16^1 = 16$	$16^0 = 1$
Digit	2	A	6
Product of digit and place value	$2 \times 16^2 = 512$	$10 \times 16^1 = 160$	$6 \times 16^0 = 6$

Adding up the values in the bottom row shows that the equivalent denary number is 678.

2 Conversion

2.1 Denary to Binary

There are two methods for converting a denary number to binary.

Method 1: division by 2

1. Divide the denary number by 2 using division with remainder and write down the quotient(result) and the remainder
2. Repeat the division until you get a result of 0
3. Write the remainders in reverse (from the last calculated remainder to the first) to get the binary number

For example, to convert the denary number 135_{10} into binary using method 1, the result would be calculated like this:

Denary	Quotient	Remainder
135 / 2	67	1
67 / 2	33	1
33 / 2	16	1
16 / 2	8	0
8 / 2	4	0
4 / 2	2	0
2 / 2	1	0
1 / 2	0	1

Therefore, $135_{10} = 10000111_2$

Method 2: place values table

1. Draw a table with the place values of each digit
2. Find the highest place value(2^n number) that is equal or less than the denary number
3. Set the digit that corresponds to that place value to 1
4. Subtract the place value from the denary number
5. Take the result of the subtraction and repeat steps 1–4 until you have a result of 0
6. Set the remaining digits to 0
7. Read the binary number from left to right

For example, to convert the denary number 135_{10} into binary using method 2, the result would be calculated like this:

For an 8-bit number, the place values table is:

128	64	32	16	8	4	2	1

The maximum place value divisor of 135 is 128 so the corresponding digit is set to 1:

128	64	32	16	8	4	2	1
1							

135 – 128 = 7. The maximum place divisor of 7 is 4, so the corresponding digit is set to 1:

128	64	32	16	8	4	2	1
1					1		

7 – 4 = 3. The maximum place divisor of 3 is 2, so the corresponding digit is set to 1:

128	64	32	16	8	4	2	1
1					1	1	

3 – 2 = 1. The maximum place divisor of 1 is 1, so the corresponding digit is set to 1:

128	64	32	16	8	4	2	1
1					1	1	1

1 – 1 = 0. The subtraction process ends, and all remaining digits are set to 0:

128	64	32	16	8	4	2	1
1	0	0	0	0	1	1	1

Hence, $135_{10} = 10000111_2$

2.2 Binary to Denary

Each digit of a binary number has a specific place value that is a power of 2.
For an 8-bit number, the place values table is:

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
128	64	32	16	8	4	2	1

To convert a binary number into denary, you add up the place values only for the binary digits that are set to 1.

Consider the binary number 11000110.

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
1	1	0	0	0	1	1	0

$$(1\times 2^7)+(1\times 2^6)+(0\times 2^5)+(0\times 2^4)+(0\times 2^3)+(1\times 2^2)+(1\times 2^1)+(0\times 2^0)=198$$

128 + 64 + 4 + 2 = 198

Its denary equivalent is 198_{10}

2.3 Hexadecimal to Binary

To convert a hexadecimal number to binary:

1. Break up the number into its individual hexadecimal digits and
2. Using the table below as a reference, replace each hexadecimal digit with its four-digit binary equivalent.
3. Read the binary number from left to right.

Hexadecimal digit	0	1	2	3	4	5	6	7
Binary equivalent	0000	0001	0010	0011	0100	0101	0110	0111
Hexadecimal digit	8	9	A	B	C	D	E	F
Binary equivalent	1000	1001	1010	1011	1100	1101	1110	1111

To convert $B03_{16}$ into binary:

Hexadecimal digit	B	0	3
Binary equivalent	1011	0000	0011

Therefore, the binary number is 101100000011_2.

2.4 Binary to Hexadecimal

To convert a binary number to its hexadecimal equivalent:

1. Split the number into groups of four binary digits, starting from the right.
2. If there is a group on the left that does not have the full set of four binary digits, add leading zeros to that group until it is a full set of four binary digits.
3. Using the table below as a reference, convert each four-digit group of binary numbers into its corresponding hexadecimal digit.

Hexadecimal digit	0	1	2	3	4	5	6	7
Binary equivalent	0000	0001	0010	0011	0100	0101	0110	0111
Hexadecimal digit	8	9	A	B	C	D	E	F
Binary equivalent	1000	1001	1010	1011	1100	1101	1110	1111

Consider converting 10 1000 0110 1011_2 to hexadecimal.

