ANDERSON SERANGOON JUNIOR COLLEGE
2026 JC1 H2 MATHEMATICS

CHAPTER 01: STANDARD CURVES

Lesson Objectives
At the end of the chapter, you should be able to

- relate the following equations: $y = |x|$, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, $\frac{y^2}{b^2}-\frac{x^2}{a^2}=1$ and equations of parabolas with their graphs, identifying their characteristics such as symmetry, axial intercepts, turning points, asymptotes and restrictions on the possible values of x and/or y.
- use the graphic calculator to graph a curve with equations stated above.
- sketch simple parametric equations, and convert parametric equation into its Cartesian equation.

Pre-Requisites
You should already know:
Standard graphs learnt previously at the O-Levels, such as straight lines, quadratic curves, circles, the logarithmic curve (i.e. $y=\ln x$) and the exponential curve (i.e. $y=\mathrm{e}^x$).

Pre-Lesson Activity

- Watch video 'Using the Conics App' on https://education.ti.com/en/resources/test-preparation#lightbox=Test-Prep-tips-for-TI-84-Plus-CE to familarise with GC keystrokes.

- Read Section 1 and complete exercises (1) to (3).

(1) Find the equation of the line which passes through the point (2, –3) and makes an angle of $\frac{\pi}{3}$ with the positive x-axis.
[Solution]

[$y = x\sqrt{3}-2\sqrt{3}-3$]

(2) Sketch the graph of $y=4-\mathrm{e}^{\frac{1}{2}x}$, giving the **exact coordinates** where the curve cuts the axes, and the equation of the asymptotes. State any increasing and/or decreasing properties.
[Solution]

[(4ln 2, 0), (0, 3), $y = 4$]

(3) Sketch the curve of $y=-x^2+4x-2$, indicating the coordinates of intercepts and the turning point.

[GC steps in Annex 1]

Section 1: Standard Curves

1.1 Polynomial Curves

<table>
<tr><th>Type of Graph</th><th>Common Form of Equation</th><th>Graph</th></tr>
<tr><td>Straight Lines</td><td>

- $y = mx + c$; or
- $y - y_0 = m(x - x_0)$,

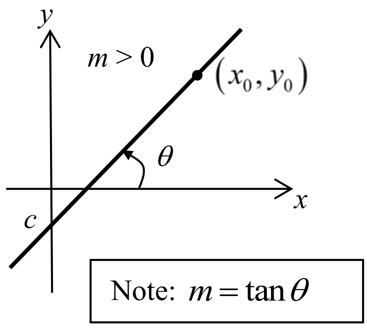
where m is the gradient, c is the y-intercept and (x_0, y_0) is a point on the line.

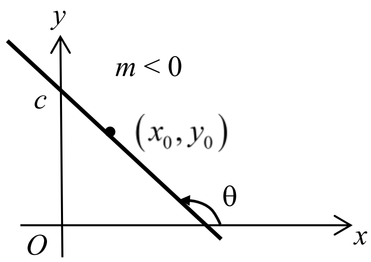
<u>Note</u>

(1) $m = \dfrac{y_1 - y_0}{x_1 - x_0}$ or $\tan\theta$, where θ is the angle that the line makes with the **positive** x-axis in the anti-clockwise direction.

(2) Vertical line: $x = h$
Horizontal line: $y = k$

</td><td>





BIG IDEA

Equivalence - is a relationship that express the equality of 2 math objects. In this case, the gradient of a straight line can be represented when you have either

1. coordinates of 2 points, or
2. Angle that the line makes with the positive x-axis.

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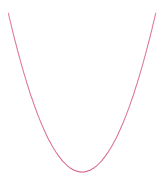
<tr><td>Quadratic Curves</td><td>

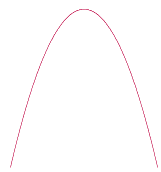
- $y = ax^2 + bx + c$ $(a \neq 0)$

</td><td>

<u>$a > 0$</u>



<u>$a < 0$</u>

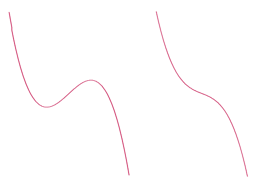


</td></tr>
<tr><td>Cubic Curves</td><td>

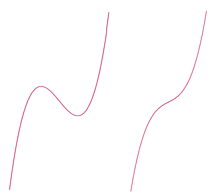
- $y = ax^3 + bx^2 + cx + d$
$(a \neq 0)$

</td><td>

<u>$a < 0$</u>

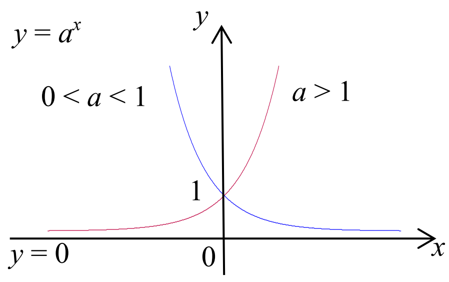
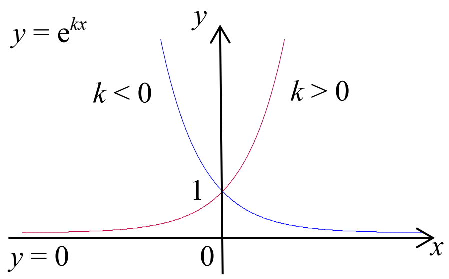
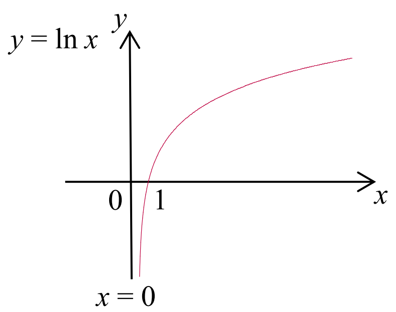


<u>$a > 0$</u>



</td></tr>
</table>

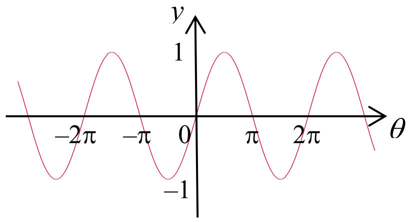
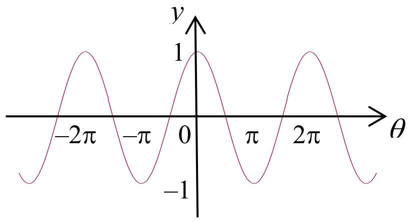
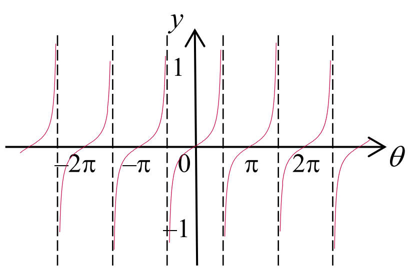
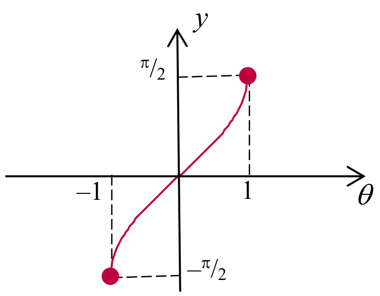
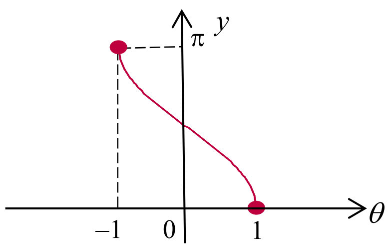
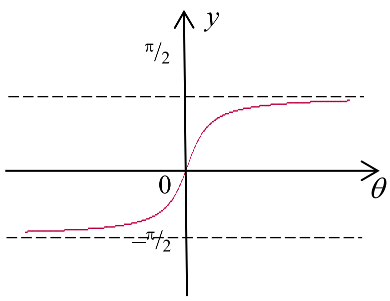
1.2 Exponential and Logarithm Curves

