Revision for Common Test

1 A polynomial $\mathrm{p}(x)$ is of degree 3 such that $x=-1$ and $x=1$ are two of the roots for $\mathrm{p}(x)=0$. If given further that $\mathrm{p}(3)=4\mathrm{p}(2)$, show that x is a factor of $\mathrm{p}(x)$.

2 The function f is defined such that $\mathrm{f}(x)=|x-1|+2$.

(a) Sketch the graph of $y=\mathrm{f}(x)$ for $-1 \leq x \leq 4$.

(b) Hence, or otherwise, solve for x when $\mathrm{f}(x)<\mathrm{g}(x)$, where $\mathrm{g}(x)=\dfrac{11-x}{3}$.

(c) Sketch on a separate graph, the function $\mathrm{h}(x)=|x-1|+x$.

3 It is given that $px^3-x^2+18x-7 \equiv (2x-1)(x+3)(k-x)+mx-1$ for some real constants p, k, and m.

(a) Explain why p must be -2.

(b) Find the values of k and m.

4 Show that both $x-\sqrt{2}$ and $x+\sqrt{2}$ are factors of $\mathrm{g}(x)=2x^3-8x^2-4x+16$.

(a) Factorise $\mathrm{g}(x)$ completely and sketch the graph of $y=\mathrm{g}(x)$, showing clearly the intersections with the axes.

(b) Solve the inequality $x^3+8>2x(2x+1)$.

5 A polynomial $\mathrm{P}(x)$ is such that $\mathrm{P}(x)=px^4+4x^3-3x^2+qx-5$. When $\mathrm{P}(x)$ is divided by x^2-1 the remainder is $5x-6$.

(a) Find the values of p and q.

(b) Solve for x when $\mathrm{P}(x)=2x^4+7x-10$.

6 Solve the simultaneous equations $\mathrm{e}^x(\mathrm{e}^2)=(\mathrm{e}^2)^y$ and $\lg(x^2-1)=\lg y+\lg(2x-y)$.

7 Solve for x when:

(i) $6^x = 5^{x+1}$

(ii) $\log_2(x+6) + \log_2(x-1) = 1 + \log_2(3x)$

8 It is given that $22x^3 + 15x^2 - 4x - 6$ and $2 + 14x - 12x^2 - 5x^3$ have the same remainder when divided by $x - a$.

(a) Show that $27a^3 + 27a^2 - 18a - 8 = 0$.

(b) By substituting $3a$ by b, or otherwise, solve the above equation in a.

9 It is given that $\mathrm{f}(x) = ax^3 - (a+3b)x^2 + 2bx + c$ is exactly divisible by $x^2 - 2x$. When $\mathrm{f}(x)$ is divided by $x - 1$, the remainder is 8 more than when it is divided by $x + 1$.

(a) Find the values of a, b and c. Hence factorise $\mathrm{f}(x)$ completely.

(b) Solve for x, when $\mathrm{f}(x) > 0$.

10 A function f is defined by $\mathrm{f} : x \mapsto \dfrac{kx+1}{x}$, $x \neq 0$, where k is a positive real constant.

(a) Sketch the graph of $y = \mathrm{f}(x)$.

(b) State the range of f.

(c) Find an expression for $\mathrm{f}^{-1}(x)$, in terms of k, and state the domain of f^{-1}.

11 A function f is defined by $\mathrm{f} : x \mapsto \mathrm{e}^{-2x} + k$, where k is a positive real constant.

(a) Sketch the graph of $y = \mathrm{f}(x)$.

(b) State the range of f.

(c) Find an expression for $\mathrm{f}^{-1}(x)$, in terms of k, and state the domain of f^{-1}.

12 Solve for x for each of the following:

(i) $\ln(\lg x) = -0.4$

(ii) $\log_3(x-2) = \log_9 729 - \log_3(x+4)$

(iii) $\log_9[2+\log_2(x-4)] = \log_{25} 5$

13 The mass, m grams, of a radioactive substance X produced as a result of an experiment, present at time t hours after first being observed, is given by the formula $m = 16e^{-0.4t}$.

(a) Sketch the graph of m against t.

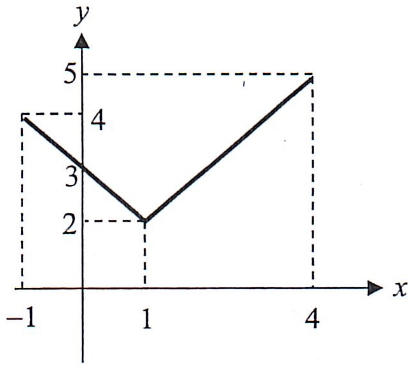
(b) Determine the amount of radioactive substance left 5 hours after first being observed.

