Name : ____________________________

Class	Index Number

METHODIST GIRLS' SCHOOL

Founded in 1887

PRELIMINARY EXAMINATION 2025
Secondary 4

Tuesday
26 August 2025

ADDITIONAL MATHEMATICS
Paper 1

4049/01
2 h 15 min

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST
Write your class, index number and name in the spaces at the top of this page.
Write in dark blue or black pen
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

	90

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)...(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 A home-based sticker company that prints stickers has calculated its profit, \$$P$, for each order using the equation $P = 120x - 40 - 30x^2$, where x is the number of stickers produced in **hundreds**.

(a) Express $P = 120x - 40 - 30x^2$ in the form $P = a(x-b)^2 + c$, where a, b and c are constants. [2]

$P = 120x - 40 - 30x^2$
$= -30(x^2 - 4x) - 40$
$= -30[x^2 - 4x + (-2)^2 - (-2)^2] - 40$
$= -30[x-2]^2 + 80$

(b) Using your answer in **(a)**, explain clearly if the company should accept an order for printing 400 stickers. [2]

No. As he will be making a loss of $40 if he accept the order.

2 Find the exact coordinates of the stationary point on the curve $y = \frac{e^{2x}}{3x^2}$, $x \neq 0$. [6]

$$\frac{dy}{dx} = \frac{(3x^2)(e^{2x})(2) - (e^{2x})(6x)}{(3x^2)^2}$$

$$= \frac{6xe^{2x}(x-1)}{9x^4}$$

$$= \frac{2e^{2x}(x-1)}{3x^3}$$

$$0 = \frac{2e^{2x}(x-1)}{3x^3}$$

$2e^{2x} > 0, x \neq 0$

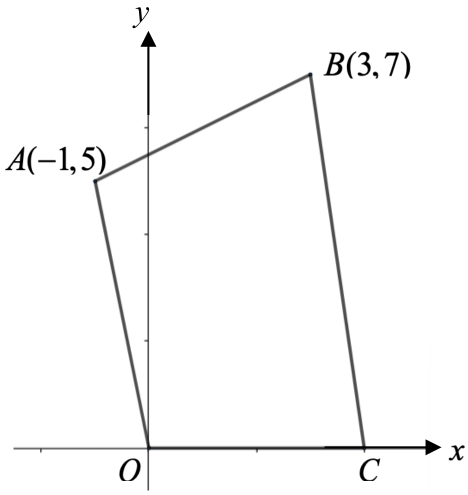
$x - 1 = 0$

$x = 1$

$y = \frac{e^2}{3}$

Stationary point = $(1, \frac{e^2}{3})$

3 **Solutions to this question by accurate drawing will not be accepted.**



The diagram shows a quadrilateral $OABC$ where O is the origin. The point A is $(-1,5)$ and the point B is (3, 7). The perpendicular bisector of AB meets the x-axis at C.

(a) Find the coordinates of C. [4]

Gradient of $AB = \dfrac{7-5}{3+1} = \dfrac{1}{2}$

Gradient of Perpendicular to AB = -2

Midpoint of AB = $\left(\dfrac{-1+3}{2}, \dfrac{5+7}{2}\right) = (1,6)$

Equation of perpendicular bisector of AB:

$y-6=-2(x-1)$

$y=-2x+8$ [M1]

At x-axis, $y = 0$

$0=-2x+8$

$x=4$

Coordinates of C = (4, 0)

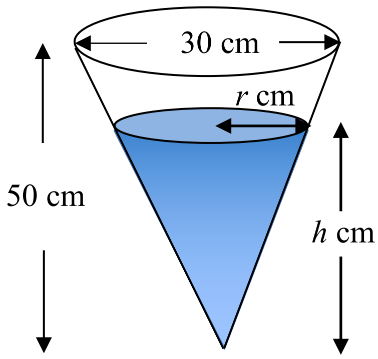
(b) Find the area of the quadrilateral $OABC$. [2]

