EUNOIA JUNIOR COLLEGE

JC2 Preliminary Examination 2025

General Certificate of Education Advanced Level

Higher 2

CANDIDATE NAME

CIVICS GROUP | INDEX NO.

FURTHER MATHEMATICS **9649/02**

Paper 2

15 September 2025

3 hours

Additional Materials: Printed Answer Booklet
List of Formulae (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **6** printed pages and **2** blank pages.

Section A: Pure Mathematics [50 marks]

1 Use the trapezium rule with 6 equally-spaced ordinates to approximate the value of $\int_{-1}^{1.5} \sqrt{\ln(x+2)}\,dx$, giving your answer correct to 3 significant figures. With the help of a sketch, explain whether this approximation is an overestimate or underestimate. [6]

2 By finding the equation of the tangent plane to the surface $z=\sqrt{x^2+y^2}$ at (x_0, y_0) where $(x_0, y_0) \neq (0,0)$, show that

(a) the tangent plane passes through the origin, and [4]

(b) the tangent plane makes an angle of $45°$ with the z-axis. [3]

3 A circular concert hall is modelled on the coordinate plane by the region $x^2+y^2 \leq 4$ following an appropriate scaling of real-world distances. The deviation from ideal sound intensity at any point (x, y) on the floor of the hall is given by the function

$$\mathrm{S}(x,y) = 3x^2 + y^2 - 2x$$

where $\mathrm{S}(x,y)$ is measured in decibels (dB). Positive values of S indicate that the sound is louder than ideal at that point, while negative values indicate the sound is quieter than ideal.

(a) Points where $\mathrm{S}(x,y)=0$ correspond to perfectly balanced acoustic spots, where the sound intensity is ideal. Find all such points that lie within the concert hall, that is, those satisfying $x^2+y^2 \leq 4$. Justify your reasoning. [2]

(b) Find all stationary points(s) of $\mathrm{S}(x,y)$ in the interior of the concert hall and determine the nature. [4]

(c) Find all extreme values of $\mathrm{S}(x,y)$ along the boundary of the hall. Hence, determine the absolute maximum and minimum values of $\mathrm{S}(x,y)$, and state where they occur. [5]

4 Let **A** be a 2×2 matrix where $\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 10 & 3 \end{pmatrix}$.

(a) Find the eigenvalues and eigenvectors of **A**. [5]

The sequence $\{u_0, u_1, u_2, \ldots, u_n, \ldots\}$ is given by

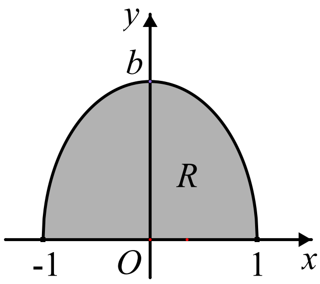
$$u_0 = 1,\ u_1 = 1, \text{ and } u_{n+1} = 3u_n + 10u_{n-1} \text{ for } n \geq 1.$$

Define $\mathbf{v}_n = \begin{pmatrix} u_n \\ u_{n+1} \end{pmatrix}$ and it can be shown that $\mathbf{v}_n = \mathbf{A}\mathbf{v}_{n-1}$.

(b) By expressing **A** in the diagonalised form $\mathbf{QDQ}^{-1}$, find an expression for u_n in terms of n. [4]

(c) Suppose $\begin{pmatrix} u_0' \\ u_1' \end{pmatrix}$ is an eigenvector of the matrix **A**, explain why the resulting sequence is geometric and how this relates to the corresponding eigenvalue. [2]

5 The diagram shows the upper-half of an ellipse $x^2 + \frac{y^2}{b^2} = 1$ where $b > 0$, and region R bounded by the part of the ellipse and x-axis.



(a) Find, in terms of b, the volume V of the solid of revolution generated by rotating region R through 2π radians about the x-axis. [2]

(b) Show that the curved surface area S of this solid of revolution is given by

$$4\pi b \int_0^1 \sqrt{1 + \left(b^2 - 1\right)x^2}\, \mathrm{d}x .$$ [3]

(c) Find the surface area S, in terms of b where applicable, when

(i) $b = 1$, and [2]

(ii) $b < 1$, using the substitution $x = \frac{\sin\theta}{\sqrt{1 - b^2}}$. [5]

When $b > 1$, it is possible to show that $S = 2\pi b^2 + \frac{2\pi b}{\sqrt{b^2 - 1}} \ln(b + \sqrt{b^2 - 1})$.

(d) In many natural and engineered systems, from soap bubbles to biological cells, the shape that minimises surface area S for a given enclosed volume V is the most efficient.

(i) On the given diagram (in the answer booklet), sketch the graph of the dimensionless isoperimetric ratio $\frac{S}{V^{\frac{2}{3}}}$, against variable length b. [2]

(ii) State the specific shape of the solid of revolution when this ratio is minimised. [1]

Section B: Probability and Statistics [50 marks]

6 The number of employees absent at a multinational company on each weekday during a particular week is tabulated as follows. The human resource manager wishes to check whether the number of absentees is independent of the day of the week.

