

NAME: _____ ()

CLASS: 3 ()



**ANGLICAN HIGH SCHOOL
SECONDARY THREE
END OF YEAR EXAMINATIONS 2024**

S3

ADDITIONAL MATHEMATICS

4049

27 September 2024

2 hours

Candidates answer on the Question Paper and Graph Paper

Additional Material: 1 sheet of Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.

Write in dark blue or black pen.

You may use a HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

If working is needed for any question, it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 80.

For Examiners' Use

For Examiners Use								
Questions	1	2	3	4	5	6	7	8
Marks								
Questions	9	10	11	12	13	14	15	
Marks								
	Units						80	
	Clarity/Logic							
	Accuracy/Precision							
	Parent's Name and Signature:							
	Date:							

This document consists of 16 printed pages.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of } \Delta = \frac{1}{2}ab \sin C$$

1 (a) Express $-2x^2 + 8x - 26$ in the form of $a(x-h)^2 + k$. [2]

(b) Hence, show that the expression $-2x^2 + 8x - 26$ is negative for all real values of x , [2]

2 The equation of a curve is $\frac{1}{x} - \frac{1}{y} = 1$.

- (i) It is given that the line $y = kx - 1$, where $k \neq 0$, is a tangent to the curve at point Q . Find the value of k . [4]

- (ii) Using this value of k , find the coordinates of Q . [2]

- (iii) Deduce the range of values of k given that $y = kx - 1$ does not intersect the curve $\frac{1}{x} - \frac{1}{y} = 1$. [2]

- 3 The polynomial $p(x) = 6x^3 + ax^2 + bx - 15$ has a factor of $2x + 3$ and leaves a remainder of 21 when divided by $(x + 2)$.

(a) Find the value of a and of b . [4]

(b) Hence, factorise the expression completely. [3]

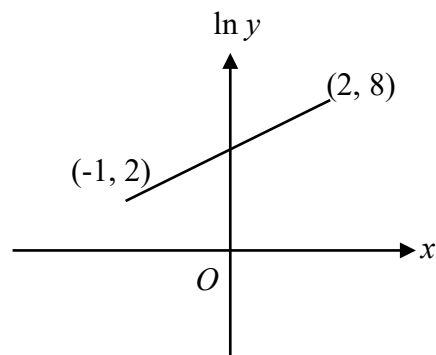
- 4 (i) Simplify the expression $\frac{\sqrt{396}}{9} - \sqrt{\frac{11}{9}}$. [2]

(ii) Hence, simplify $\frac{1}{2-\sqrt{11}} \div \left(\frac{\sqrt{396}}{9} - \sqrt{\frac{11}{9}} \right)$. [2]

5 Solve $3x-2 = \sqrt{2x^2-4x+3}$. [3]

- 6 Express $\frac{2}{x^3 - 9x^2}$ in partial fractions. [4]

- 7 The diagram shows part of a straight-line graph.



- (a) Express y in terms of x in the form $y = e^{ax+b}$. [3]

- (b) Find the value of x when $\ln y = 1.5$.

[1]

- 8 Solve the following equations.

(a) $7^{x-1} \times 3^{2x} = 5$.

[3]

(b) $\log_7(x+1) + \log_{\frac{1}{7}}(x-1) = 2 + \log_7 \frac{1}{3}$ [3]

- 9 Mr Chan makes a cup of tea and leaves it on the table to cool. The temperature of the tea, $T^{\circ}C$ is given by the equation $T = 27 + 50e^{-kx}$, where x is in minutes. The time taken for the tea to cool to $42^{\circ}C$ is 11 minutes.

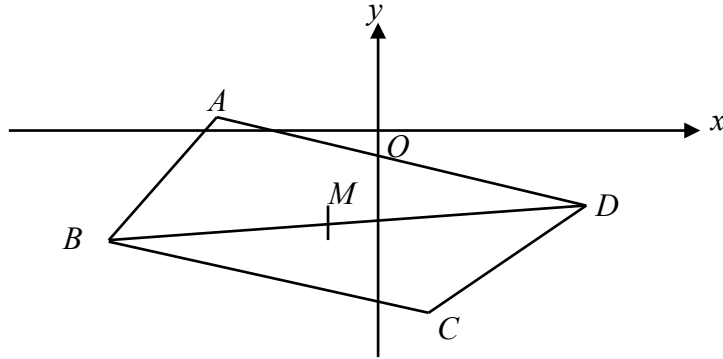
Find

- (a) the initial temperature of the tea, [1]

- (b) the time taken for the tea to cool to $35^{\circ}C$. [3]

- (c) with justification, the temperature of the tea after a long time. [1]

- 10 The diagram shows a trapezium $ABCD$ where AD is parallel to BC . The coordinates of B and C are $(-12, -6)$ and $(3, -11)$ respectively. $M\left(-\frac{3}{2}, -\frac{11}{2}\right)$ is the midpoint of BD . The line AB has a gradient of $\frac{7}{3}$. The equation of the line AD is $3y + x = -6$.



