Raffles Girls' School
(SECONDARY)

Established in 1879

Name: ______________________

Class: 4 / ________ Register No: ________

MATHEMATICS 2
YEAR FOUR

Pen-and-Paper Assessment 2

Wednesday **7 July 2021** **1 hour 30 minutes**

INSTRUCTIONS TO CANDIDATES

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction tape.

Answer **all** the questions in the space provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.

Omission of essential working will result in a loss of marks.

You are reminded of the need for clear presentation in your answers.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **60** and the weighting is **25%**.

For examiners' use	
Question	**Marks**
1	/ 9
2	/ 7
3	/ 5
4	/ 4
5	/ 6
6	/ 7
7	/ 11
8	/ 11
PAU	
Total Marks	60

Parent's / guardian's Name: ______________________

Signature: ______________ Date: ______________

MATHEMATICAL FORMULAE

1 ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2 TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\sin P + \sin Q = 2 \sin \frac{1}{2}(P+Q) \cos \frac{1}{2}(P-Q)$$

$$\sin P - \sin Q = 2 \cos \frac{1}{2}(P+Q) \sin \frac{1}{2}(P-Q)$$

$$\cos P + \cos Q = 2 \cos \frac{1}{2}(P+Q) \cos \frac{1}{2}(P-Q)$$

$$\cos P - \cos Q = -2 \sin \frac{1}{2}(P+Q) \sin \frac{1}{2}(P-Q)$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Differentiate the following with respect to x, leaving your answer in the simplest form.

(a) $\left(2+5x^3\right)\left(2x-1\right)^4$ [3]

(b) $\tan^4\left(\pi-5x\right)$ [3]

(c) $\dfrac{e^{3x+1}}{\sqrt[3]{4x+1}}$ [3]

2 Given that $x, y > 0$, solve the simultaneous equations

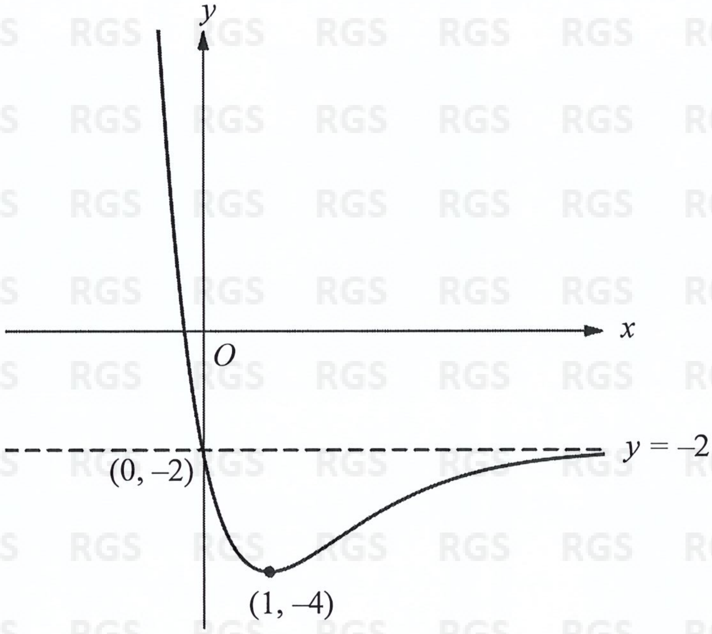
$$(\lg x)(\lg y) = 2$$

$$\lg\sqrt{\frac{xy}{10}} = 1.$$

[7]

3 Given that $xy - 2x^4 = 4\sqrt{x}$, show that $2x^2 \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} - 3x\frac{\mathrm{d}y}{\mathrm{d}x} = 3y$. [5]

4 The graph of $y = \mathrm{f}(x)$ has a horizontal asymptote, $y = -2$, a turning point at (1, –4) and intersects the y-axis at (0, –2) as shown in the diagram below.
On separate axes, sketch the graph of $y = \mathrm{f}(2x) + 4$, indicating clearly the images of the asymptote and turning point. [4]



5 The equation of a curve is $y=\sqrt{ax^4+bx}$, where a and b are constants. The curve passes through the point $P(1, 1)$ and the gradient at P is $\frac{5}{2}$.