Area = $\dfrac{1}{2}\begin{vmatrix} 0 & 4 & 3 & -1 & 0 \\ 0 & 0 & 7 & 5 & 0 \end{vmatrix}$

$= \dfrac{1}{2}[(0+28+15+0)-(0-7+0+)]$

= 25 units²

4 The diagram shows an inverted right circular cone with a diameter, 30 cm and height, 50 cm.
The cone was initially empty. It is being filled with water at a constant rate of 100 cm³/s. The surface of the water remains horizontal as it fills. At time t seconds, the radius of the horizontal water surface area is r cm and the depth of the water is h cm.



(a) Express r in terms of h. [2]

$$\frac{r}{15} = \frac{h}{50}$$
$$r = \frac{3}{10}h$$

(b) Show that the volume of water, V cm³, in the cone is $V = \frac{3\pi h^3}{100}$. [1]

$$V = \frac{1}{3}\pi r^2 h$$
$$V = \frac{1}{3}\pi\left(\frac{3}{10}h\right)^2 h$$

$$V = \frac{3\pi h^3}{100}$$

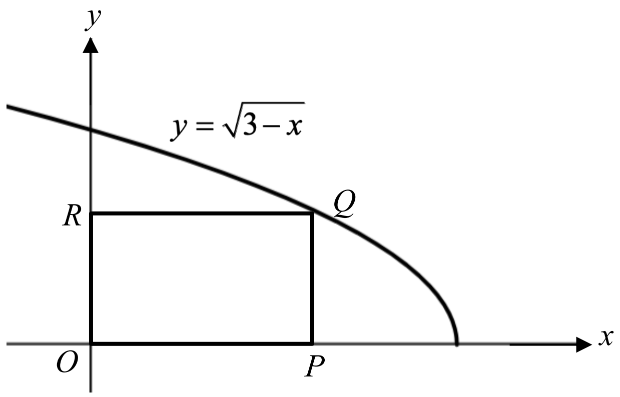
(c) Find the rate of change of the depth of the water when h = 35 cm. [3]

$$\frac{dV}{dh} = \frac{9}{100}\pi h^2$$
$$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$$

When h = 35 cm,

$$100 = \frac{9}{100}\pi(35)^2 \times \frac{dh}{dt}$$
$$\frac{dh}{dt} = 0.288716 = 0.289\ cm/s(\text{3sf})$$

5 The diagram shows part of the graph of $y = \sqrt{3-x}$, $x < 3$. O is the origin and points P and R lie on the x and y axes respectively. Point Q is a point on the curve such that $OPQR$ forms a rectangle.



(a) Given that the x-coordinate of P is h, write down an expression for the area of the rectangle, A units^2, in terms of h and show that $\dfrac{\mathrm{d}A}{\mathrm{d}h} = \dfrac{6-3h}{2\sqrt{3-h}}$. [3]

When $x = h,\ y = \sqrt{3-h}$

$A = h\sqrt{3-h}$

$$\frac{dA}{dh} = (h)(\frac{1}{2})(3-h)^{-\frac{1}{2}}(-1) + (3-h)^{\frac{1}{2}}(1)$$

$$= \frac{-h}{2\sqrt{3-h}} + \sqrt{3-h}$$

$$= = \frac{-h + 2(3-h)}{2\sqrt{3-h}}$$

$$\frac{\mathrm{d}A}{\mathrm{d}h} = \frac{6-3h}{2\sqrt{3-h}}$$

(b) Find the stationary value of A and determine the nature of this stationary value. [4]

$$\frac{\mathrm{d}A}{\mathrm{d}h} = \frac{6-3h}{2\sqrt{3-h}} = 0$$

$6 - 3h = 0$
$h = 2$

$A = 2\sqrt{3-2} = 2$ units²

h	Slightly less than 2	2	Slightly more than 2
$\frac{dA}{dh}$	+	0	-
Shape	/	-	\

Or

$$\frac{\mathrm{d}^2A}{\mathrm{d}h^2} = \frac{2\sqrt{3-h}(-3) - (6-3h)(\frac{1}{2})(3-h)^{-1/2}(-1)}{4(3-h)} = -\frac{3}{2}$$

At $h = 2$, $\frac{d^2A}{dh^2} < 0$

Therefore Stationary value $A = 2$ is a maximum value.

