Topical Revision Worksheet 4

Topic: Polynomials

Question 1 [2008 EOY, Q11]	marks
(i) **(a)** Show that $2x+1$ is a factor of $2x^3+x^2-8x-4$. **Solution:** Sub $x=-\frac{1}{2}$: $2\left(-\frac{1}{8}\right)+\frac{1}{4}-8\left(-\frac{1}{2}\right)-4=-\frac{1}{4}+\frac{1}{4}+4-4=0$ Hence $2x+1$ is a factor of $2x^3+x^2-8x-4$	[1]
(i) **(b)** Solve for *x* if $2x^3+x^2-8x-4=0$. **Solution:** By long division, $(2x+1)(x^2-4)=0$ Hence $(2x+1)(x^2-4)=0 \Rightarrow x=-\frac{1}{2}$ or ± 2	[4]
(ii) The expression $4x^3-20x^2+px+12$ leaves a remainder of $-10x+q$ when divided by $2x^2-3x+1$. Find the values of *p* and *q*. **Solution:** $4x^3-20x^2+px+12 \equiv (2x^2-3x+1)Q(x)-10x+q$ $4x^3-20x^2+px+12 \equiv (2x-1)(x-1)Q(x)-10x+q$ Sub $x=\frac{1}{2}$, $4\left(\frac{1}{8}\right)-20\left(\frac{1}{4}\right)+\frac{1}{2}p+12=-10\left(\frac{1}{2}\right)+q \Rightarrow p=2q-25$ Sub $x=1$, $4-20+p+12=-10+q \Rightarrow p=-6+q$ Hence $-6+q=2q-25 \Rightarrow q=19$ and $p=13$	[5]

Question 2 [2010 EOY, Q12]	marks
(a) Solve for x when $6x^3+7x^2-14x-8=0$. **Solution:** By trial and error, when $x=-2$, $6x^3+7x^2-14x-8=0$ hence $(x+2)$ is a factor. By long division, $6x^3+7x^2-14x-8=(x+2)(6x^2-5x-4)$ Hence, $x=-2$ or $x=\frac{5\pm\sqrt{25-4(-24)}}{12}$ [M1] $=-\frac{1}{2}$ or $\frac{4}{3}$	[3]
(b) It is given that $12x^3+ax^2+15x+b\equiv(2x^2+5x-3)Q(x)+158x-83$ where $Q(x)$ is a polynomial. (i) State the degree of $Q(x)$. **Solution:** Degree of $Q(x)$ is 1 as it is linear	[1]
(ii) Find the values of a and b. **Solution:** $12x^3+ax^2+15x+b\equiv(2x^2+5x-3)Q(x)+158x-83$ $12x^3+ax^2+15x+b\equiv(2x-1)(x+3)Q(x)+158x-83$ Sub $x=\frac{1}{2}$ to get $a+4b=-52$ Sub $x=-3$ to get $9a+b=-188$ Solve simultaneously to get $a=-20$ and $b=-8$	[3]

Question 3 [2011 EOY, Q11]	marks
(i) Given that $f(x)=x^3-2\sqrt{3}x^2+2(A-x)$ has a factor $\left(x+\sqrt{2}\right)$ and the other two roots of the equation $f(x)=0$ are $\sqrt{2}$ and k. Find the value of A and of k. **Solution:** $f\left(-\sqrt{2}\right)=0$ $\left(-\sqrt{2}\right)^3-\left(2\sqrt{3}\right)\left(-\sqrt{2}\right)^2+2\left(A+\sqrt{2}\right)=0$ $-2\sqrt{2}-4\sqrt{3}+2A+2\sqrt{2}=0$ $A=2\sqrt{3}$ $f(x) \quad =x^3-2\sqrt{3}x^2+2\left(2\sqrt{3}-x\right)$ $\quad\quad =x^3-2\sqrt{3}x^2-2x+4\sqrt{3}$ $f(x)=\left(x-\sqrt{2}\right)\left(x+\sqrt{2}\right)(x-k)$ $2k=4\sqrt{3} \Rightarrow k=2\sqrt{3}$	[4]
(ii) Solve the equation $3x^3=2x^2+7x+2$. **Solution:** $3x^3-2x^2-7x-2=0$ Found 1 linear factor Correct factorization into linear and quadratic e.g. $(x-2)\left(3x^2+4x+1\right)=0$ Correct factorization into 3 linear factors $(x-2)(3x+1)(x+1)=0$ $x=2,-\dfrac{1}{3},-1$	[4]