Day of the week	Mon	Tues	Wed	Thurs	Fri	Total
Number of absentees	121	87	87	91	114	500

(a) Find the frequencies expected according to the hypothesis that the number of absentees is independent of the day of the week. [1]

(b) Test at the 5% level whether the differences in the observed and expected data are significant. [4]

7 Scientists at a catfood company are considering adding a new preservative (E999) to their product. In order to test the efficacy of E999, 12 tins were chosen at random from the production line and opened. Each tin is split into two equal portions :

- One portion is treated with E999
- The other portion is left untreated

The 24 portions are stored under identical laboratory conditions. The lengths of time for which the contents of each portion remained safe to eat were then measured and recorded in coded units. The results are tabulated below :

Tin		1	2	3	4	5	6	7	8	9	10	11	12
Length of time	Treated (E999)	35	37	23	38	36	36	27	32	35	41	29	33
	Untreated	34	35	26	34	35	33	25	31	31	37	30	29

(a) Making suitable assumptions, which should be stated, provide a 95% confidence interval, in terms of the coded units, for the population mean difference in safe lifetime between the treated and untreated portions. [5]

(b) Using the same assumptions made in **(a)**, test, at the 1% significance level, whether there is any significant evidence of an increase in the mean safe lifetime as a result of the addition of E999. [4]

8 An engineer is evaluating whether a new calibration procedure improves the accuracy of a precision instrument. For each of 15 components, the measurement error (in micrometres) is recorded before and after calibration.

The data below shows the error for each component before and after calibration:

Component	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Before	10.2	10.5	9.9	11.0	10.8	9.8	10.4	10.0	10.2	10.3	10.6	10.0	10.7	9.8	10.0
After	12.2	9.5	10.0	10.8	9.7	7.6	9.0	10.7	7.8	9.9	10.0	8.7	13.2	8.1	9.2

(a) Using the data from the 15 components above, perform a sign test at the 5% significance level to determine whether the new calibration procedure leads to a reduction in measurement error. [4]

Let W denote the Wilcoxon test statistic, defined as the sum of the ranks of the positive differences:

$$W = \sum_{i=1}^{n} X_i R_i$$

where

- R_i is the rank of the i-th paired difference,
- $X_i = 1$ if the difference is positive and $X_i = 0$ otherwise.

(b) Assuming the null hypothesis that the differences are symmetrically distributed about zero, and that there are no ties or zero differences, show that

$$\mathrm{E}(W) = \frac{1}{4}n(n+1) \qquad \text{and} \qquad \mathrm{Var}(W) = \frac{1}{24}n(n+1)(2n+1).$$ [4]

The same expectation and variance are obtained if W is defined as the sum of ranks of the negative differences.

(You are given that $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$)

The study was later expanded to include 10 additional components. Their measurement errors before and after calibration are shown below:

Component	16	17	18	19	20	21	22	23	24	25
Before	10.3	9.9	10.0	10.2	9.8	10.1	9.8	10.4	10.1	9.9
After	10.8	8.4	11.6	9.9	10.7	8.0	8.0	9.2	8.2	7.6

(c) For large sample sizes, the Wilcoxon signed rank test statistic W can be approximated by a normal distribution. Using the expressions for the expectation and variance derived in **(b)**, perform the Wilcoxon signed rank test at the 5% significance level based on the full sample of 25 components, to assess whether the calibration procedure leads to a reduction in measurement error. [4]

(d) Based on your findings, which result would you place greater confidence in when deciding whether to adopt the new calibration procedure? Justify your answer. [1]

9 The number of casualties arriving at a hospital each day is denoted by N.

(a) State two assumptions needed for N to be well modelled by a Poisson distribution. [2]

Now assume that N follows a Poisson distribution with mean 8, that is

$$\mathrm{P}(N = n) = \frac{e^{-8}8^n}{n!}, \quad n = 0,\ 1,\ 2,\ ...$$

Casualties require surgery with probability $\frac{1}{4}$ and whether a casualty requires surgery is independent of whether other casualties require surgery.

(b) Find the probability that, on a day when exactly n casualties arrive, exactly r of them require surgery, in terms of n and r. [2]

(c) Using your result in **(b),** show algebraically that the number of casualties requiring surgery each day also follows a Poisson distribution $\mathrm{Po}(2)$. [4]

(d) Given that in a particular randomly chosen week a total of 12 casulties require surgery on Monday and Tuesday, find the probability that 8 casualties require surgery on Monday. [3]

10 The random variable X has probability density function

$$\mathrm{f}(x) = \begin{cases} kx^n(1-x) & 0 \le x \le 1\ , \\ 0 & \text{otherwise}\ , \end{cases}$$

where n is an integer greater than 1.

(a) Show that $k = (n+1)(n+2)$. [3]

(b) Find $\mathrm{E}(X)$ in terms of n. [2]

(c) Find m in terms of n, where m is the mode of X. [3]

(d) You are given that, for positive x, $\left(\frac{x}{x+1}\right)^{x+1}$ is an increasing function of x.

By considering $\mathrm{P}(X \le m)$, or otherwise, show that the mode of X is greater than its median. [4]

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