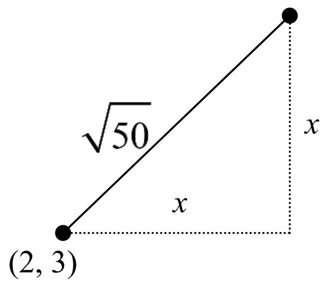
2024 Sec 3 E Math Practice Paper A Solution

Qn	**Solutions**
1(i)	x is negative
1(ii)	$x = 8a\sqrt{bx^2+1}$ $x^2 = 64a^2\left(bx^2+1\right)$ $\left(1-64a^2b\right)x^2 = 64a^2$ $x = -\sqrt{\dfrac{64a^2}{1-64a^2b}}$ (reject +ve since $x<0$) OR $= \dfrac{8a}{\sqrt{1-64a^2b}}$ (since $a<0$)
2(a)	$9a^2-12ab+4b^2-6a+4b$ $=(3a-2b)^2-2(3a-2b)$ $=(3a-2b)(3a-2b-2)$
2(b)	$\dfrac{1}{3(x-2)}-\dfrac{1}{2(x+2)}-\dfrac{3}{(x-2)^2}$ $=\dfrac{2(x-2)(x+2)-3(x-2)^2-18(x+2)}{6(x-2)^2(x+2)}$ $=\dfrac{2x^2-8-3x^2+12x-12-18x-36}{6(x-2)^2(x+2)}$ $=\dfrac{-x^2-6x-56}{6(x-2)^2(x+2)}$
2(c)	$\left(a^3+b^3\right)^2=\left(a^2+b^2\right)^3$ $a^6+2a^3b^3+b^6=a^6+3a^4b^2+3a^2b^4+b^6$ $3a^4b^2+3a^2b^4=2a^3b^3$ $3a^2+3b^2=2ab$ $\dfrac{a^2+b^2}{ab}=\dfrac{2}{3}$

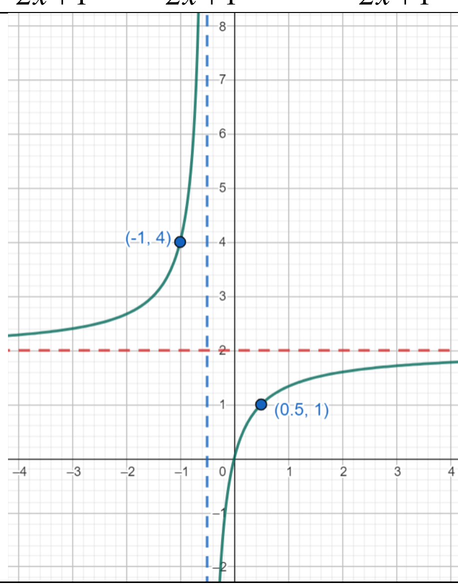
3	$$\frac{\left(9^{\frac{2}{3}}x^{\frac{2}{3}}\right)^9}{\left(27^{\frac{8}{3}}x^{\frac{4}{3}}\right)^6}=\left(\frac{1}{81}x^{\frac{1}{m}}\right)^k$$ $$\frac{\left(3^{\frac{4}{3}}x^{\frac{2}{3}}\right)^9}{\left(3^8x^{\frac{4}{3}}\right)^6}=\left(3^{-4}x^{\frac{1}{m}}\right)^k$$ $$\frac{3^{12}x^6}{3^{48}x^8}=3^{-4k}x^{\frac{k}{m}}$$ $$3^{-36}x^{-2}=3^{-4k}x^{\frac{k}{m}}$$ $k=9;\ m=-4.5$
4	$-x-10<3x-2$ AND $3x-2\le 5-\frac{3x-4}{3}$ $-8<4x$ AND $9x-6\le 15-3x+4$ $x>-2$ AND $x\le 2\frac{1}{12}$ i.e. $-2<x\le 2\frac{1}{12}$ Thus, largest integer is 2.
5	$x-2y=3$ $3x^2-xy+y^2=5$ Sub. $x=2y+3$ into $3x^2-xy+y^2=5$ $3(2y+3)^2-(2y+3)y+y^2=5$ $27+36y+12y^2-3y-2y^2+y^2-5=0$ $11y^2+33y+22=0$ $y^2+3y+2=0$ $(y+1)(y+2)=0$ $y=-2$ or $y=-1$ $x=-1$ or $x=1$ Points of intersection: $(-1,-2)$ and $(1,-1)$

