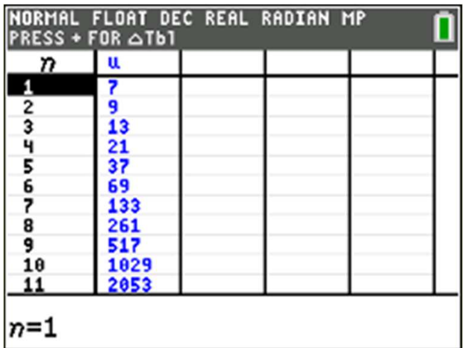
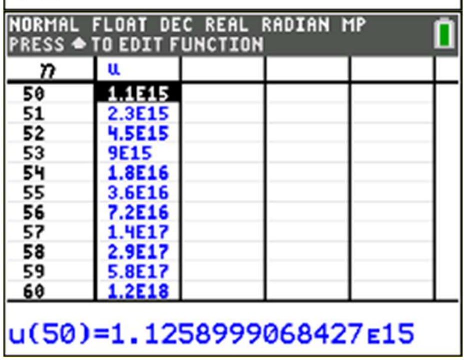
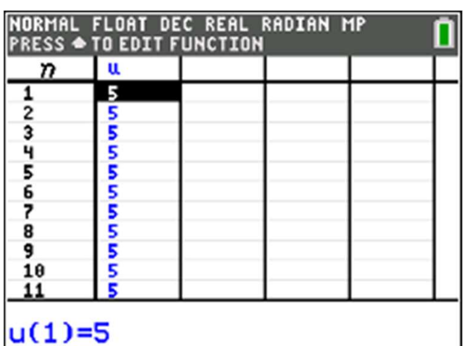
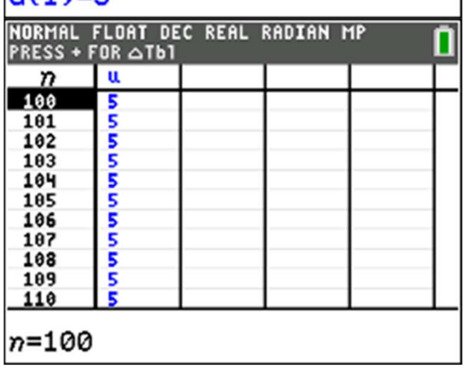
Raffles Institution
H2 Mathematics (9758)
Solution for 2020 A-Level Paper 2

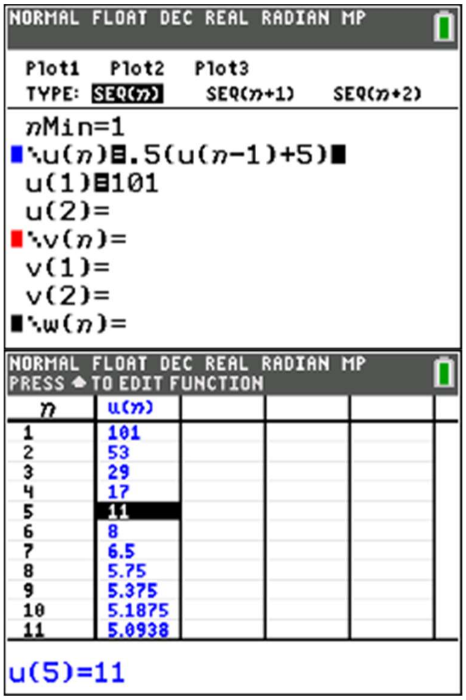
Section A: Pure Mathematics

Question 1

No.	Suggested Solution	Remarks for Student
	Let $y = ax^2 + bx + c$ $\frac{dy}{dx} = 2ax + b$ $y = -2$ when $x = 1 \Rightarrow a + b + c = -2$ $\frac{dy}{dx} = 0$ when $x = 1 \Rightarrow 2a + b = 0$ $\frac{dy}{dx} = 5$ when $x = 2 \Rightarrow 4a + b = 5$ Using GC, $a = \frac{5}{2},\ b = -5,\ c = \frac{1}{2}$ $\therefore y = \frac{5}{2}x^2 - 5x + \frac{1}{2}$ Alternative: Let $y + 2 = k(x-1)^2$ for some constant $k \in \mathbb{R}$ $\frac{dy}{dx} = 2k(x-1)$ $\frac{dy}{dx} = 5$ when $x = 2 \Rightarrow 2k = 5 \Rightarrow k = \frac{5}{2}$ $\therefore y = \frac{5}{2}(x-1)^2 - 2$	Do ensure you get everything correct for such straight forward question