Split the binary number into groups of four digits, starting from the right

Binary	10	1000	0110	1011

Notice that the leftmost group only contains two digits, we add two leading zeros to its left to form a full set of four digits.

Binary	**00**10	1000	0110	1011
Hexadecimal	2	8	6	B

Each group of four binary digits is then converted into a hexadecimal digit.
Hence, the hexadecimal equivalent of 10 1000 0110 1011_2 is $286B_{16}$.

2.5 Denary to Hexadecimal

There are two methods for converting a denary number to its hexadecimal equivalent.

Method 1: division by 16

This method follows the same principle of repeated divisions with remainder used for converting a denary number into binary:

1. Divide the denary number by 16, and write down the result and the remainder
2. Repeat the division until you get a result of 0
3. Convert the denary remainders to their hexadecimal equivalent
4. Write the remainders in reverse (from the last calculated remainder to the first) to get the final result

For example, to convert the denary number 1899_{10} into hexadecimal using method 1, the result would be calculated like this:

Denary	Quotient	Remainder	Hexadecimal
1899 / 16	118	11	B
118/16	7	6	6
7/16	0	7	7

Therefore, 1899_{10} = $76B_{16}$

Method 2: place values table

1. Draw a table with the place values of each digit
2. Find the highest place value(16^n number) that is equal or less than the denary number
3. Find largest hexadecimal digit below this place value such that its product with this place value is equal to or less than the denary number.
4. Subtract this product from the denary number
5. Take the result of the subtraction and repeat steps 1–4 until you have a result of 0
6. Set the remaining digits to 0
7. Read the hexadecimal number from left to right starting from the first non-zero digit.

For example, to convert the denary number 1899_{10} into hexadecimal using method 2, the result would be calculated like this:

The place values table is:

Place value	65536	4096	256	16	1
	16^4	16^3	16^2	16^1	16^0

The maximum place value that is equal or less than 1899 is 256. The largest hexadecimal digit such that its product with 256 is equal to or less than 1899 is 7_{16}.

Place value	65536	4096	256	16	1
	16^4	16^3	16^2	16^1	16^0
Hexadecimal			7		

Subtract the product 7 x 16^2 = 1792 from 1899 will give 107 which is a non-zero result. Hence, we repeat the previous step.
The maximum place value that is equal or less than 107 is 16. The largest hexadecimal digit such that its product with 16 is equal to or less than 107 is 6_{16}.

Place value	65536	4096	256	16	1
	16^4	16^3	16^2	16^1	16^0
Hexadecimal			7	6	

Subtract the product 6 x 16^1 = 96 from 107 will give 11 which is a non-zero result. Hence, we repeat the previous step again.
The maximum place value that is equal or less than 11 is 1. The largest hexadecimal digit such that its product with 1 is equal to or less than 11 is 11_{10}. Its hexadecimal equivalent is B_{16}.

Place value	65536	4096	256	16	1
	16^4	16^3	16^2	16^1	16^0
Hexadecimal			7	6	B

Subtract the product 11 x 16^0 = 11 from 11 will give 0. As the result is 0, we can proceed to the last step of the algorithm. In this case, all the place values after the first non-zero hexadecimal digit are already filled.

Hence, reading the first non-zero hexadecimal digit from the left, the hexadecimal equivalent of 1899_{10} = $76B_{16}$.

Method 3: via binary

1. Convert the denary number into binary.
2. Then perform the steps for the binary-to-hexadecimal conversion

2.6 Hexadecimal to Denary

To convert a hexadecimal number to its denary equivalent:

1. Convert each hexadecimal digit to its equivalent denary number.
2. Multiply each denary number by its place value.
3. Sum the product of each denary number and its place value.

Consider converting $C7F_{16}$ to denary:

Place value	256	16	1
	16^2	16^1	16^0
Hexadecimal digit	C	7	F
Denary equivalent	12	7	15

Multiply the denary equivalent of each digit with its place value as shown in the table above and sum up the products.

$(12 \times 16^2) + (7 \times 16^1) + (15 \times 16^0) = 3199$

Hence, the denary equivalent of $C7F_{16}$ is 3199.