Type of Graph	**Common Form of Equation**	**Graph**	**Properties**
Exponential Curves	$y = a^x$, where $a > 0$ and $a \neq 1$ If $a > 1$, then a^x increases at an increasing rate, without limit If $0 < a < 1$, then a^x decreases and approaches 0		Asymptote: $y = 0$
	$y = \mathrm{e}^{kx}$ (base $\mathrm{e} \approx 2.72 > 0$) If $k > 0$, then $y = \mathrm{e}^{kx}$ is increasing at an increasing rate If $k < 0$, then $y = \mathrm{e}^{kx}$ is decreasing and approaches 0		Asymptote: $y = 0$
Logarithm Curves	$y = \ln x, \quad x > 0$		Asymptote: $x = 0$ Relationship between $y = \ln x$ and $y = \mathrm{e}^x$ **:** Reflection in the line $y = x$

💡 BIG IDEA

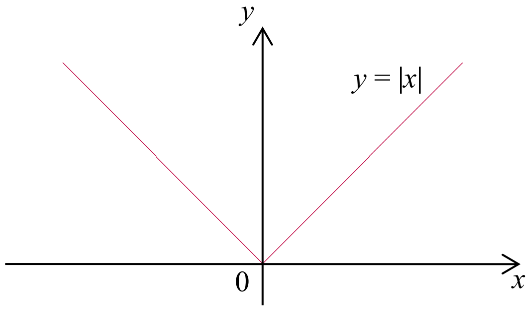
Diagrams – Cartesian graphs provide a way to visualize the behavior of relationship between two variables. It highlights features of the graphs such as showing the asymptotic behavior of a function/ cartesian equation $y = \mathrm{f}(x)$

1.3 Trigonometric Curves

Type of Graph	Equation	Graph	Properties
Trigonometric Curves	$y = \sin\theta$		(1) $-1 \le \sin\theta \le 1$ (2) Period of 360^0 [or 2π]. (3) Symmetrical about the origin.
	$y = \cos\theta$		(1) $-1 \le \cos\theta \le 1$ (2) Period of 360^0 [or 2π]. (3) Symmetrical about the y-axis.
	$y = \tan\theta$		(1) $-\infty < \tan\theta < \infty$. (2) Vertical asymptote at $\theta = \ldots, -\frac{3}{2}\pi, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3}{2}\pi, \ldots$ (3) Period of 180° [or π]
	$y = \sin^{-1} x$		Principal range is $-\frac{1}{2}\pi \le \sin^{-1} x \le \frac{1}{2}\pi$
	$y = \cos^{-1} x$		Principal range is $0 \le \cos^{-1} x \le \pi$
	$y = \tan^{-1} x$		Principal range is $-\frac{1}{2}\pi < \tan^{-1} x < \frac{1}{2}\pi$

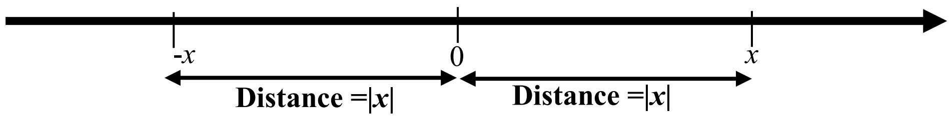
1.4 The Modulus Function $y = |x|$

Definition: The **modulus** of a real number x, denoted by $|x|$, is the non-negative numerical value (or **absolute** value) of x, i.e. the value of x without regard to its sign.

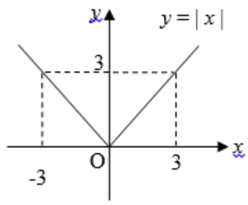


Significance of $|x|$

$|x|$ = distance of the point representing x from the point representing 0 on the number line.



Basic Properties:

	Property	**Example**
1	$\lvert x\rvert = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$	• $\lvert 3\rvert = 3$ • $\lvert -2\rvert = 2$
2	$\lvert x\rvert \geq 0$ for all real values of x	
3	$\lvert -a\rvert = \lvert a\rvert$	• $\lvert -2\rvert = \lvert 2\rvert = 2$
4	$\lvert a.b\rvert = \lvert a\rvert.\lvert b\rvert$	• $\lvert 2x\rvert = \lvert 2\rvert \times \lvert x\rvert = 2\lvert x\rvert$
5	$\left\lvert \frac{a}{b} \right\rvert = \frac{\lvert a\rvert}{\lvert b\rvert}$	• $\left\lvert \frac{a}{2} \right\rvert = \frac{\lvert a\rvert}{\lvert 2\rvert} = \frac{\lvert a\rvert}{2}$
6	$\lvert x^2\rvert = \lvert x\rvert^2 = x^2$	• $\lvert 2^2\rvert = \lvert 2\rvert^2 = 4$
7	$\sqrt{x^2} = \lvert x\rvert$	• $\sqrt{3^2} = 3$ but $\sqrt{(-3)^2} = 3 = \lvert -3\rvert$
8	If a is a positive constant, then $\lvert x\rvert = a \iff x = \pm a$	• If $\lvert x \rvert = 3$, then $x = \pm 3$ 

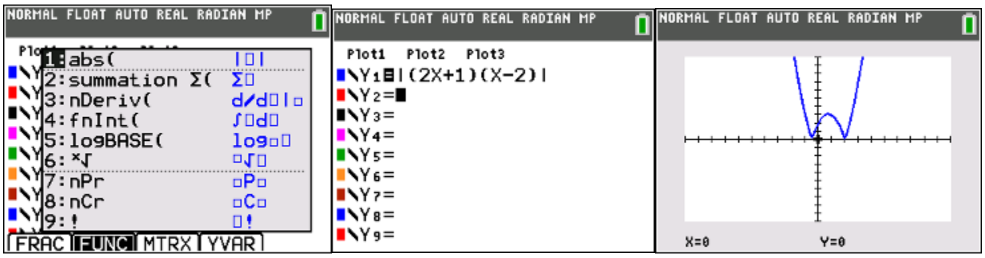
Example 1

Sketch $y=|(2x+1)(x-2)|$, clearly labeling the axial intercept and turning point.

[Solution]

GC steps for sketching Modulus function

(A) Press $\boxed{\text{y=}}$. To enter the modulus function, press $\boxed{\text{ALPHA}}$ and $\boxed{\text{window}}$ for [f2], followed by 1: abs (. Key in the rest of the equation.



(B) Key in the rest of the equation. Press $\boxed{\text{GRAPH}}$ to view the graph.

(Tip: If you cannot see the full graph, select zoom, to zoom in or out until you can see the whole graph)

Refer to Annex A for keystrokes to find x intercept, y intercept and turning point.

Note: you need to label the turning point, and x and y intercept for a quadratic function.

