

Section A: Pure Mathematics [40 marks]

- 1 Do not use a calculator in answering this question.

Find the complex numbers z and w which satisfy the simultaneous equations

$$(3+2i)z-3w=3-11i \quad \text{and} \quad (1-i)z-6iw=10+8i. \quad [4]$$

- 2 The curve C has equation $y = \frac{\cos x}{\sqrt{1+\sin x}}$ for $0 \leq x \leq \frac{\pi}{2}$. The region R is bounded by the curve C , the x -axis, and the lines $x=0$ and $x=\frac{\pi}{6}$.

- (a) Use the substitution $u=1+\sin x$ to find the exact area of R . [4]

The region S is bounded by the curve C , the line $y=\frac{1}{\sqrt{2}}$, and the y -axis.

- (b) Verify that $y=\frac{1}{\sqrt{2}}$ when $x=\frac{\pi}{6}$. Hence find the volume of the solid formed when S is rotated through 2π radians about the x -axis. [3]

- 3 A curve C has equation $y = \frac{5x^2+16x+18}{x^2-4x}$.

- (a) State the equations of the asymptotes. [2]

- (b) Using an algebraic method, show that $\frac{5x^2+16x+18}{x^2-4x}$ cannot lie between two certain values (to be determined). [4]

- (c) Hence sketch the curve C , indicating clearly any turning point(s). [3]

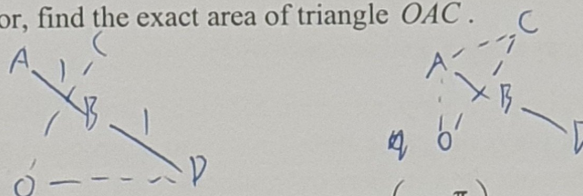
- 4 Referred to the origin O , the points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively. O , A and B are **not** collinear. Point C has position vector $2\mathbf{a} + \mathbf{b}$ and point D lies on AB extended such that $AB = BD$.

(a) Show that the position vector of D is $2\mathbf{b} - \mathbf{a}$. [2]

(b) Given that $OC = OD$, use suitable scalar products to show that $8\mathbf{a} \cdot \mathbf{b} = 3|\mathbf{b}|^2 - 3|\mathbf{a}|^2$. [2]

(c) Given also that $|\mathbf{b}| = 2|\mathbf{a}|$, show that $\cos \theta = \frac{9}{16}$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$. [2]

(d) Given further that $\mathbf{a}$ is a unit vector, find the exact area of triangle OAC . [4]



- 5 (a) Given that x is sufficiently small, find the series expansion of $\tan\left(x - \frac{\pi}{3}\right)$ up to and including the term in x^2 . [4]

(b) Hence find an approximation for the value of $\tan\left(-\frac{7\pi}{24}\right)$, giving your answer in the form $a + b\pi + c\pi^2$, where a , b and c are exact real constants to be determined. [3]

(c) Find the series expansion of $\ln\left|\sec\left(x - \frac{\pi}{3}\right)\right|$ up to and including the term in x^2 . [3]

Section B: Probability and Statistics [60 marks]

- 6 A fair tetrahedral die has its four faces numbered 1, 2, 3 and 4. A game is played by rolling the die twice, and the number on the face touching the table is recorded each time. Let X be the score obtained from the two rolls, defined as follows:

- If the two numbers recorded are different, X is the absolute difference between them.
- If the two numbers recorded are the same, X is their common value.

(a) Find the probability distribution of X . [2]

(b) Find the expectation and variance of X . [2]

To encourage citizens to adopt an active lifestyle, a town council awards fitness cash vouchers to participants. Each participant plays the game three times and is awarded a \$2 cash voucher for every point scored. The participant cashes out the vouchers only after playing the game three times, where a one-time administration fee of \$0.50 is deducted. Let W be the random variable denoting the value, in dollars, of the cash vouchers received by a participant after the administration fee has been deducted.

(c) Find $E(W)$ and $\text{Var}(W)$. [3]

1 2 3 4

3 3

1 4 4 1

4 4

1 1

1 2

3 4

2 3

3 2

2 1

4 3

2 2

1 3

3 1

2 4

4 2

- 7 A set of twins, Eugene and Eunice, is involved in a photoshoot with four other sets of twins.

(a) Find the number of ways in which ten people can be arranged in a row such that everyone is standing next to his or her twin. [2]

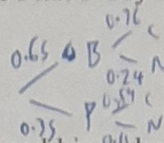
(b) The ten people are now arranged in two rows, with 4 people in the front row and 6 people in the back row. Calculate the number of ways this can be done if Eugene and Eunice cannot be next to each other in the same row. [3]

The ten people are now seated at two round tables which can each seat five people. One table is black and the other is white.

(c) Find the number of ways in which they can be seated such that no one is seated at the same table as his or her twin. [3]

- 8 A meal delivery company, FoodforEu, offers two subscription plans, Basic Plan and Premium Plan, with a 3day free trial for both plans. Over time, it was found that 76% on the Basic Plan continued their subscription after their trial, while 89% of customers on the Premium Plan continued their subscription after their trial.

In a particular month, 65% of new customers signed up for the Basic Plan 3-day free trial while the rest signed up for the Premium Plan 3-day free trial.