Question 4 [2012 EOY, Q8]	marks
The cubic polynomial $f(x)$ is such that the coefficient of x^3 is 2 and the roots of $f(x)=0$ are -1, $2k$ and k^2. It is given that $f(x)$ has a remainder of 168 when divided by $x-2$. i) Show that $k^3-k^2-2k-12=0$. **Solution:** $f(x)=2(x+1)(x-2k)(x-k^2)$ *Ability to express f as a product of three linear factors and a constant 2.* $f(2)=2(3)(2-2k)(2-k^2)=168$ $4-2k^2-4k+2k^3=28$ $k^3-k^2-2k-12=0$	[3]
(ii) Hence find a value for *k* and show that there are no other real values of *k* which satisfy this equation. **Solution:** $f(k)=k^3-k^2-2k-12$ $f(3)=3^3-3^2-6-12=0$ *By trial and error, k = 3* $f(k) \quad =(k-3)(ak^2+bk+c)\ M1$ $=(k-3)(k^2+2k+4)\quad M1$ $k^2+2k+4=(k+1)^2+3>0$ *Hence, there is only one real value of k.*	[4]
(iii) Use your answer to part **(ii)** to solve the equation $12e^{-3x}+2e^{-2x}+e^{-x}-1=0$. **Solution:** $1-e^{-x}-2e^{-2x}-12e^{-3x}=0$ $e^{3x}-e^{2x}-2e^{x}-12=0$ *Let* $e^{x}=3 \Rightarrow x=\ln 3$	[2]

Question 5 [2013 EOY, Q5]	marks
Given that x^2+x-6 is a factor of $2x^4 + x^3 - ax^2 + bx + a + b = 1$, find the value of a and of b. **<u>Solution:</u>** $x^2+x-6=(x-2)(x+3)$ Let $x=2$ and subs into the eqn. $\Rightarrow \quad a-b=13 \quad ---- (1)$ Let $x=-3$ and subs into the eqn. $\Rightarrow \quad 4a+b=67 \quad ---- (2)$ Solve the above two equations, $a=16$ and $b=3$	[4]
Question 6 [2014 EOY, Q7]	marks
(i) Solve the equation $2x^3=5x^2-x-2$. **<u>Solution:</u>** $2x^3=5x^2-x-2$ $2x^3-5x^2+x+2=0$ Let $\mathrm{f}(x)=2x^3-5x^2+x+2$ $\because \mathrm{f}(1)=0, \ \therefore (x-1)$ is a factor of $\mathrm{f}(x)$. Using long division, $(x-1)(2x^2-3x-2)=0$ $(x-1)(2x+1)(x-2)=0$ $x=1,-\frac{1}{2}$ or 2	[4]
(ii) Find the remainder when $(x-3)^{200}-(x-2)^{100}$ is divided by x^2-5x+6. **<u>Solution:</u>** Let $\mathrm{f}(x)=(x-3)^{200}-(x-2)^{100}$. When divided by x^2-5x+6, $\mathrm{f}(x)=(x^2-5x+6)\,\mathrm{Q}(x)+ax+b$, where a and b are real constants. $\mathrm{f}(x)=(x-2)(x-3)\,\mathrm{Q}(x)+ax+b$ By Remainder Theorem, $\mathrm{f}(2)=(2-3)^{200}-(2-2)^{100}=1$ $\Rightarrow 2a+b=1$ $\mathrm{f}(3)=(3-3)^{200}-(3-2)^{100}=-1$ $\Rightarrow 3a+b=-1$ Solving the above two equations, $a=-2, b=5$ $\therefore$ The remainder is $-2x+5$.	[4]

Question 7 [2015 EOY, Q9]	marks
(i) Solve for x when $2(e^{3x}+2)=e^{x}(7e^{x}+5)$. **Solution:** $2(e^{3x}+2)=e^{x}(7e^{x}+5)\Rightarrow 2y^{3}-7y^{2}-5y+4=0$ where $y=e^{x}$ $y=-1$ is a root so $(y+1)(2y-1)(y-4)=0\Rightarrow e^{x}=-1$ (rej.) or $\frac{1}{2}$ or 4 Hence $x=-\ln 2$ or $\ln 4$	[5]
(ii) Given that $f(x)=8x^{4}+px^{3}-30x^{2}+x+q$, where p and q are unknown constants, leaves a remainder of 2 when divided by $x-2$ and a remainder of 9 when divided by $2x+3$, find the values of p and q and hence find the remainder when $f(x)$ is divided by $2x^{2}-x-6$. **Solution:** $f(2)=2\Rightarrow 8+8p+q=0$ and $f\left(-\frac{3}{2}\right)=9\Rightarrow -27p+8q=300$ Solving, $p=-4$ and $q=24$ Hence $f(x)=8x^{4}-4x^{3}-30x^{2}+x+24$ By long division, remainder is $6-2x$	[6]