6 **(a)** The expansion of $(1+kx)^n$ in ascending powers of x is $1+\frac{7}{3}x+\frac{7}{3}x^2+....$

Show that $k=\frac{1}{3}$ and calculate the value of n. [4]

$$(1+kx)^n = 1+\binom{n}{1}(kx)+\binom{n}{2}(kx)^2+...$$

$$\binom{n}{1}k=\frac{7}{3} \qquad \binom{n}{2}k^2=\frac{7}{3}$$

$$nk=\frac{7}{3} \qquad \frac{n(n-1)}{2}k^2=\frac{7}{3}$$

$$k=\frac{7}{3n}$$

$$\frac{n(n-1)}{2}\left(\frac{7}{3n}\right)^2=\frac{7}{3}$$

$$\frac{n(n-1)}{2}\left(\frac{7}{3n^2}\right)=1$$

$$\frac{7(n-1)}{6n}=1$$

$$7n-7=6n$$

$$n=7$$

$$k=\frac{7}{3(7)}=\frac{1}{3} \text{ (shown)}$$

(b) Explain why all terms in the expansion of $\left(px^2 - \frac{1}{x^4}\right)^{12}$ contain only even powers of x. [4]

General Term = $\binom{12}{r}\left(px^2\right)^{12-r}\left(-\frac{1}{x^4}\right)^r$

$=\binom{12}{r}p^{12-r}x^{24-2r}(-1)^r\left(x^{-4}\right)^r$

$=\binom{12}{r}p^{12-r}(-1)^r x^{24-6r}$

Power of x: $24-6r=2(12-3r)$
Therefore all powers of x is 2(12-3r), all terms contain only even powers of x.

7 **(a)** Find all the angles between $-\pi$ and π that satisfy the equation $2\cos 2x = 4\sin x + 3$. [3]

$$2\cos 2x = 4\sin x + 3$$
$$2(1-2\sin^2 x) = 4\sin x + 3$$
$$2-4\sin^2 x - 4\sin x - 3 = 0$$
$$4\sin^2 x + 4\sin x + 1 = 0$$
$$\left(2\sin x + 1\right)^2 = 0$$
$$\sin x = -\frac{1}{2}$$
$\alpha = \dfrac{\pi}{6}$ $x = -\dfrac{\pi}{6}, -\dfrac{5\pi}{6}$ [Accept -0.524 and -2.62 rad]

(b) Solve the equation $3\operatorname{cosec}^2 x - \operatorname{cosec} x = 2 - \cot^2 x$ for $0° \le x \le 360°$. [5]

$$3\operatorname{cosec}^2 x - \operatorname{cosec} x = 2 - \cot^2 x$$
$$3\operatorname{cosec}^2 x - \operatorname{cosec} x = 2 - (\operatorname{cosec}^2 x - 1)$$
$$4\operatorname{cosec}^2 x - \operatorname{cosec} x - 3 = 0$$
$$(4\operatorname{cosec} x + 3)(\operatorname{cosec} x - 1) = 0$$