- (a) Find the probability that
- (i) a randomly chosen new customer in that month continued his or her subscription after the 3-day free trial, [2]
 - (ii) a randomly chosen new customer in that month chose the Premium Plan given that he or she continued the subscription after the 3-day free trial, [2]
 - (iii) out of 5 randomly chosen new customers in that month who continued their subscription plans, at least one of them signed up for the Premium Plan. [3]
- (b) Explain why the probability in part (a)(ii) is less than 0.5, even though customers on the Premium Plan are more likely to continue their subscription than those on the Basic Plan. [1]

- 9 A company manufactures protein bars. The manager claims that the mean mass of the protein bars is 50 grams. A random sample of 40 protein bars is taken, and their masses x grams, are summarised below.

$$\sum (x - 50) = -28 \text{ and } \sum (x - 50)^2 = 252.4$$

- (a) Calculate unbiased estimates of the population mean and variance of the mass of the protein bars. [2]
- (b) Test, at the 5% level of significance, whether the manager has overstated the mean mass of the protein bars. [4]
- (c) The manager now wants to determine at 5% level of significance whether the mean mass of a protein bar is different from 50 grams. Without doing a new test, explain how the conclusion will change. [1]

The manufacturer also produces yoghurt tubs and claims that the mean and variance of the yoghurt tubs are 500 grams and 8.24 grams² respectively. It is assumed that the masses of these yoghurt tubs have a normal distribution. In a study, a random sample of n tubs was taken and the sample mean was 500.50 grams.

- (d) Find the least value of n such that the null hypothesis, $H_0 : \mu = 500$ would be rejected at the 5% level of significance against the alternative hypothesis, $H_1 : \mu > 500$. [3]
- (e) State the meaning of the p -value of this test in the context of this question. [1]

- 10** An automotive analyst is tracking the resale price (P thousand dollars), of used cars over time (t years), after its initial purchase. The following data is collected from several used car listings:

t	0.5	1	2	5	8	12	15	18
P	98	65.2	28.5	14.8	11.2	10.4	10.1	9

- (a) Draw a scatter diagram of the data. Use your diagram to explain whether the relationship between P and t is likely to be well modelled by an equation of the form $P = at + b$, where a and b are constants. [2]
- (b) The analyst proposes two models (A) and (B) for the relationship between P and t , where a , b , c , and d are constants:

$$(A) P = a + b \left(\frac{1}{t} \right) \qquad (B) P = c + d \ln t.$$

- (i) By calculating the relevant product moment correlation coefficients, determine whether the relationship between P and t is modelled better by (A) or (B). Explain how you decide which is the better model and state the equation in this case. [5]
- (ii) Use your equation in part (b)(i) to estimate the resale price of a used car 30 years after its initial purchase. Explain whether your estimation is reliable. [2]
- (c) The analyst wants to publish his report in a Hong Kong car magazine. Re-write your equation from part (b)(i) so that it can be used when the price is given in Hong Kong dollars.

[1 Singapore dollar = 6.13 Hong Kong dollars]

[1]

The analyst later discovers that the prices listed are inclusive of a \$1000 administrative fee.

- (d) Explain whether the product moment correlation coefficient would differ if the analyst removed this administrative fee from his data. [1]

- 11 The masses of mangoes in kg have the distribution $N(0.3, 0.1^2)$.

- (a) Find the probability that two randomly selected mangoes weigh at least 0.5 kg in total. [2]

Mark sells fruit boxes, each of which contain 10 randomly selected blueberries and 2 randomly selected mangoes. Let the number of unblemished blueberries in a randomly selected fruit box be U . On average, the probability of a blueberry being unblemished is p .

- (b) State a reason, in the context of the question, why U might **not** be well-modelled by a binomial distribution. [1]

Assume now that U has the distribution $B(10, p)$. The probability that a randomly selected fruit box contains at least 8 unblemished blueberries is 0.9.

- (c) Find the value of p . [2]

Each fruit box is classified as grade A, B or C according to the following criteria:

Grade	Criteria
A	Fruit boxes that meet <u>both</u> the following criteria: 1. Mangoes weigh at least 0.6 kg in total 2. The fruit box contains at least 9 unblemished blueberries.
B	Fruit boxes that do not meet the criteria for grade A, but meet <u>both</u> the following criteria: 1. Mangoes weigh at least 0.5 kg in total 2. Contains at least 8 unblemished blueberries.
C	All fruit boxes that are not classified as grade A nor B

- (d) Show that the probability that a randomly chosen fruit box is classified as grade C is 0.316, correct to 3 significant figures. [2]
- (e) Hence find the probability that a randomly selected grade C fruit box contains at least 8 unblemished blueberries. [3]
- (f) The selling prices of a grade A, grade B and grade C fruit box are \$12, \$10 and \$6 respectively. Find the probability that the total selling price of 5 randomly selected fruit boxes is less than \$37. [3]

Mark decides to amend the criteria for grade A boxes as follows:

- **Each** mango in the box weighs at least 0.3 kg **and** the box contains at least 9 unblemished blueberries.

- (g) State how the percentage of fruit boxes classified as grade A will change, and explain your answer. [2]