

SUMMARY: INTRODUCTION TO COMPLEX NUMBERS

Representations of Complex Numbers

- **Cartesian form of complex number:** $z = x + iy$

- x and y are real numbers
- $i = \sqrt{-1}$
- Real part of z , $\text{Re}(z) = x$
- Imaginary part of z , $\text{Im}(z) = y$

- **Argand diagram representation:**

- As point $P(x, y)$
- As position vector $\overrightarrow{OP} = x\mathbf{i} + y\mathbf{j}$

- **Modulus of z :** $|z| = r = \sqrt{x^2 + y^2}$

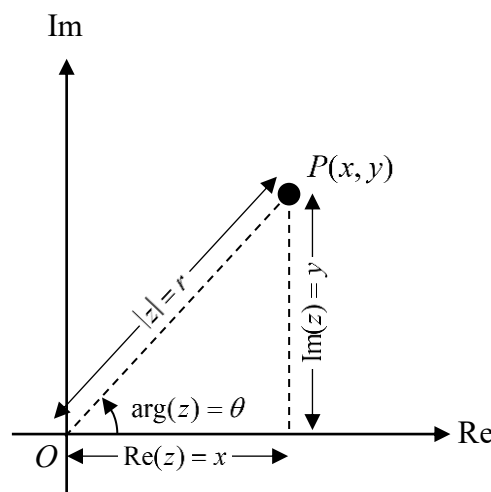
- **Argument of z :** $\arg(z) = \theta$

(1) Find basic angle $\alpha = \tan^{-1} \left| \frac{y}{x} \right|$

(2) Find argument based on quadrant that complex number lies in. (Refer to lecture notes page 17 for more details on finding argument for complex numbers that lies in 4 different quadrants)

- **Find $\text{Re}(z)$ and $\text{Im}(z)$ given modulus and argument:**

- $\text{Re}(z) = x = r \cos \theta$
- $\text{Im}(z) = y = r \sin \theta$



Solving Polynomial Equations

- **Fundamental Theorem of Arithmetic:**

- Any polynomial equation $P(z) = 0$, where $P(z)$ is a polynomial of degree n , has exactly n roots. These roots may be real or complex.

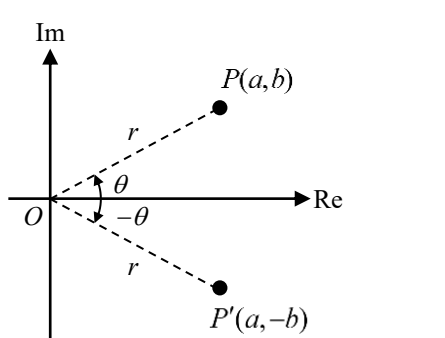
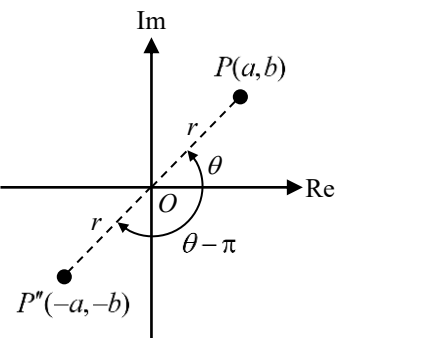
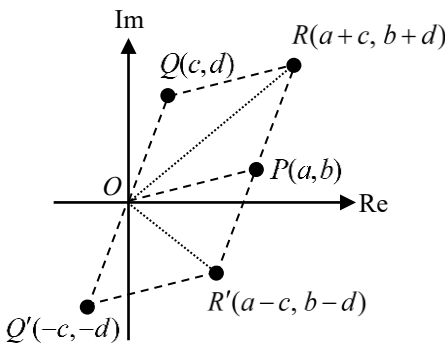
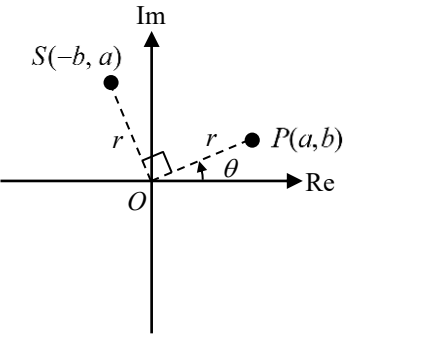
$\Rightarrow$ It is possible to express any polynomial $P(z)$ of degree n as a product of n linear factors $(z - \alpha)$, where α is a root of the equation $P(z) = 0$.

- **Complex conjugate roots:**

- If a polynomial $P(z)$ only contains real coefficients, and the complex number $z = a + ib$ is a root of the polynomial equation $P(z) = 0$, then the conjugate $z^* = a - ib$ is also a root of the equation.

Geometrical Interpretations of Algebraic Operations on Complex Numbers

Let $z = a + ib$ be denoted by point $P(a, b)$ and $w = c + id$ be denoted by point $Q(c, d)$ on the Argand diagram.

	Conjugation $z^* = a - ib$ represented by point $P'(a, -b)$ - Point P' is a reflection of point P in the real axis $z^* = z = r$ $\arg(z^*) = -\arg(z) = -\theta$
	Negation $-z = -a - ib$ represented by point $P''(-a, -b)$ - Point P'' is a reflection of point P about the origin $-z = z = r$ $\arg(-z) = \arg(z) - \pi = \theta - \pi$
	Addition and Subtraction $z + w = (a + c) + i(b + d)$ represented by point $R(a + c, b + d)$ $z - w = (a - c) + i(b - d)$ represented by point $R'(a - c, b - d)$
	Multiplication by i $iz = -b + ia$ represented by point $S(-b, a)$ - Point S is obtained by rotating point P anticlockwise about the origin by $\frac{\pi}{2}$ radians $iz = z = r$ $\arg(iz) = \arg(z) + \frac{\pi}{2} = \theta + \frac{\pi}{2}$