$4\operatorname{cosec} x + 3 = 0$ $\quad$ $\operatorname{cosec} x - 1 = 0$

$\operatorname{cosec} x = -\frac{3}{4}$ $\quad$ $\operatorname{cosec} x = 1$

$\sin x = -\frac{4}{3}$ (rej) $\quad$ $\sin x = 1$

$x = 90°$

Alternate Solution:

$$\frac{3}{\sin^2 x} - \frac{1}{\sin x} = 2 - \frac{\cos^2 x}{\sin^2 x}$$

$$3 - \sin x = 2\sin^2 x - \cos^2 x$$
$$3 - \sin x = 2\sin^2 x - (1 - \sin^2 x)$$
$$3\sin^2 x + \sin x - 4 = 0$$
$$(3\sin x + 4)(\sin x - 1) = 0$$

$\sin x = -\frac{4}{3}$ (NA) *or* $\sin x = 1$

$x = 90°$

8 **(a)** Express $\dfrac{1-6x}{(x+2)(3x^2+1)}$ in the form of $\dfrac{a}{x+2}+\dfrac{bx+c}{3x^2+1}$, where a, b and c are integers. [4]

$$\frac{1-6x}{(x+2)(3x^2+1)}=\frac{a}{x+2}+\frac{bx+c}{3x^2+1}$$
$$1-6x=a(3x^2+1)+(bx+c)(x+2)$$

When $x=-2$, $a=1$

When $x=0$,
$1=1(1)+(c)(3)$
$c=0$

When $x=1$,
$1-6=1(3+1)+(b)(1+2)$
$3b=-9$
$b=-3$

$$\frac{1-6x}{(x+2)(3x^2+1)}=\frac{1}{x+2}-\frac{3x}{3x^2+1}$$

(b) Differentiate $\ln(3x^2+1)$ with respect to x. [2]

$$\frac{d}{dx}\ln(3x^2+1)=\frac{6x}{3x^2+1}$$

(c) Using your answer to **parts (a)** and **(b)**, show that
$\int_0^2 \frac{1-6x}{(x+2)(3x^2+1)}\,\mathrm{d}x = \ln 2 - \frac{1}{2}\ln 13$. [4]

$$\int_0^2 \frac{1-6x}{(x+2)(3x^2+1)}\,\mathrm{d}x = \int_0^2 \frac{1}{x+2} - \frac{3x}{3x^2+1}\,\mathrm{d}x$$

$$= \int_0^2 \frac{1}{x+2}\,dx - \int_0^2 \frac{3x}{3x^2+1}\,\mathrm{d}x$$

$$= \left[\ln(x+2)\right]_0^2 - \frac{1}{2}\int_0^2 \frac{6x}{3x^2+1}\,\mathrm{d}x$$

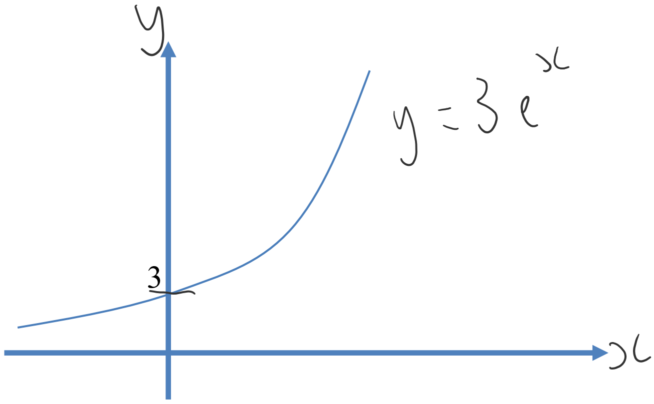
$$= \ln(4) - \ln(2) - \frac{1}{2}\left[\ln(3x^2+1)\right]_0^2$$

$$= \ln(4) - \ln(2) - \frac{1}{2}\left[\ln 13 - \ln 1\right]$$

$$= \ln\left(\frac{4}{2}\right) - \frac{1}{2}\ln 13$$

$$= \ln 2 - \frac{1}{2}\ln 13 \quad \text{(shown)}$$

9 **(a)** **(i)** Sketch the graph of $y = 3e^x$. [1]