Test yourself

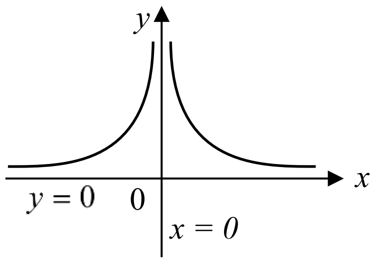
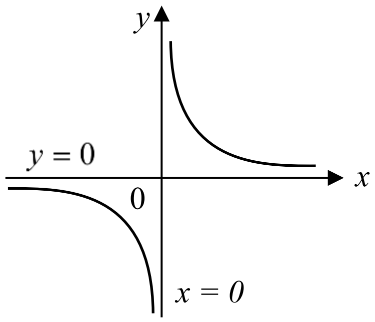
Determine whether each of the following statements is 'True' or 'False'.
Explain why it is so and/or give the correct statement.

1	$\lvert x+y\rvert=\lvert x\rvert+\lvert y\rvert$	True/False	
2	$\lvert x\rvert>2x+1 \Rightarrow (\lvert x\rvert)^2>(2x+1)^2$	True/False	

1.5 Power Function

A power function is a function $y = ax^n$, where n is any real constant number.

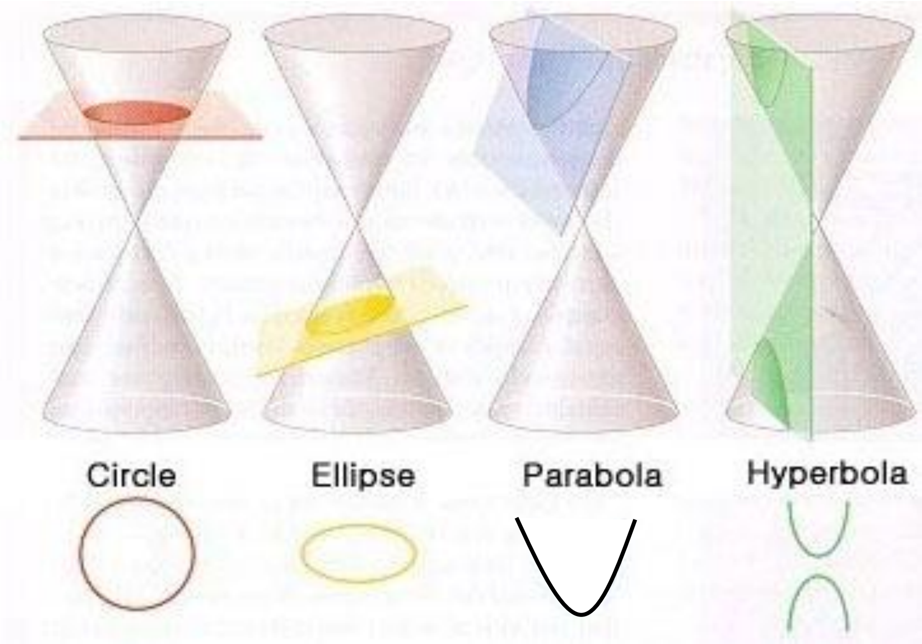
When n = 0, 1, 2 or 3, you will get a constant, linear graph, a quadratic graph and a cubic graph respectively. (Refer to Section 1.1)

Type of Graph	**Common Form of Equation**	**Graph**	**Properties**
Graphs of $y = ax^n$	When $n = -2$ and $a = 1$ $y = \frac{1}{x^2}$		**Asymptotes** $y = 0$, $x = 0$
	When $n = -1$ and $a = 1$ $y = \frac{1}{x}$		**Asymptotes** $y = 0$, $x = 0$

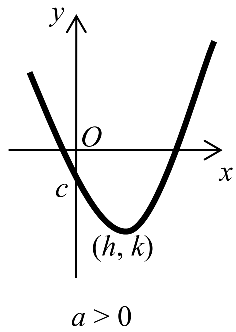
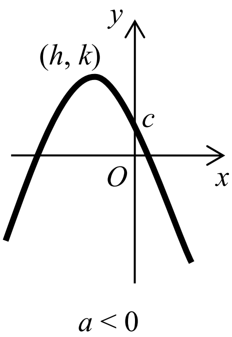
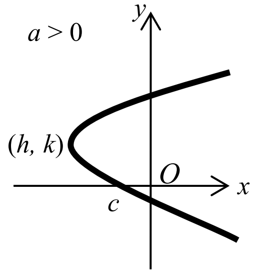
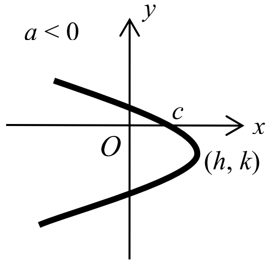
Note:
An **asymptote** is a line which a curve approaches as x and y gets very large (i.e. x or $y \to \pm\infty$).
You will learn more about this feature of graph in the chapter of Curve Sketching.

Section 2 Conics: Circles, Ellipses, Hyperbolas and Parabolas

A conic is a curve obtained by intersecting a double cone with a plane.

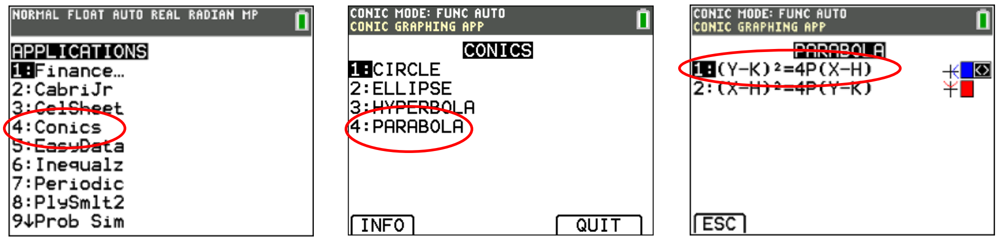


2.1 The Parabolas: $\boxed{y = ax^2 + bx + c \quad or \quad x = ay^2 + by + c, \quad (a \neq 0)}$

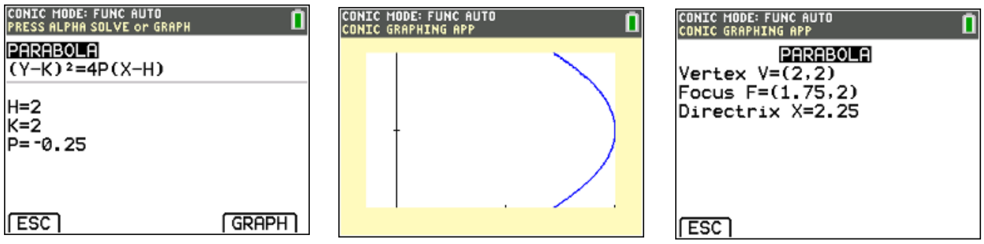
Equation	Graph
▪ $y = ax^2 + bx + c \quad (a \neq 0)$, c is the y-intercept. ▪ *Completing squares:* $y = a(x-h)^2 + k \quad (a \neq 0)$, (h, k) is the **<u>turning point.</u>** ▪ The curve is symmetrical about the line $x = h$.	 
▪ $x = ay^2 + by + c \quad (a \neq 0)$, c is the x-intercept. ▪ $x = a(y-k)^2 + h \quad (a \neq 0)$, (h, k) is called the **<u>vertex</u>** of the parabola. ▪ The curve is symmetrical about the line $y = k$.	 