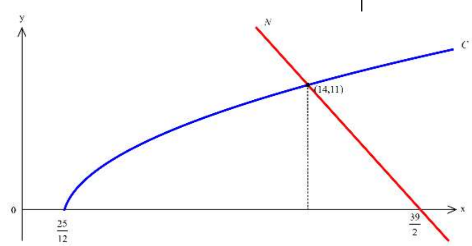
Question 2

No.	Suggested Solution	Remarks for Student
(a)(i) **(A)**	Using GC, it can be observed that the sequence strictly increases without bound.  	While some may notice that the difference of the terms form a GP 2, 4, 8, … Key word is increasing here.
(i) **(B)**	Using GC, it is a constant sequence.  	

(ii)	$u_{n+1} = 2u_n - 5 \Rightarrow u_n = \frac{1}{2}\left(u_{n+1} + 5\right)$  From GC, $u_5 = 11$ <u>Alternative method (Without using GC):</u> $u_1 = p$ $u_2 = 2u_1 - 5 = 2p - 5$ $u_3 = 2u_2 - 5 = 2(2p-5) - 5 = 4p - 15$ $u_4 = 2u_3 - 5 = 2(4p-15) - 5 = 8p - 35$ $u_5 = 2u_4 - 5 = 2(8p-35) - 5 = 16p - 75 = 101$ $p = 11$	
(b)(i)	$v_1 = a,\ v_2 = b$ $v_3 = v_1 + 2v_2 - 7 = a + 2b - 7$ $v_4 = v_2 + 2v_3 - 7 = b + 2(a + 2b - 7) - 7 = 2a + 5b - 21$ Since $v_4 = 2v_3$, $2a + 5b - 21 = 2(a + 2b - 7)$ $b = 7$ **Alternatively,** $v_4 = v_2 + 2v_3 - 7$ $2v_3 = v_2 + 2v_3 - 7$ $v_2 = 7$ $b = 7$	

(ii)	$v_1 = a,\ v_2 = b$ $v_3 = v_1 + 2v_2 - 7 = a + 2b - 7 = a + 7$ since $b = 7$ $v_5 = v_3 + 2v_4 - 7$ $= a + 7 + 2(2a + 14) - 7$ $= 5a + 28$	
(c)(i)	For $n \geq 2$, n^{th} term $= S_n - S_{n-1}$ $= (n^3 - 11n^2 + 4n) - ((n-1)^3 - 11(n-1)^2 + 4(n-1))$ $= [n^3 - (n-1)^3] + 11[(n-1)^2 - n^2] + 4[n - (n-1)]$ $= [n^3 - (n^3 - 3n^2 + 3n - 1)] + 11[(n^2 - 2n + 1) - n^2] + 4$ $= (3n^2 - 3n + 1) - 22n + 11 + 4$ $= 3n^2 - 25n + 16$ 1st term $= S_1 = 1 - 11 + 4 = -6 = 3 - 25 + 16$ $\therefore n^{th}$ term $= 3n^2 - 25n + 16$	Note that the 1st term, -6, satisfies the expression for the *n*th term.
(ii)	$S_m = S_3$ $m^3 - 11m^2 + 4m = 3^3 - 11(3)^2 + 4(3) = -60$ $m^3 - 11m^2 + 4m + 60 = 0$ $m = -2, 3$ or 10 $\therefore m = 10$ since $m > 3$	GC Plysmlt 2 can be used to solve the cubic equation.