(ii) Find the equation of a suitable straight line that can be inserted in **part (a)(i)** to solve the equation $6e^x - x = 6$. [2]

$6e^x = 6 + x$

$3e^x = 3 + \frac{1}{2}x$

Insert: $y = 3 + \frac{1}{2}x$

(b) **(i)** Without using a calculator, show that $\tan 75° = a + b\sqrt{3}$, where a and b are integers. [3]

$$\begin{aligned}
\tan 75° &= \tan(30° + 45°) \\
&= \frac{\tan 30° + \tan 45°}{1 - \tan 30° \tan 45°} \\
&= \frac{\frac{1}{\sqrt{3}} + 1}{1 - \frac{1}{\sqrt{3}}} \\
&= \left(\frac{1+\sqrt{3}}{\sqrt{3}}\right) \div \left(\frac{\sqrt{3}-1}{\sqrt{3}}\right) \\
&= \left(\frac{1+\sqrt{3}}{\sqrt{3}}\right) \times \left(\frac{\sqrt{3}}{\sqrt{3}-1}\right) \\
&= \frac{1+\sqrt{3}}{\sqrt{3}-1} \\
&= \frac{\left(1+\sqrt{3}\right)\left(1+\sqrt{3}\right)}{3-1} \\
&= \frac{4+2\sqrt{3}}{2} \\
&= 2+\sqrt{3}
\end{aligned}$$

(ii) Hence, find the exact value of $\cot^2 75°$. [3]

$$\begin{aligned}
\cot^2 75° &= \frac{1}{\tan^2 75°} \\
&= \frac{1}{\left(2+\sqrt{3}\right)^2} \\
&= \frac{1}{4+4\sqrt{3}+3} \\
&= \frac{1}{7+4\sqrt{3}} \\
&= \frac{7-4\sqrt{3}}{49-48} \\
&= 7-4\sqrt{3}
\end{aligned}$$

10 The maximum height that a roller coaster can reach is 90 meters above sea level, and the lowest possible height is 10 meters above sea level. The roller coaster takes 10 seconds to move from its highest position to its lowest position.
The height, h m, of the roller coaster above sea level can be modelled by the function $h = a\cos\left(\frac{\pi}{b}t\right) + c$, where t is the time in seconds.

(a) Show that $h = 40\cos\left(\frac{\pi}{10}t\right) + 50$. [3]

Period: $\dfrac{2\pi}{\frac{\pi}{b}} = 20$

$b = 10$

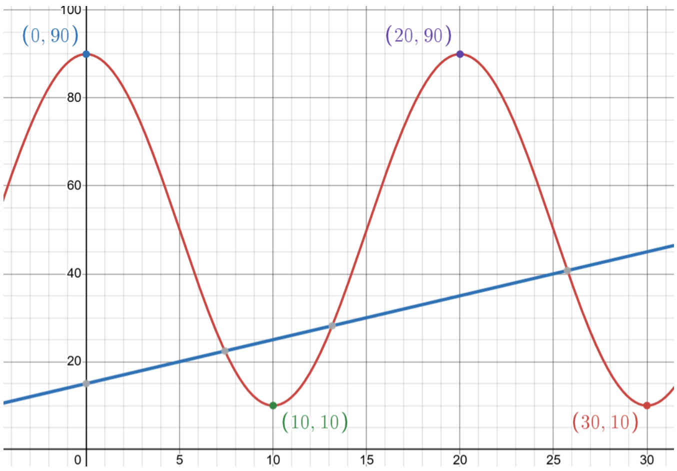
Max $h = 90$
Min $h = 10$

$$\begin{aligned} a + c &= 90 \\ -a + c &= 10 \\ 2c &= 100 \\ c &= 50 \end{aligned}$$

$a = 90 - 50 = 40$ or $a = \dfrac{90-10}{2} = 40$

$\therefore h = 40\cos\left(\frac{\pi}{10}t\right) + 50$

(b) Sketch the graph of $h = 40\cos\left(\frac{\pi}{10}t\right) + 50$ for $0 \le t \le 30$. [2]