GC steps for sketching a Parabola

Example: Sketch the parabola $(y-2)^2 = 2 - x$.
(A) Select Conics under [apps] and choose 4:PARABOLA.

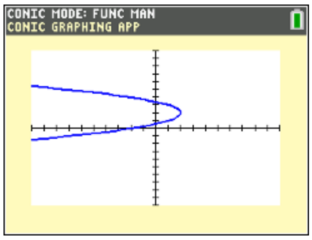


(B) Press [1] and key in the values of H, K and P.
Note that pressing [alpha][solve] gives the coordinates of the vertex



(C) Press [graph] to display your curve

Zooming in Conics
1. Press [mode] to select window settings: MAN (manual). Press [y=] to exit and now press [graph] again to view the parabola.
2. Press [zoom] select 3: Zoom Out
3. You may centralize cursor and then press [enter].



Note: Although you can still trace, you cannot solve for intersection points accurately for graphs under Conics mode.

Some notes on using GC to graph conics

The GC provides two main menus for graphing purposes: **GRAPH** and **CONICS**.

- In GRAPH menu, we may
  - input the Cartesian equation in the form $y = \mathrm{f}(x)$.
  - input the Parametric equations in the form $x = \mathrm{f}(T), y = \mathrm{g}(T)$
- In CONICS menu,
  - we select the form required (circle, ellipse, hyperbola, parabola) and input the corresponding information for the standard forms.
  - we can obtain centers and vertices, but we are unable to activate **Calc** to obtain points of intersections with the axes.

Example 2

Sketch the curve of $x = -y^2 + 4y - 2$, showing clearly its points of intersection with the axes. State the equation of its line of symmetry.

[Solution]

$x = -y^2 + 4y - 2$

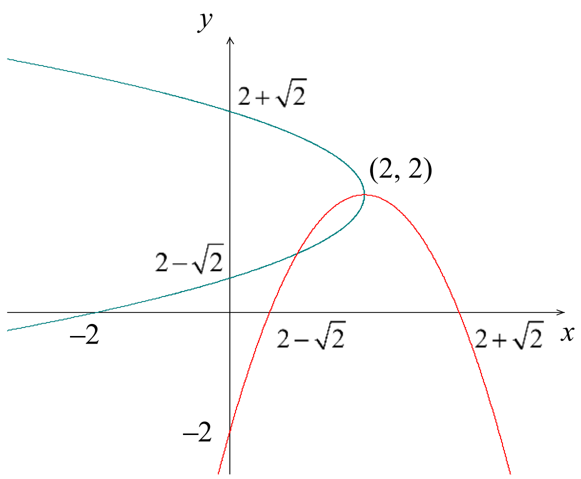
To find the intercepts:

Intercepts: .

The line of symmetry is

Remarks for Example 1

1. The graph $x = -y^2 + 4y - 2$ can be obtained from the graph of $y = -x^2 + 4x - 2$ by interchanging x and y. These 2 graphs can be obtained from each other by a reflection in the line $y = x$.
2. *Mathematical Notation:* $\sqrt{a}$ or $\surd a$, is the positive square root of the real number a. i.e
   (i) $\sqrt{4} = 2$
   (ii) If $x^2 = 4$, then $x = \pm\sqrt{4} = \pm 2$



2.2 Ellipses and Circles

Consider the equation: $x^2 + 2x + 4y^2 - 8y + 1 = 0$

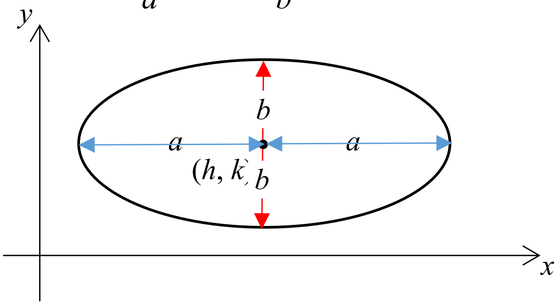
Completing the square, we will get $(x+1)^2 + 4(y-1)^2 = 4$

$$\text{i.e. } \frac{(x+1)^2}{2^2} + \frac{(y-1)^2}{1^2} = 1$$

So in general, an equation of the form $Ax^2 + By^2 + Cx + Dy + E = 0$ (where A and B are of the same sign) is also an ellipse since we will get an equation of the form $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ after completing the square.

An ellipse is a plane curve with the following equation:

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1 \ (a,\, b>0).$$

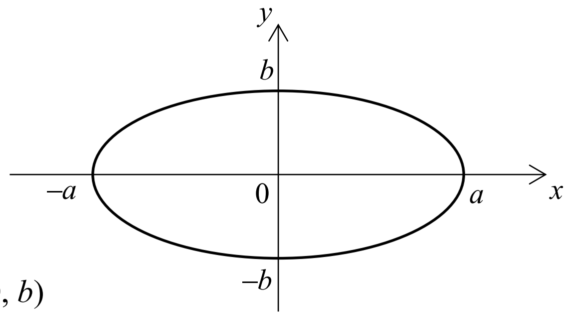


The centre of the ellipse is (h, k) and

a = horizontal length from centre to left and right vertices
b = vertical length from centre to bottom and top vertices

We can actually obtain $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ from $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. This will be discussed further under the chapter of **Transformation of Curves**.

If $\boldsymbol{h = k = 0}$, we will have $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$



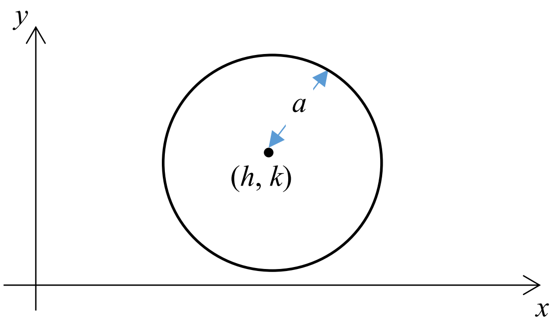
This is the equation of an ellipse with

- centre $(0,0)$
- 4 vertices $(-a, 0)$, $(a, 0)$, $(0, -b)$ and $(0, b)$
- horizontal length from centre to left OR right vertices = a
- and vertical length from centre to bottom OR top vertices = b
- two lines of symmetry: $y = 0$ and $x = 0$

If $a = b$, the equation will become $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{a^2}=1$.

i.e. $(x-h)^2+(y-k)^2=a^2$

which is the equation of a circle with centre (h, k) and radius a.



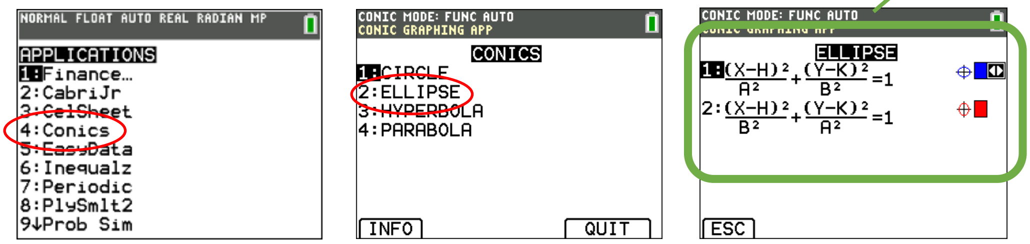
There are infinitely many lines of symmetry which pass through the centre of the circle.