(c) Determine, also, the time taken, to the nearest 0.1 hour, for the radioactive substance to be reduced to 5% of the original amount.

14 Express $\ln\left(3e^2\right) - \frac{1}{2}\ln\left(\frac{9}{e}\right) - \ln\left(\frac{\sqrt{e}}{5}\right)$ in the form $r + \ln s$ where r and s are rational numbers.

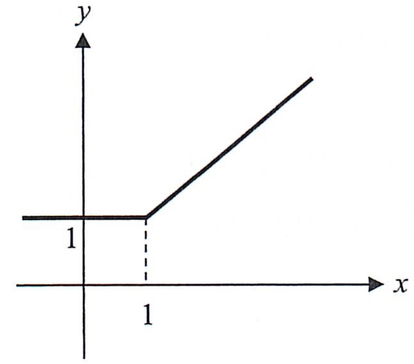
Answer Key

2 **(a)**



(b) $-1 < x < 2$

(c)



3 **(b)** $k = 2$, $m = 5$

4 **(a)** $g(x) = 2(x^2 - 2)(x - 4)$ **(b)** $-\sqrt{2} < x < \sqrt{2}$ or $x > 4$

5 **(a)** $p = 2$, $q = 1$ **(b)** $x = 1$ or $x = -\frac{5}{4}$

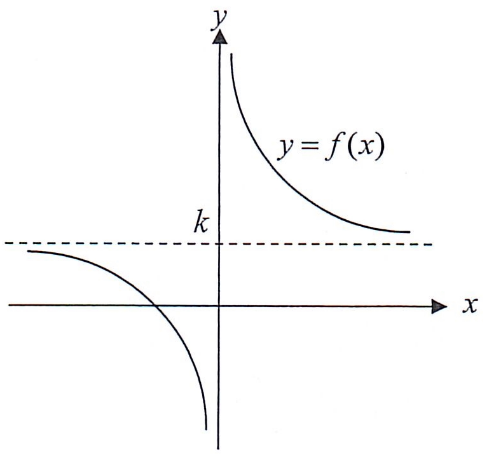
6 $x = 4$, $y = 3$

7 **(i)** $x = 8.83$ **(ii)** $x = 3$

8 **(b)** $a = -\frac{1}{3}$, $-\frac{4}{3}$ or $\frac{2}{3}$

9 **(a)** $a = 2$, $b = 1$, $c = 0$ **(b)** $0 < x < \frac{1}{2}$ or $x > 2$

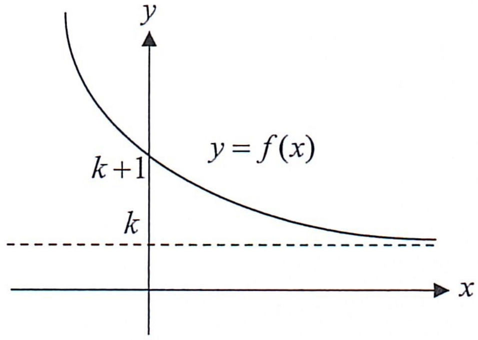
10 **(a)**



(b) $f(x) \neq k$

(c) $f^{-1}(x) = \dfrac{1}{x-k}$, domain is $x \neq k$

11 **(a)**



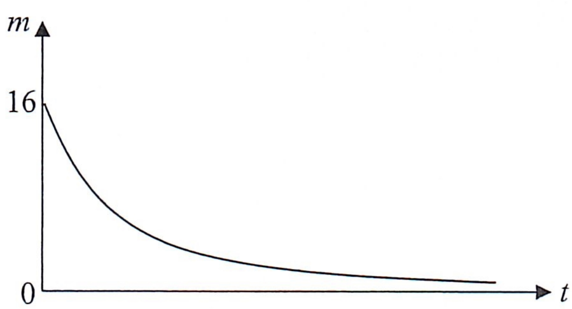
(b) $f(x) > k$

(c) $f^{-1}(x) = -\dfrac{1}{2}\ln(x-k)$, domain is $x > k$

12 **(i)** $x = 4.68$ (3 s.f.) **(ii)** $x = 5$ ($x = -7$ is rejected)

(iii) $x = 6$

13 **(a)**



(b) 2.17 grams (3 s.f.)

(c) 7.5 hours (nearest 0.1 hr)

14 $2 + \ln 5$