(c) Find the time when the roller coaster first reaches a height of 62 m. [2]

$62 = 40\cos\left(\frac{\pi}{10}t\right) + 50$

$40\cos\left(\frac{\pi}{10}t\right) = 12$

$\cos\left(\frac{\pi}{10}t\right) = \frac{12}{40}$

$\alpha = 1.26610$

$\frac{\pi}{10}t = 1.26610$

$t = 4.03\text{s} \quad (3\text{sf})$

(d) **(i)** On your graph in **part (b)**, draw the graph of $h = t + 15$. [1]

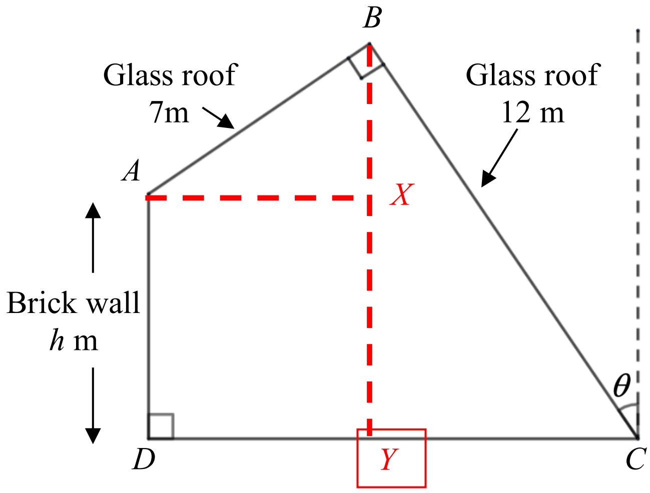
(ii) State for the interval $0 \le t \le 30$, the number of solutions of $t = 40\cos\left(\frac{\pi}{10}t\right) + 35$. [1]

$t = 40\cos\left(\frac{\pi}{10}t\right) + 35$

$t + 15 = 40\cos\left(\frac{\pi}{10}t\right) + 50$

3 solutions

11 An architect is designing a building, *ABCD,* using the model shown below.
To allow more natural light into the building, a 12 m glass roof, *BC*, is installed at an angle of θ to the vertical. Another glass roof, *AB*, of 7 m, is installed such that it is perpendicular to *BC*. The vertical height of the brick wall, *AD*, is given as *h* m.



(a) Show that $h = 12\cos\theta - 7\sin\theta$. [2]

$$\sin\theta = \frac{BX}{7}$$
$$BX = 7\sin\theta$$

$$\cos\theta = \frac{BY}{12}$$
$$BY = 12\cos\theta$$

$$h = BY = BX$$
$$= 12\cos\theta - 7\sin\theta$$

(b) Express *h* in the form $R\cos(\theta+\alpha)$, where $R>0$ and $0<\alpha<\frac{\pi}{2}$. [3]

$$R = \sqrt{12^2+7^2} = \sqrt{193}$$
$$\alpha = \tan^{-1}\left(\frac{7}{12}\right) = 0.52807$$
$$h = \sqrt{193}\cos(\theta+0.528)$$

(c) Find the value of θ for which $h = 8\,\text{m}$. [2]

$\sqrt{193}\cos(\theta + 0.52807) = 8$

$\cos(\theta + 0.52807) = \dfrac{8}{\sqrt{193}}$

Basic angle $= 0.95714$

$\theta + 0.52807 = 0.95714$

$\theta = 0.429\,(3\text{sf})$

(d) The building owner makes a request to have the brick wall to have a height of at least 14 m. Is the architect able to meet the building owner's request? Explain your answer. [2]