GC steps for sketching an Ellipse/Circle

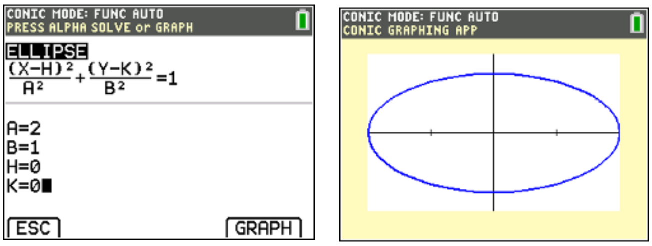
Example: Sketch the ellipse $\frac{x^2}{2^2}+y^2=1$

(A) Select Conics under [apps] and choose 2:ELLIPSE.

Since the value below x^2 is larger than that below y^2, we select option 1



(B) Press [1]* and key in the values of A, B, H and K



*Note that GC will only accept values of $A>B>0$.

(C) Press [graph] to display your curve

Example 3

The ellipse E has equation $\frac{x^2}{9}+\frac{y^2}{16}=1$.

(a) Sketch the ellipse, indicating clearly the coordinates of centre and the 4 vertices.

(b) Determine the equation of the lower half of the ellipse.

[Solution]

(a) $\frac{x^2}{3^2}+\frac{y^2}{4^2}=1$

(b) $\frac{x^2}{9}+\frac{y^2}{16}=1$

The equation of the lower half of the ellipse is

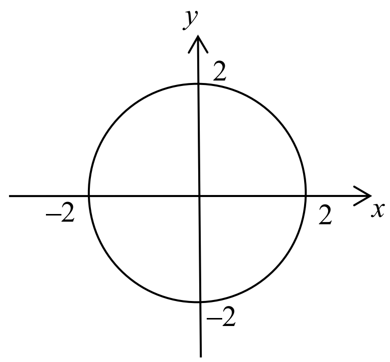
Example 4

Sketch the circle $x^2 + y^2 = 4$. Find the equation of

(i) the top half of the circle

(ii) the right half of the circle

[Solution]



(i) $x^2 + y^2 = 4 \Rightarrow y = \pm\sqrt{4-x^2}$

Equation of the top half of the circle:

(ii) $x^2 + y^2 = 4 \Rightarrow x = \pm\sqrt{4-y^2}$

Equation of the right half of the circle is

What about the equations of

(i) the bottom half of the circle; (ii) the left half of the circle?

Example 5

Sketch the graph of $x^2 + 2x + y^2 - 4y = 0$

[Solution]

$x^2 + 2x + y^2 - 4y = 0$

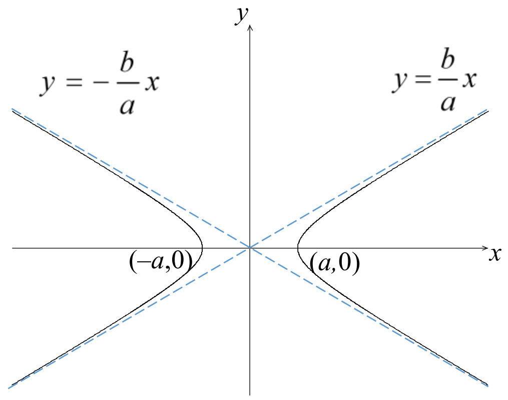
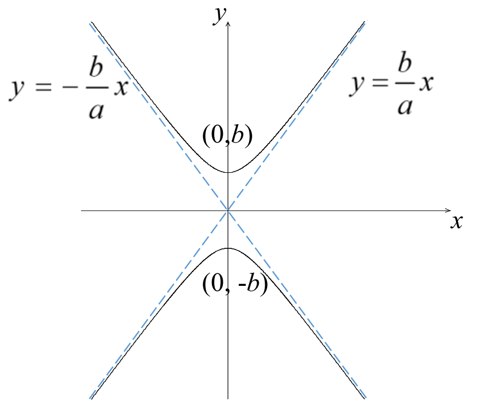
Test Yourself

Sketch the following ellipses:

(a) $9x^2 + 9y^2 = 1$ (b) $x^2 + y^2 = 12$ (c) $4x^2 + 9y^2 = 16$

2.3 Hyperbola

2.3.1 The Hyperbola $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$ or $\dfrac{y^2}{b^2}-\dfrac{x^2}{a^2}=1$ with centre at the origin

$\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1,\quad a,b>0$	$\dfrac{y^2}{b^2}-\dfrac{x^2}{a^2}=1,\quad a,b>0$
	
Asymptotes: $y=\pm\dfrac{b}{a}x$ (Oblique asymptote) When $x\to\pm\infty$, $y\to\pm\dfrac{b}{a}x$	Asymptotes: $y=\pm\dfrac{b}{a}x$ (Oblique asymptote) When $x\to\pm\infty$, $y\to\pm\dfrac{b}{a}x$

Notes
(a) The orientation of the graph depends on the form of the equation.
(b) The **distance from the centre to the vertices** will be given by **either *a* or *b*** (depending on the orientation of the graph).
(c) What is needed in your sketch:
- Centre
- Equations of asymptotes
- Equation of hyperbola
- Coordinates of vertices

BIG IDEA

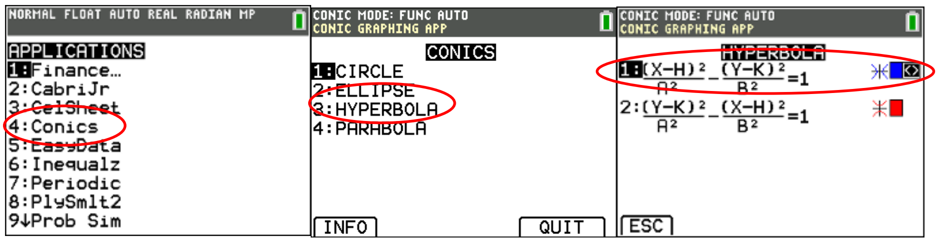
Limits – describe the behavior of a mathematical object tht varies with a parameter, as the parameter approaches a certain value or infinity. When studying graphs with asymptotes, the idea of limits as x approaches infinity (i.e. horizontal asymptote) or a singular point (i.e. vertical asymptote) are required.

GC steps for sketching a Hyperbola

Example: Sketch the hyperbola $\dfrac{x^2}{3^2}-\dfrac{y^2}{8^2}=1$

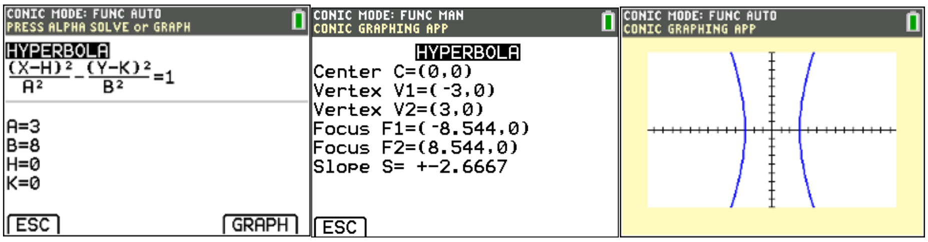
(A) Select Conics under [apps] and choose 3:HYPERBOLA.

(B) Select 1: $\dfrac{(X-H)^2}{A^2}-\dfrac{(Y-K)^2}{B^2}=1$ (since coefficient of x^2 is positive in standard form)



(C) Input A=3, B=8, H=0 and K=0. Press [alpha][enter] for centre, vertices, and slope (make sure mode is in "AUTO").