Find the value of a and of b. [6]

6 A curve has the equation $y = 16\sin^2 x\cos^2 x\ (2\cos^2 x - 1)^2$.

(i) Show that $16\sin^2 x\cos^2 x\ (2\cos^2 x - 1)^2 = \sin^2 4x$. [1]

(ii) Find the equation of the normal to the curve at $A\left(\frac{\pi}{16}, \frac{1}{2}\right)$. [3]

(iii) A particle moves along the curve such that t seconds after it started, its coordinates are (x, y). Given that y decreases at a constant rate of 0.5 units per second, find the rate of change of x at the instant when the particle reaches A. [3]

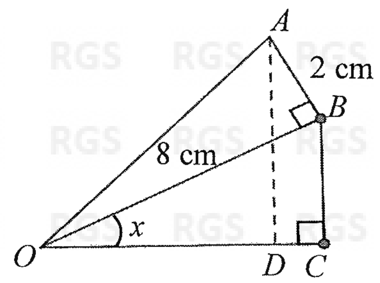
7 The diagram shows two triangles OAB and OBC with their right angles at B and C.

It is given that $AB = 2$ cm, $OB = 8$ cm, $\angle BOD = x$, where $0° \le x \le 90°$ and AD is perpendicular to OC.

(i) Show that $OD = (8\cos x - 2\sin x)$ cm and hence express it in the form $R\cos(x + \alpha)$ where $R > 0$ and $0° \le \alpha \le 90°$. [6]

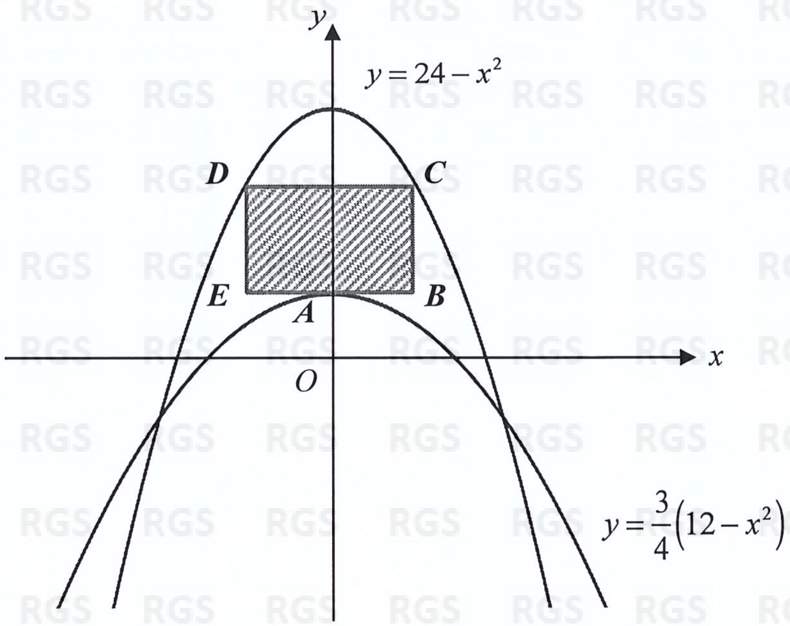
(ii) Show that the area of triangle OAD is $17\cos k(x + \alpha)$ cm^2, where k is an integer. [2]

(iii) Given that x can vary, state the maximum area of triangle OAD and the corresponding value of x. Explain why this area is the maximum. [3]



8 The parabolas with equations $y = 24 - x^2$ and $y = \frac{3}{4}\left(12 - x^2\right)$ are shown in the diagram.

A rectangle *ABCDE* is placed between the two parabolas as shown such that *C* and *D* both lie on the upper parabola, *DC* and *EAB* are parallel to the *x*-axis, and *A* is the turning point of the lower parabola that lies on the line *EB*.



(a) **(i)** If $AB = x$ units, show that the area, T unit², of rectangle *ABCDE* is given by $T = 30x - 2x^3$. [2]

(ii) Find the exact value of x for which the area of the rectangle has a stationary value. [3]

(iii) Determine whether the area of the rectangle in part (ii) is a maximum or minimum value. Support your claim with suitable working. [2]

(b) Suppose a point P lies on the upper parabola and point Q lies on the lower parabola such that both P and Q are not stationary points.

(i) If the tangent to the curve at P is parallel to the tangent to the curve at Q, find the coordinates of a ***possible*** point P and of Q. [3]

(ii) Marvin claimed that "*since the tangent at P is parallel to the tangent at Q, then the length of PQ is the distance between the two parallel lines*". Do you agree with Marvin? Without calculations, explain your answer. [1]

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