$\sqrt{193}\cos(\theta + 0.52807) \geq 14$

$\cos(\theta + 0.52807) \geq \dfrac{14}{\sqrt{193}}$

$\cos(\theta + 0.52807) \geq 1.0077$

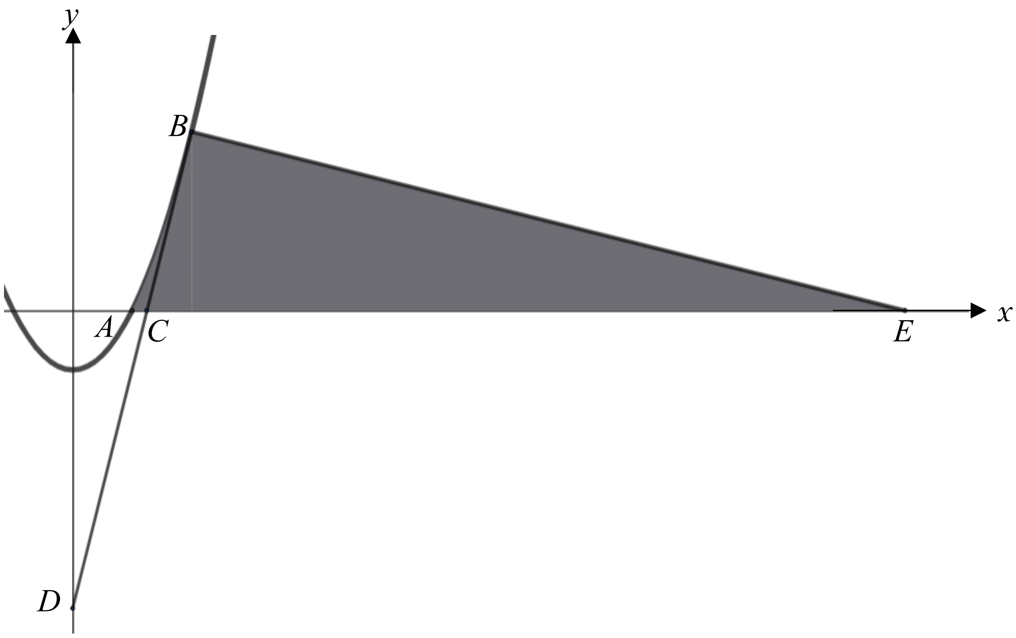
Since $-1 \leq \cos(\theta + 0.52807) \leq 1$, Not possible.

Alternate Solution:

Since max $h = \sqrt{193} = 13.9 < 14,$ it is not possible.

12 The diagram shows part of the curve $y = x^2 - 1$, cutting the x-axis at $A(1, 0)$. The tangent at point B on the curve cuts the x-axis at $C\left(\frac{5}{4}, 0\right)$ and y-axis at $D(0, -5)$ respectively.

The normal at B meets the x-axis at E.



(a) Find the coordinates of B. [3]

Gradient of $BD = \dfrac{0-(-5)}{\frac{5}{4}-0} = 4$

$y = x^2 - 1$

$\dfrac{dy}{dx} = 2x$

At B, gradient = 4

$2x = 4$

$x = 2$

$y = (2)^2 - 1 = 3$

Coordinates of B = (2, 3)

(b) Find the coordinates of E. [2]

Gradient of $BE = -\dfrac{1}{4}$

Equation of normal:

$$y-3=-\frac{1}{4}(x-2)$$
$$y=-\frac{1}{4}x+\frac{7}{2}$$

At x-axis, $y = 0$

$$0=-\frac{1}{4}x+\frac{7}{2}$$
$$x=14$$

Coordinates of E = (14, 0)

(c) Find the area of the shaded region. [3]

$$\text{Area} = \int_1^2 \left(x^2-1\right)dx+\frac{1}{2}(12)(3)$$
$$=\left[\frac{x^3}{3}-x\right]_1^2+18$$
$$=19\frac{1}{3}\text{ units}^2$$

END OF PAPER.

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