(D) Press [graph] to display your curve



Example 6

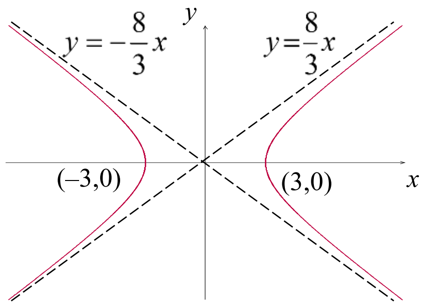
Sketch the following hyperbolas:

(a) $\dfrac{x^2}{3^2}-\dfrac{y^2}{8^2}=1$ (b) $\dfrac{y^2}{25}-\dfrac{x^2}{16}=1$

[Solution]

(a) When $y=0$, $x=\pm 3$

As $x\to\pm\infty$, $y\to\pm\dfrac{8}{3}x$



(b)

Test Yourself

Sketch (a) $3x^2-6y^2-18=0$ (b) $x^2-4y^2=-4$

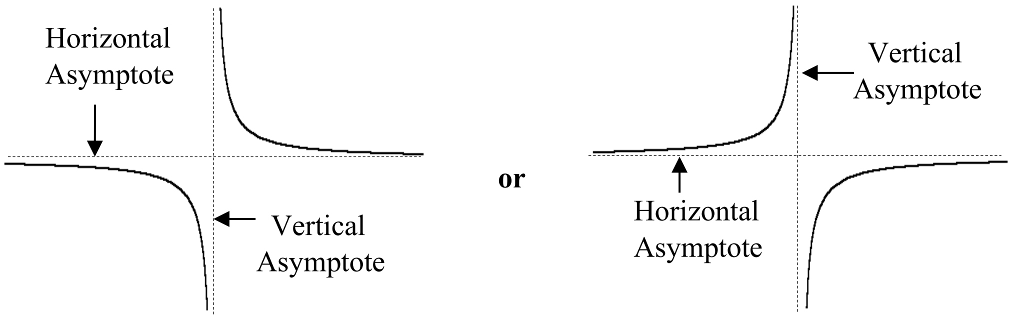
2.3.2 Rectangular Hyperbola

A hyperbola is rectangular if its asymptotes are perpendicular to each other. In particular, if the asymptotes are parallel to the axes, the equation of the hyperbola will take the form:

$$y=\frac{ax+b}{cx+d}\text{, where } c\neq 0$$

Using long division, we can always rewrite the above equation as,

$$y=A+\frac{B}{cx+d} \quad \text{where } B\neq 0\ ,\ c\neq 0$$



Note:

- When $x\to -\frac{d}{c}$, $cx+d\to 0\Rightarrow y\to \pm\infty$

  $\therefore$ The curve has vertical asymptote $x=-\frac{d}{c}$.

- When $x\to\pm\infty$, $\frac{B}{cx+d}\to 0\Rightarrow y\to A$

  $\therefore$ The curve has horizontal asymptote is $y=A$.

Note: $y=\frac{ax+b}{cx+d}$ is one form of **rational function**. There are other forms of rational function such as $y=\frac{a}{bx+c}$ and $y=\frac{ax^2+bx+c}{dx+e}$ which you have learnt in the chapter of **Curve Sketching.**

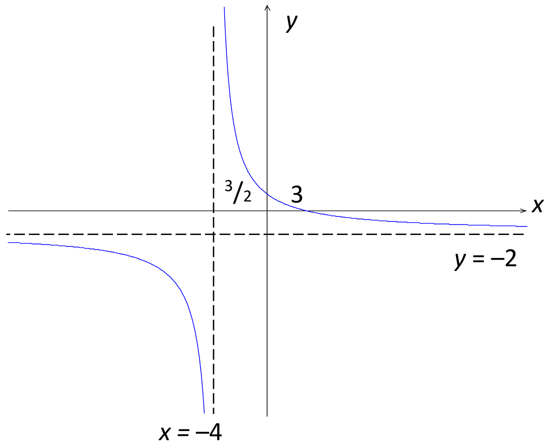
Example 7

For each of the curves below, sketch its graph showing clearly its intercepts and asymptotes:

(a) $y=\dfrac{6-2x}{x+4}$ (b) $y=2-\dfrac{6}{3+x}$

Solution:

(a)



When $x=0,\ y=\dfrac{3}{2}$

When $y=0,\ 6-2x=0 \Rightarrow x=3$

(b) $y=2-\dfrac{6}{3+x}$

Asymptotes: $x=-3$ and $y=2$

When $x=0,\ y=0$

Section 3: Some Standard Curves in Parametric Forms

The equation of a curve can also be expressed using a pair of ***parametric equations***.

For example, $x=3t^2+2,\ y=6t-1,\ t\in\mathbb{R}$

represent a pair of parametric equations where t is known as the ***parameter***. [If the values of t is not specified, then t takes all ***real*** numbers]

Each value of $t\in\mathbb{R}$ gives the coordinates of **one** point (x, y) on the curve where $(x,y)=(3t^2+2, 6t-1)$.

t	–2	–1	0	0.5	1	2	3.2	…
x	14	5	2	2.75	5	14	32.72	
y	–13	–7	–1	2	5	11	18.2	

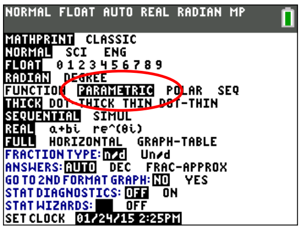
To sketch this curve, we can
1. make use of GC and draw the graph using parametric equations; or
2. convert to Cartesian equation, then draw the graph in GC using Cartesian equation.

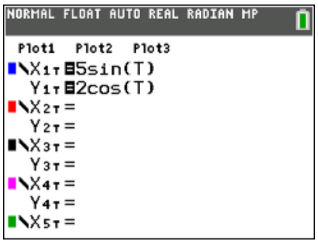
Basic GC steps for sketching a parametric curve

Example: Use your graphing calculator and draw the graph obtained for the following pair of parametric equations.

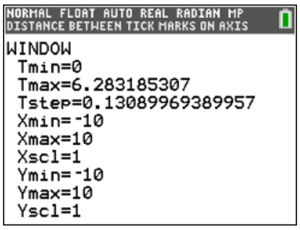
$$x = 5\sin\theta,\ y = 2\cos\theta$$

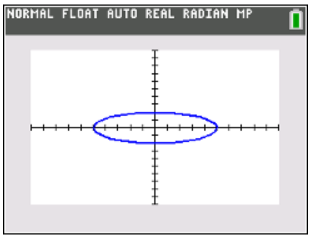
Step 1: Press [mode] and select PARAMETRIC mode.
Step 2: Press [y=] to key in the pair of equations.





*Step 3: Press [window] to see that the default setting for Tmin/Tmax is changed to a suitable *t* value depending on the range of values of *t* given in the question. Press [graph] to see the graph.