- (a) Show that the coordinates of A are $(-9, 1)$. [2]

Find

(b) the coordinates of D , [1]

(c) the coordinates of point, F , that is the foot of the perpendicular from C to AD , and [2]

(d) the area of the triangle ABC . [1]

11 The points $H(-18, 0)$, $K(5, 7)$ and $L(-10, 32)$ lie on a circle, C . The line $y = 32$ is a tangent to the circle.

(a) Find the equation of the perpendicular bisector of the chord HK . [3]

(b) Show that the coordinates of the centre of the circle are $(-10, 15)$. [1]

(c) Find the equation of the circle. [2]

12 (a) By using the general term, find the term with x^7 in the binomial expansion of $\left(x - \frac{3}{x}\right)^{15}$. [3]

(b) Hence, or otherwise, find the coefficient of x^8 in the expansion of $\left(2x + \frac{1}{x}\right)\left(x - \frac{3}{x}\right)^{15}$. [3]

- 13** Given that $\tan P = \frac{2}{5}$ and that $\cos P$ is negative, without finding the value of P , find the exact value of $\sin P$ and of $\cos P$. [3]

- 14** Given the function $y = 4 \cos x - 2$ and $0^\circ \leq x \leq 360^\circ$,
(a) State the period and amplitude of the function. [1]

- (b) Find the maximum and minimum values of y and the corresponding values of x . [3]

- (c) Sketch the graph of $y = 4 \cos x - 2$ for $0^\circ \leq x \leq 360^\circ$. [2]

- 15 The variables x and y are related by the equation $y = \frac{p}{x} + qx$, where p and q are constants. The table shows experimental values of x and of y .

x	1.10	1.20	1.40	1.60	2.00
y	-1.86	-2.67	-4.14	-5.50	-8.00

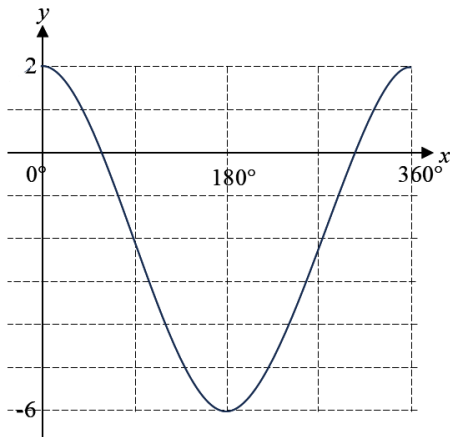
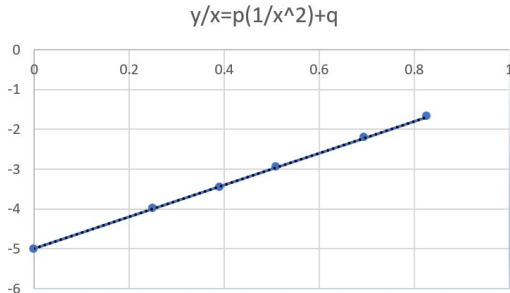
- (a) On a piece of graph paper, plot $\frac{y}{x}$ against $\frac{1}{x^2}$ to obtain a straight line graph. [3]

- (b) Use your graph to estimate the value of p and the value of q . [3]

- (c) By adding a suitable straight line, use your graph to find the value of x that satisfies the pair of simultaneous equations $y = \frac{p}{x} + qx$ and $y = -2x$. [2]

End of Paper

Answer Key

1	(a) $-2(x-2)^2 - 18$	2	(i) $k = 0$ (NA) or $k = 4$ (ii) $\left(\frac{1}{2}, 1\right)$ (iii) $0 < k < 4$
3	(a) $a = 37, b = 32$ (b) $(2x+3)(3x-1)(x+5)$	4	(i) $\frac{\sqrt{11}}{3}$ (ii) $\frac{-6\sqrt{11}-33}{77}$
5	$x = \frac{1}{7}$ (NA) or $x = 1$	6	$-\frac{2}{81x} - \frac{2}{9x^2} + \frac{2}{81(x-9)}$
7	(a) $y = e^{2x+4}$ (b) $x = -1.25$	8	(a) $x = 0.858$ (b) $x = \frac{26}{23}$
9	(a) 77°C (b) 16.7 minutes (c) 27°C	10	(b) $(9, -5)$ (c) $\left(\frac{27}{5}, -\frac{19}{5}\right)$ or $\left(5\frac{2}{5}, -3\frac{4}{5}\right)$ (d) 60 square units
11	(a) $y = -\frac{23}{7}x - \frac{125}{7}$ (c) $(x+10)^2 + (y-5)^2 = 289$	12	(a) $T_5 = 110565x^7$ (b) 208845
13	$\sin P = -\frac{2}{\sqrt{29}}$ $\cos P = -\frac{5}{\sqrt{29}}$	14	(a) Period = 360° , Amplitude = 4 (b) Maximum value of $y = 2$, $x = 0^\circ, 360^\circ$ Minimum value of $y = -6$, $x = 180^\circ$
14	(c) 	15	(a)  (b) $p = 4 \pm 0.2$, $q = 5 \pm 0.2$ (c) $x = 1.15 \pm 0.1$ ($x > 0$)