6a	$\mathbf{Q}-\mathbf{P}=\begin{pmatrix}1 & 0\\ 4+2y & 1\end{pmatrix}-\begin{pmatrix}1 & x\\ y & 0\end{pmatrix}$ $=\begin{pmatrix}0 & -x\\ 4+y & 1\end{pmatrix}$ $\mathbf{P}^2=\begin{pmatrix}1 & x\\ y & 0\end{pmatrix}\begin{pmatrix}1 & x\\ y & 0\end{pmatrix}$ $=\begin{pmatrix}1+xy & x\\ y & xy\end{pmatrix}$
6b(i)	$\begin{pmatrix}10 & 24\\ 12 & 20\\ 8 & 18\end{pmatrix}\begin{pmatrix}40 & 30\\ 60 & 50\end{pmatrix}$ $=\begin{pmatrix}1840 & 1500\\ 1680 & 1360\\ 1400 & 1140\end{pmatrix}$
6b(i)	Alternatively: $\begin{pmatrix}40 & 60\\ 30 & 50\end{pmatrix}\begin{pmatrix}10 & 12 & 8\\ 24 & 20 & 18\end{pmatrix}$ $=\begin{pmatrix}1840 & 1680 & 1400\\ 1500 & 1360 & 1140\end{pmatrix}$
6b(ii)	$\begin{pmatrix}1840 & 1500\\ 1680 & 1360\\ 1400 & 1140\end{pmatrix}\begin{pmatrix}6\\ 10\end{pmatrix}$ $=\begin{pmatrix}26040\\ 23680\\ 19800\end{pmatrix}$
6b(ii)	Alternatively: $\begin{pmatrix}6 & 10\end{pmatrix}\begin{pmatrix}1840 & 1680 & 1400\\ 1500 & 1360 & 1140\end{pmatrix}$ $=\begin{pmatrix}26040 & 23680 & 19800\end{pmatrix}$
7a(i)	$\cos\angle ABC=\dfrac{40^2+35^2-70^2}{2\times40\times35}$ $\angle ABC=137.8°$ (1 d.p)
7a(ii)	Area of $\Delta ABC=\dfrac{1}{2}\times35\times40\times\sin137.822°$ $= 470\text{ m}^2$ (3 s.f)

7(b)	Let x be the difference of height between the two poles, therefore $\tan 3.6° = \dfrac{x}{40}$ $x = 40\tan 3.6°$ Height of pole at A = $5 + 40\tan 3.6°$ m $= 7.5$ m (1 d.p)
7(c)	BH is the shortest distance from B to path AC $\dfrac{1}{2}\times 70\times BH = \dfrac{1}{2}\times 35\times 40\times \sin 137.822°$ $BH = 13.4285$ m Let largest angle of depression be θ, $\tan\theta = \dfrac{3}{13.4285}$ $\theta = 12.6°$ (1 d.p)
7(d)	$ZC = \dfrac{2}{3}AC$ $\dfrac{\text{Area } \Delta ZCB}{\text{Area } \Delta ABC} = \dfrac{2}{3}$ Probability = $\dfrac{2}{3}$
8(i)	Centre of circle, $C = (2, 3)$ Radius $r = \sqrt{2^2 + 3^2 + 23}$ $= 6$ units
8(ii)	$x^2 + y^2 - 4x - 6y - 23 = 0$ At $x = -4$, $16 + y^2 + 16 - 6y - 23 = 0$ $y^2 - 6y + 9 = 0$ $(y-3)^2 = 0$ Repeated root of $y = 3$ $\Rightarrow$ line $x = 4$ is a tangent to the circle, C. or Discriminant $= 36 - 4(1)(9) = 0$ $\Rightarrow x = 4$ is a tangent to the circle, C. Alternatively, the distance from $x = -4$ to $(2,3)$ is 6 units, which is equal to the radius. Therefore $x = -4$ is a tangent line.