Question 3

No.	Suggested Solution	Remarks for Student
(i)	$\frac{dx}{dt}=6t,\ \frac{dy}{dt}=6$ $\frac{dy}{dx}=\frac{1}{t}$ $y=11\Rightarrow 6t-1=11\Rightarrow t=2,\ \therefore \frac{dy}{dx}=\frac{1}{2}$ Cartesian equation of $N: y-11=-2\left(x-14\right)$ $2x+y=39$	Equation needs to be in the form $ax+by=c$
(ii)	When $y=0,\ t=\frac{1}{6}$; When $y=11,\ t=2$  <u>Method 1: Parametric</u> Area $=\int_{\frac{1}{6}}^{2}\left(y\right)\frac{dx}{dt}dt+\frac{1}{2}\left(\frac{39}{2}-14\right)\left(11\right)$ $=\int_{\frac{1}{6}}^{2}\left(6t-1\right)6t\ dt+30.25$ $=\frac{2057}{18}$ <u>Method 2: Cartesian form</u> $y=6t-1=6\sqrt{\frac{x-2}{3}}-1$ Area $=\int_{\frac{25}{12}}^{14}\left(6\sqrt{\frac{x-2}{3}}-1\right)dx+\frac{1}{2}\left(\frac{39}{2}-14\right)\left(11\right)$ $=\frac{2057}{18}$	GC can be used to evaluate the area.
(iii)(a)	$\frac{2057}{18}\times 6=\frac{2057}{3}$ (why multiply by 6? Just imagine you are given area of a triangle. You scale the base by 2, and the height by 3, the resulting area will be 6 times the original area)	

(b)	$y=6t-1=6\sqrt{\frac{x-2}{3}}-1,\ x\ge\frac{25}{12}$ and $y\ge 0$ After transformation, $\frac{y}{3}=6\sqrt{\frac{\frac{x}{2}-2}{3}}-1$ $y=18\sqrt{\frac{x-4}{6}}-3=3\left(\sqrt{6(x-4)}-1\right)$ $y=3\left(\sqrt{6(x-4)}-1\right)$ Equivalent to $54x=y^2+6y+225,\ x\ge\frac{25}{6}$ and $y\ge 0$	

Question 4

No.	Suggested Solution	Remarks for Student
(i)	$2h+a=30$ $h=\dfrac{30-a}{2}$ $H^2=h^2-\left(\dfrac{a}{2}\right)^2$ $=\left(\dfrac{30-a}{2}\right)^2-\left(\dfrac{a}{2}\right)^2$ $=\dfrac{(30-a)^2-a^2}{4}$ $=\dfrac{(30-a-a)(30-a+a)}{4}$ $=\dfrac{900-60a}{4}$ $=225-15a$	
(ii)	$V=\dfrac{1}{3}a^2H=\dfrac{1}{3}a^2\sqrt{225-15a}$ $V^2=\dfrac{1}{9}a^4(225-15a)=25a^4-\dfrac{5}{3}a^5$ $2V\dfrac{\mathrm{d}V}{\mathrm{d}a}=100a^3-\dfrac{25}{3}a^4$ $\dfrac{\mathrm{d}V}{\mathrm{d}a}=0\Rightarrow 100a^3-\dfrac{25}{3}a^4=0$ $\Rightarrow a^3\left(100-\dfrac{25}{3}a\right)=0$ $a=\dfrac{300}{25}=12$ since $a>0$ $V=\dfrac{1}{3}(12^2)\sqrt{225-15(12)}=48\sqrt{45}=144\sqrt{5}$	
(iii)(a)	$S=4\left(\dfrac{1}{2}ah\right)$ $=2a\left(\dfrac{30-a}{2}\right)$ $=a(30-a)$ which is quadratic (factorised), so max S occurs when $a=15$	
(b)	Square (2 layers of the base)	