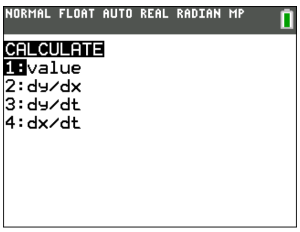


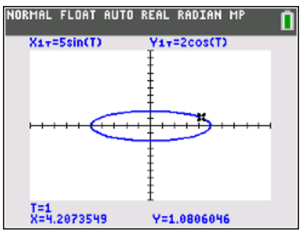
Gradient of Curve

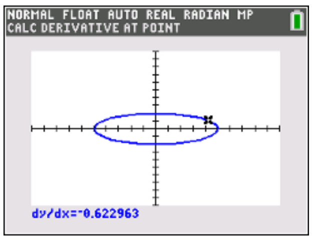
If the gradient of the curve is required, you can use your GC to find it using the following steps. (More on how to calculate $\frac{\mathrm{d}y}{\mathrm{d}x}$ for parametric equations in Differentiation)

Step 1: Press [trace] and type '1' for *t*-value and press [enter], as shown by GC, $x = 4.21$ and $y = 1.08$.

Step 2: Press [2nd][calc], select [2] for d*y*/d*x* and type '1' for *t*-value. Press [enter] to get d*y*/d*x* = -0.623.





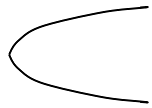


Conversion between Parametric equations and Cartesian equations

Method 1 (by elimination of *t*)

Example: Find the Cartesian equation of the curve defined by the parametric equations

$$x = 3t^2 + 2,\ y = 6t - 1.$$



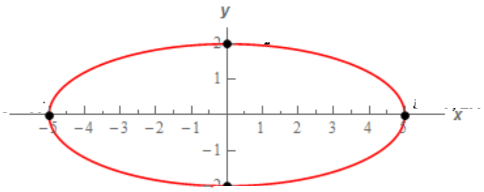
$$y = 6t - 1 \quad \Rightarrow\ t = \frac{y+1}{6} \quad \ldots\ldots(1)$$

[Solution] Sub (1) into equation for x : $x = 3\left(\frac{y+1}{6}\right)^2 + 2 \quad \Rightarrow$ Parabola

Method 2 (by using trigonometric identities)

Example: Obtain the Cartesian equation of the curve defined by the parametric equations

$$x = 5\sin\theta,\ y = 2\cos\theta.$$



[Solution]

$\sin\theta = \frac{x}{5}$ ……(1)

$\cos\theta = \frac{y}{2}$ ……(2)

Sub (1) and (2) into *identity* $\sin^2\theta + \cos^2\theta = 1$,

$$\Rightarrow \left(\frac{x}{5}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$$

Actually it's the Cartesian equation of an ellipse with centre (0,0).

Important Note: Not all parametric equations can be easily converted to Cartesian equation. It is **not** necessary to convert unless we are told to do so.

BIG IDEA

Equivalence – Equations defined by a parameter (parametric equations) can also be represented in its cartesian form IF we are able to 'get rid of' the parameter. The conversion from one form to another requires manipulation skills, and sometimes can be helpful for analysis and the finding of solutions.

Example 8

*Important: Remember to check the range of values for t.

Sketch the curve defined by the parametric equations using the GC.

(a) $x = 3t^2 + 2,\ y = 6t - 1$ (b) $x = \sin^2\theta,\ y = 2\cos\theta$

(c) $x = 2\operatorname{cosec}\theta,\ y = 3\cot\theta,\ 0 \le \theta \le \pi$

By finding the Cartesian equations of the curve above, verify the accuracy of your sketch.

Solution:

(a) To find the x-intercept:

From GC,

To find the vertex:

Note that $x = 3t^2 + 2 \ge 2$, the smallest x occurs when $t = 0$.

$\therefore$ the vertex is (2, –1).

This is a Cartesian equation of a parabola. (Verified)

(b) Using the parametric equations, we obtain the graph in Figure 1. However, in parametric mode, we cannot obtain axial intercept from GC.

Note that x and y are defined by sine and cosine, which have limits to the range of values it takes. To find the axial intercepts, find the limits on x and y.

$-1 \le \sin\theta \le 1 \quad \Rightarrow$

$-1 \le \cos\theta \le 1 \quad \Rightarrow$

Fig. 1

Using the identity $\sin^2\theta + \cos^2\theta = 1$

$$x + \left(\frac{y}{2}\right)^2 = 1 \quad \Rightarrow \quad x = 1 - \frac{y^2}{4}$$

This is an equation of a parabola. A sketch of this parabola from Cartesian equation is shown in Figure 2. Why is the sketch different from Figure 1?

A sketch of the parabola in the Cartesian equation form will show a curve for all possible values of x and y.

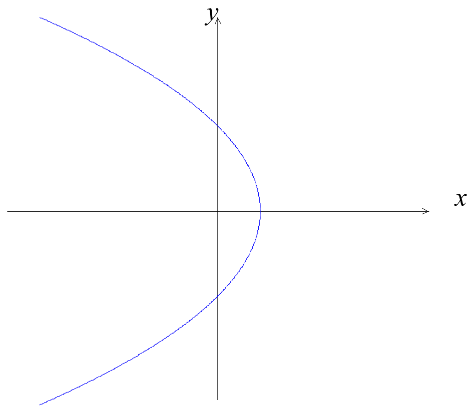
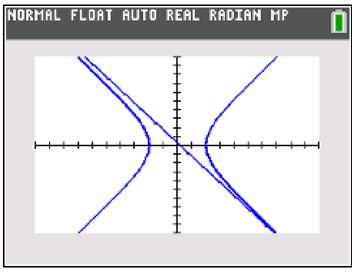


Fig. 2

(c)

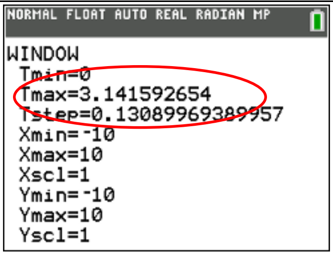
Using the GC in parametric mode to sketch $x = 2\operatorname{cosec}\theta$, $y = 3\cot\theta$, you will see this.

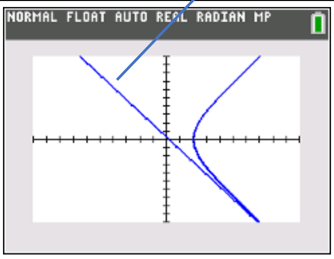


What might this line be?

Since $0 \le \theta \le \pi$, press window to change the default setting for Tmin and Tmax accordingly. Press graph.

You will still not get a 'nice graph'. In fact the 'extra line' is the 'asymptote' and not part of the graph.





In this case, we should convert to Cartesian equation before sketching the graph.

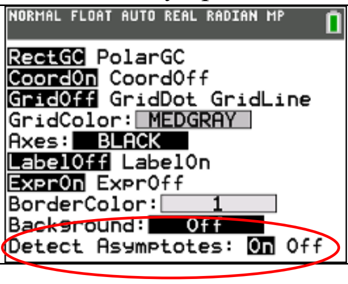
From the Cartesian equation, this is a hyperbola.

Hence, sketch only the part of hyperbola where $x \ge 2$.

Note

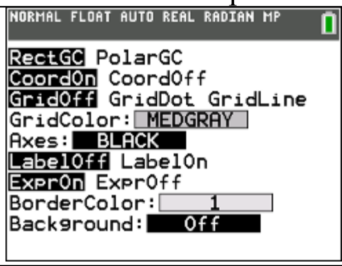
Cartesian Mode

Select 2nd ZOOM to select [FORMAT]. Make sure that Detect Asymptotes is turn on.



Parametric Mode

Unfortunately, under Parametric mode, there is no such option.