8 (iii)	 $x^2 + x^2 = 50$ (Pythagoras Theorem) $x = 5$ Centre of new circle = $(7,8)$ $(x-7)^2 + (y-8)^2 = 36$ $x^2 - 14x + 49 + y^2 - 16y + 64 - 36 = 0$ $x^2 + y^2 - 14x - 16y + 77 = 0$
9(i)	Gradient of line = 3 Equation of line $Y = 3X + c$ and (1, 2) lies on the line, $2 = 3 + c$ $c = -1$ Hence, the equation of the line: $\lg y = 3 \lg (x + 1) - 1$ $\lg y = 3 \lg (x + 1) - \lg 10$ $y =$
9(ii)	$\tan \theta = 3$ $\theta = 1.25$ (3 s.f)
10 (i)	$\angle OXY = \cos^{-1}\left(\frac{3.1}{4}\right)$ $= 0.684$ (3 s.f)
10(ii)	Area of sector XYZ $= \frac{1}{2}(6.2)^2 (\cos^{-1}\left(\frac{3.1}{4}\right) \times 2)$ cm^2 $= 26.3$ cm^2 (3 s.f)

10(iii)	$\angle YOZ = 2\times 2\times \cos^{-1}\left(\frac{3.1}{4}\right)$ $\angle$ at centre of circle $= 2\angle$ at circumference $YQZ = r\theta$ $= (4)\left(4\cos^{-1}\left(\frac{3.1}{4}\right)\right)$ cm $= 10.9$ cm
11(i)	Mid-point PQ = $\left(\frac{-9-1}{2}, \frac{9-3}{2}\right) = (-5, 3)$ Gradient PQ = $\frac{9+3}{-9+1} = \frac{3}{-2}$ $\therefore$ grad $\perp$ bisector=$\frac{2}{3}$ $\frac{y-3}{x+5} = \frac{2}{3}$ Eqn of $\perp$ bisector is $y-3 = \frac{2}{3}(x+5)$ $y = \frac{2}{3}x + \frac{19}{3}$
11(ii)	Eqn of QR is $y = 8x+5$ Let R$(x, 8x+5)$ $8x+5 = \frac{2}{3}x + \frac{19}{3}$ $24x+15 = 2x+19$ $22x = 4$ $x = \frac{4}{22} = \frac{2}{11}$ $\therefore y = 8\left(\frac{2}{11}\right)+5 = \frac{16}{11}+5$ $= \frac{71}{11}$ $= 6\frac{5}{11}$ $\therefore R\left(\frac{2}{11}, 6\frac{5}{11}\right)$

11(iiii)	Let $S(x, y)$ Mid-pt *RS*=mid-pt *PQ* $\left(\dfrac{x+\frac{2}{11}}{2}, \dfrac{y+\frac{71}{11}}{2}\right)=\left(\dfrac{-9-1}{2}, \dfrac{9-3}{2}\right)$ $x+\dfrac{2}{11}=-10$ and $y+\dfrac{71}{11}=6$ $x=-10\dfrac{2}{11}$ and $y=-\dfrac{5}{11}$ Hence, *S* is $\left(-10\dfrac{2}{11}, -\dfrac{5}{11}\right)$.
11(iv)	Eqn of line *l* is $33y-22x-867=0$ $33y-22x-867=0$ $y=\dfrac{22}{33}x+\dfrac{867}{33}$ $y=\dfrac{2}{3}x+\dfrac{289}{11}$ Gradient of *l* = Gradient of of $\perp$ bisector and the lines have different *y*-intercept, hence the two lines do not intersect. (two lines are parallel and distinct lines)
12 (a)	$\dfrac{4x}{2x+1}=\dfrac{2(2x+1)-2}{2x+1}=2-\dfrac{2}{2x+1}$
12 (b)	
12 (c)	$II: y=6\lg(x-2) ----> y_1=3\lg(x-2)$ $I: y_1=3\lg(x-2) ----> y_2=3\lg(x-2+4)$ $h(x)=3\lg(x+2)$