Differentiation Formula with Function $f(x)$

Given $y = f(x)$ and $\frac{dy}{dx} = f'(x)$

a) $\frac{d}{dx}(f(x))^n = n(f(x))^{n-1} \bullet f'(x)$

b) $\frac{d}{dx}(\sin f(x)) = \cos f(x) \bullet f'(x)$

c) $\frac{d}{dx}(\cos f(x)) = -\sin f(x) \bullet f'(x)$

d) $\frac{d}{dx}(\tan f(x)) = \sec^2 f(x) \bullet f'(x)$

e) $\frac{d}{dx}(\ln f(x)) = \frac{f'(x)}{f(x)}$

f) $\frac{d}{dx}(e^{f(x)}) = e^{f(x)} \bullet f'(x)$

Product Rule

Given that $y = uv$, then $\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

Quotient Rule

Given that $y = \frac{u}{v}$, then $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Tangents and Normals of a Curve

- $\left.\frac{dy}{dx}\right|_{x=a}$ = Gradient of tangent at $x = a$ on the curve
- $\left.-\frac{1}{\left(\frac{dy}{dx}\right)}\right|_{x=a}$ = Gradient of normal at $x = a$ on the curve

Increasing and Decreasing Function

- If y **increases** as x increases, then $y = f(x)$ is an **increasing** function
  Increasing function $\Rightarrow \frac{dy}{dx} > 0$
- If y **decreases** as x increases, then $y = f(x)$ is a **decreasing** function
  Decreasing function $\Rightarrow \frac{dy}{dx} < 0$

Nature of Stationary Points

If a point (x_0, y_0) is a stationary point of a curve $y = f(x)$, then $\frac{dy}{dx} = 0$ when $x = x_0$.
(i.e. the gradient to a stationary point is zero.)

Checking the Nature of Stationary Point - First Derivative Test

Given that $x = x_1$ is a stationary point.

x	$x = {x_1}^-$	$x = x_1$	$x = {x_1}^+$	Conclusion
Sign of $\frac{dy}{dx}$	$+$	0	$-$	Maximum
Sign of $\frac{dy}{dx}$	$-$	0	$+$	Minimum
Sign of $\frac{dy}{dx}$	$-$	0	$-$	Point of Inflexion
Sign of $\frac{dy}{dx}$	$+$	0	$+$	Point of Inflexion

Checking the Nature of Stationary Point - Second Derivative Test

Given that $x = x_1$ is a stationary point.

Step 1: Evaluate $\frac{d^2y}{dx^2}$ when $x = x_1$

$\frac{d^2y}{dx^2}$ when $x = x_1$	> 0	Minimum
$\frac{d^2y}{dx^2}$ when $x = x_1$	< 0	Maximum
$\frac{d^2y}{dx^2}$ when $x = x_1$	$= 0$	Inconclusive

Note: If Second Derivative Test is Inconclusive, then the First Derivative Test must be used to conclude the nature of the stationary point.

Application - Increasing and Decreasing Function

- If y **increases** as x increases, then $y = f(x)$ is an **increasing** function
  Increasing function $\Rightarrow \frac{dy}{dx} > 0$
- If y **decreases** as x increases, then $y = f(x)$ is a **decreasing** function
  Decreasing function $\Rightarrow \frac{dy}{dx} < 0$

Integration Formula

1. If $n \neq -1$, $\int x^n \, dx = \frac{1}{n+1} x^{n+1} + c$, where c is a constant.
2. If $n = -1$, $\int \frac{1}{x} dx = \ln x + c$, where c is a constant.
3. $\int \sin x \, dx = -\cos x + c$, where c is a constant.
4. $\int \cos x \, dx = \sin x + c$, where c is a constant.
5. $\int \sec^2 x \, dx = \tan x + c$, where c is a constant.
6. $\int e^x \, dx = e^x + c$, where c is a constant.
7. $\int \cos(ax + b) \, dx = \frac{1}{a} \sin(ax + b) + c$, where c is a constant.
8. $\int \sin(ax + b) \, dx = -\frac{1}{a} \cos(ax + b) + c$, where c is a constant.
9. $\int \sec^2(ax + b) \, dx = \frac{1}{a} \tan(ax + b) + c$, where c is a constant.
10. $\int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + c$, where c is a constant.
11. $\int (ax + b)^n \, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$, where c is a constant.
12. $\int (ax + b)^{-1} \, dx = \frac{\ln(ax+b)}{a} + c$, where c is a constant.