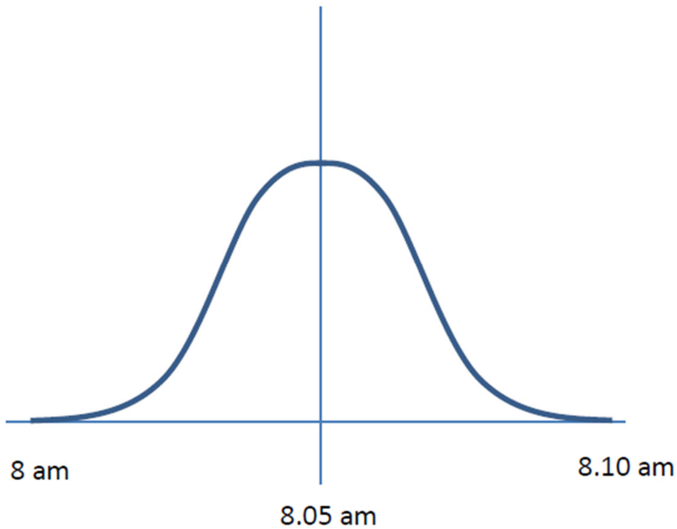
Section B: Statistics

Question 5

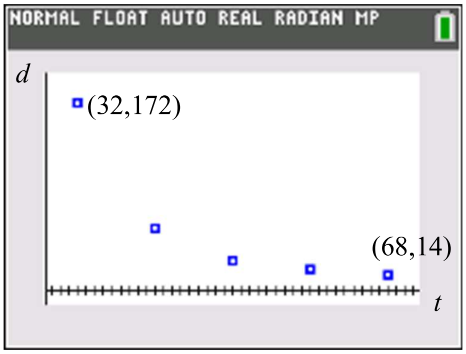
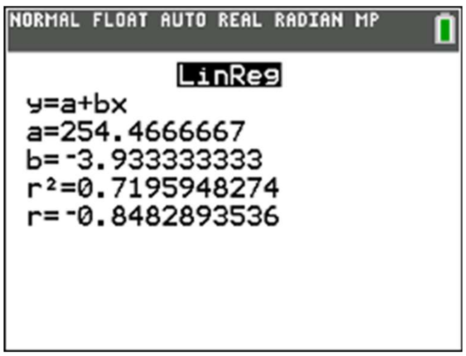
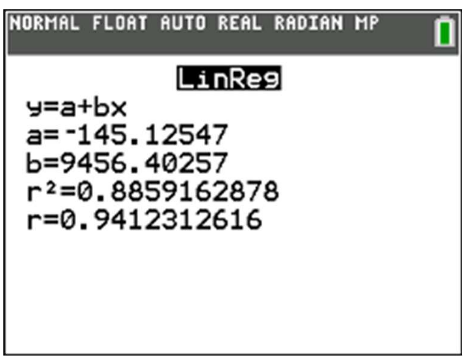
No.	Suggested Solution	Remarks for Student
(i)	0,4,10,25	
(ii)	$\mathrm{P}(S=0)=\frac{{}^{1}C_1\,{}^{3r}C_1}{{}^{3r+1}C_2}=\frac{3r\times 2}{(3r+1)(3r)}=\frac{2}{3r+1}$ $\mathrm{P}(S=4)=\frac{{}^{2r}C_2}{{}^{3r+1}C_2}=\frac{(2r)(2r-1)}{(3r+1)(3r)}=\frac{\frac{2}{3}(2r-1)}{3r+1}$ $\mathrm{P}(S=10)=\frac{{}^{r}C_1\,{}^{2r}C_1}{{}^{3r+1}C_2}=\frac{r\times 2r\times 2}{(3r+1)(3r)}=\frac{\frac{4}{3}r}{3r+1}$ $\mathrm{P}(S=25)=\frac{{}^{r}C_2}{{}^{3r+1}C_2}=\frac{(r)(r-1)}{(3r+1)(3r)}=\frac{\frac{1}{3}(r-1)}{3r+1}$	Good to check sum of prob =1 $\frac{2}{3r+1}+\frac{\frac{2}{3}(2r-1)}{3r+1}+\frac{\frac{4}{3}r}{3r+1}+\frac{\frac{1}{3}(r-1)}{3r+1}$ $=\frac{1}{3r+1}\left(2+\frac{4}{3}r-\frac{2}{3}+\frac{4}{3}r+\frac{1}{3}r-\frac{1}{3}\right)$ $=1$
	$\mathrm{E}(S)$ $=\sum s\mathrm{P}(S=s)$ $=\frac{1}{3r+1}\left(0\times 2+4\left(\frac{4}{3}r-\frac{2}{3}\right)+10\left(\frac{4}{3}r\right)+25\left(\frac{1}{3}r-\frac{1}{3}\right)\right)$ $=\frac{1}{3r+1}\left(\frac{16}{3}r-\frac{8}{3}+\frac{40}{3}r+\frac{25}{3}r-\frac{25}{3}\right)$ $=\frac{1}{3r+1}(27r-11)$	
	$\mathrm{E}(S^2)$ $=\sum s^2\mathrm{P}(S=s)$ $=\frac{1}{3r+1}\left(4^2\left(\frac{4}{3}r-\frac{2}{3}\right)+10^2\left(\frac{4}{3}r\right)+25^2\left(\frac{1}{3}r-\frac{1}{3}\right)\right)$ $=\frac{1}{3r+1}\left(\frac{64}{3}r-\frac{32}{3}+\frac{400}{3}r+\frac{625}{3}r-\frac{625}{3}\right)$ $=\frac{1}{3r+1}(363r-219)$	
	$\mathrm{var}(S)$ $=\mathrm{E}(S^2)-[\mathrm{E}(S)]^2$ $=\frac{1}{3r+1}(363r-219)-\left[\frac{1}{3r+1}(27r-11)\right]^2$ $=\frac{1}{(3r+1)^2}\left[(363r-219)(3r+1)-(27r-11)^2\right]$ $=\frac{1}{(3r+1)^2}\left[360r^2+300r-340\right]$	

(iii)	$\operatorname{var}(S) = 38$ $\frac{1}{(3r+1)^2}\left[360r^2 + 300r - 340\right] = 38$ $GC: r = 3$	

Question 6

No.	Suggested Solution	Remarks for Student
(i)		Your sketch should show the symmetry about 8.05am
(ii)	$\mathrm{P}(T>6)=0.202$	
(iii)	$T+W \sim \mathrm{N}\left(5+21,\ 1.2^2+3^2\right)$ $T+W \sim \mathrm{N}(26,\ 10.44)$ $\mathrm{P}(T+W>30)=0.108$	
(iv)	$T+D \sim \mathrm{N}(24,\ 37.44)$ $\mathrm{P}(\text{Day is fine} \mid \text{James is late})$ $=\dfrac{\mathrm{P}(\text{Day is fine AND James is late})}{\mathrm{P}(\text{James is late})}$ $=\dfrac{0.7\times\mathrm{P}(T+W>30)}{0.7\times\mathrm{P}(T+W>30)+0.3\times\mathrm{P}(T+D>30)}$ $=0.606$	

Question 7

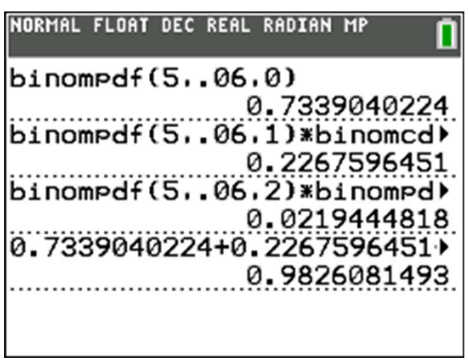
No.	Suggested Solution	Remarks for Student
(i)	  $r = -0.848$	
(ii)	 $a = 145$ $b = 9460$ $r = 0.941$	
(iii)	$d = -145.12547 + 9456.40257\left(\frac{1}{86}\right) = -35.167$ Lim's model gives a negative value which obviously doesn't fit this additional data	

(iv)	$T=\frac{5}{9}(t-32)\Rightarrow t=\frac{9T}{5}+32=\frac{9T+160}{5}$ $\begin{aligned} d &= -a+bu \\ &= -a+b\left(\frac{1}{t}\right) \\ &= -a+\frac{5b}{9T+160} \end{aligned}$ where a,b are values in part (ii)	

Question 8

No.	Suggested Solution	Remarks for Student
(i)	$\mathrm{P}(R=1)=\dfrac{{}^{17}C_1\,{}^{11}C_{11}}{{}^{28}C_{12}}<\dfrac{{}^{17}C_2\,{}^{11}C_{10}}{{}^{28}C_{12}}=\mathrm{P}(R=2)$ since ${}^{17}C_1<{}^{17}C_2$ and ${}^{11}C_{11}<{}^{11}C_{10}$	
(ii)	${}^{17+r}C_4\,{}^{11}C_8=15\,{}^{17+r}C_3\,{}^{11}C_9$ GC: $r=6$	Alternative: An algebraic but more tedious method using $\binom{n}{r}=\dfrac{n!}{r!(n-r)!}$ (available in MF27) may also be applied

Question 9

No.	Suggested Solution	Remarks for Student
(i)	All pens have equal probability of being chosen and the selection of one pen is independent of the selection of another.	
(ii)	Let X be the number of faulty pens out of 10. $X \sim \mathrm{B}(10, 0.06)$ Required probability $=\mathrm{P}(X \leq 2) = 0.981162 \approx 0.981$	
(iii)	P(a box is rejected) = $1-0.981162 \approx 0.018838$ Let Y be the number of boxes out of 75 which are rejected. $Y \sim \mathrm{B}(75, 0.018838)$ $\mathrm{P}(Y > 5\% \text{ of } 75)$ $= \mathrm{P}(Y > 3.75)$ $= \mathrm{P}(Y \geq 4)$ $= 1-\mathrm{P}(Y \leq 3)$ $= 0.053454$ ≈ 0.0535	
(iv)	Let T be the number of faulty pens out of 5. $T \sim \mathrm{B}(5, 0.06)$ $\mathrm{P}(\text{a box is accepted})$ $= \mathrm{P}(T_1 = 0) + \mathrm{P}(T_1 = 1)\mathrm{P}(T_2 \leq 1) + \mathrm{P}(T_1 = 2)\mathrm{P}(T_2 = 0)$ $= \mathrm{P}(T = 0) + \mathrm{P}(T = 1)\mathrm{P}(T \leq 1) + \mathrm{P}(T = 2)\mathrm{P}(T = 0)$ $= 0.983$ 	Key in the expressions into the GC carefully
(v)	Probability in (iv) is similar or slightly higher than (ii) with less testing.	

Question 10

No.	Suggested Solution	Remarks for Student
(i)	Let Y be the percentage of of carbon in a bar of carbon steel and μ_Y be the population mean percentage of carbon in the carbon steel bars. Null hypothesis, $\mathrm{H}_0 : \mu_Y = 1.5$ Alternative hypothesis, $\mathrm{H}_1 : \mu_Y \neq 1.5$ Perform a 2-tailed test at 5% significance level. Under H0, $\overline{Y} \sim \mathrm{N}\left(1.5, \frac{0.09^2}{15}\right)$ To find critical region for this test, we consider the following conditions on p-value in order to reject H0. $2\mathrm{P}\left(\overline{Y} > \overline{y}\right) < 0.05$ or $2\mathrm{P}\left(\overline{Y} < \overline{y}\right) < 0.05$ $\overline{y} \geq 1.54555$ or $\overline{y} \leq 1.45445$ $\overline{y} \geq 1.55$ or $0 < \overline{y} \leq 1.45$ (answers to 3 s.f.)	From context, $\overline{y} > 0$
(ii)	He has no information of the distribution of the amount of carbon in the new line of flat bars. So he would need to use a large sample size, 40, in order to approximate the distribution of the sample mean amount of carbon with a Normal distribution by applying Central Limit Theorem	
(iii	An unbiased estimate of population mean is $\overline{x}$ $= 0.254$ An unbiased estimate of population variance is s^2 $= \frac{1}{39}\left(2.586342 - \frac{10.16^2}{40}\right)$ $= \frac{0.005702}{39}$ $= 0.000146$ (3sf)	
(iv	Let μ be the population mean percentage of carbon in the flat bars. Null hypothesis, $\mathrm{H}_0 : \mu = 0.25$ Alternative hypothesis, $\mathrm{H}_1 : \mu > 0.25$ Perform an 1-tailed test at 2.5% significance level. Under H0, $\overline{X} \sim \mathrm{N}\left(0.25, \frac{1}{40}\left(\frac{0.005702}{39}\right)\right)$ approximately by Central Limit Theorem since $n = 40$ is large p-value $= \mathrm{P}\left(\overline{X} > 0.254\right) = 0.0182 < 0.025$	

	Since p-value < 0.025, we reject H_0 and conclude that there is sufficient evidence at 2.5% level of significance that the mean amount of carbon in the flat bar is more than 0.25%	