Test Yourself

Sketch the curves defined by each of the following pairs of parametric equations:

(i) $x = 4\cos^2\theta, y = 2\sin\theta$ (ii) $x = 3t,\ y = \dfrac{3}{t}$ (iii) $x = \dfrac{\sin\theta}{2},\ y = 3\cos\theta,\quad 0 \le \theta \le \pi$

<u>Example 9</u> **(**N2009/I/6)

The curve C_1 has equation $y = \dfrac{x-2}{x+2}$. The curve C_2 has equation $\dfrac{x^2}{6}+\dfrac{y^2}{3}=1$.

(i) Sketch C_1 and C_2 on the same diagram, stating the exact coordinates of any points of intersection with the axes and the equations of any asymptotes. **[4]**

(ii) Show algebraically that x – coordinates of the points of intersection of C_1 and C_2 satisfy the equation $2(x-2)^2 = (x+2)^2(6-x^2)$. **[2]**

(iii) Use your calculator to find these x – coordinates. **[2]**

Suggested Solution

(i) Given C_1: $y = \dfrac{x-2}{x+2}$ $\Rightarrow$

C_2 : $\dfrac{x^2}{6}+\dfrac{y^2}{3}=1$ $\Rightarrow$

(C_2 is an Ellipse)

<u>C1</u>

Vertical asymptotes:

Horizontal asymptotes:

<u>C1</u>	<u>C2</u>
When x=0, y=–1	When x=0, y=$\pm\sqrt{3}$
When y=0, x=2	When x=0, y=$\pm\sqrt{6}$

(ii) Given C_1: $y = \dfrac{x-2}{x+2}$ ———— (1)

C_2: $\dfrac{x^2}{6} + \dfrac{y^2}{3} = 1$

$x^2 + 2y^2 = 6$ ———— (2)

Sub (1) into (2), $x^2 + 2\dfrac{(x-2)^2}{(x+2)^2} = 6$

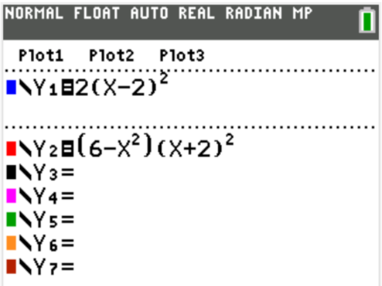
$x^2(x+2)^2 + 2(x-2)^2 = 6(x+2)^2$

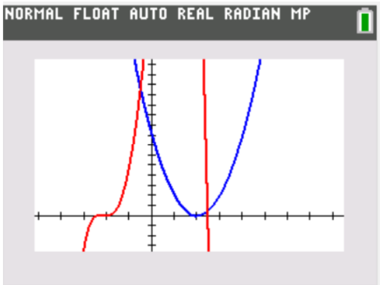
$2(x-2)^2 = 6(x+2)^2 - x^2(x+2)^2$

$2(x-2)^2 = (6-x^2)(x+2)^2$ (shown)

(iv) Using G.C: x=

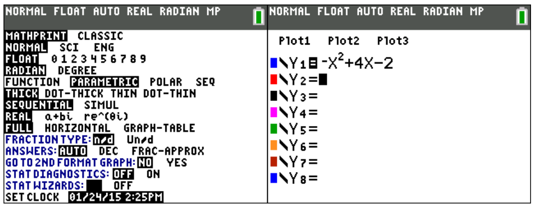
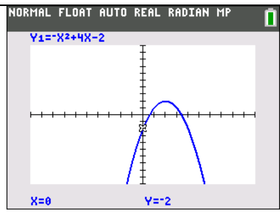
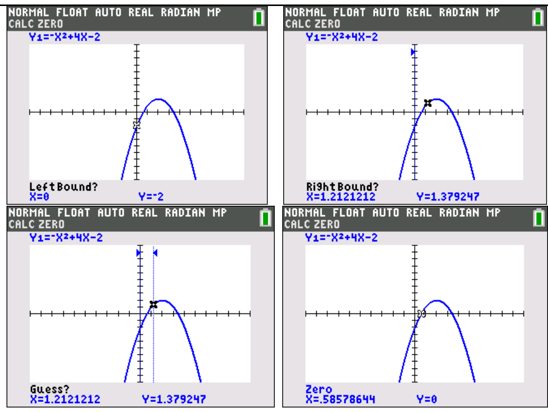
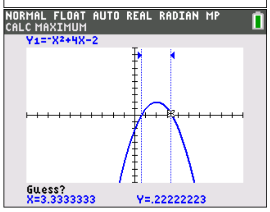
(corrected to 3 sig fig)



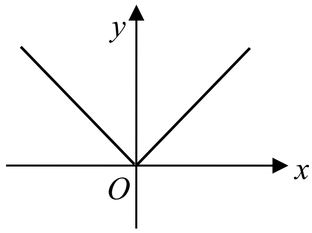
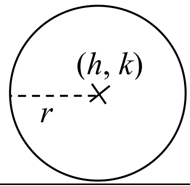
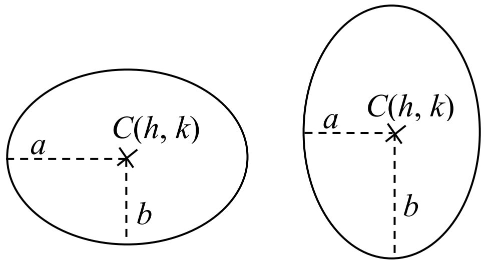
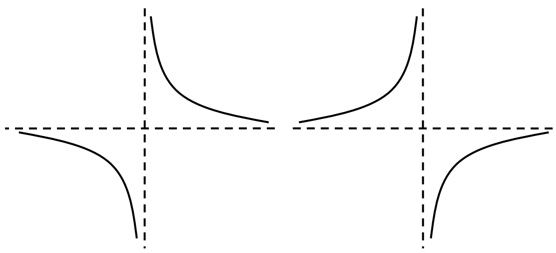
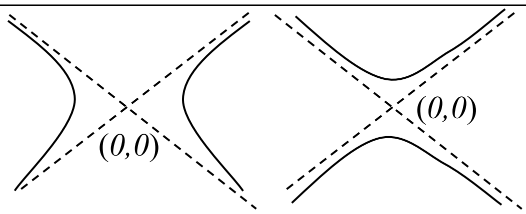
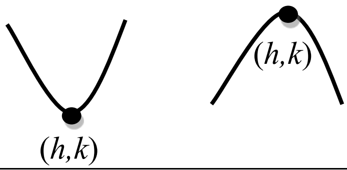


ANNEX 1: GC Keystrokes

Pre-Lesson Exercise Q3

Keystrokes	Screen
• To sketch Cartesian equations: 1. Press [mode] and make sure the calculator is in FUNC (Function mode). 2. Press [y=] to key in the equation. (Note: to key in negative, press [(-)] and not [−]) 3. Press [GRAPH] view the graph.	
• To find y-intercept: From the graph, press [2nd] [TRACE] to select [CALC] then select 1: value, then enter $x = 0$. Point of intersection with y-axis ≡ (0,–2)	
• To find x-intercept: 1. Press [2nd] [TRACE] to select [CALC], then select 2: zero. 2. Upon prompt "**Left** Bound?", move cursor to the **left** of the x-intercept using the arrow keys. Press enter to confirm. 3. Upon prompt "**Right** Bound?", move cursor to the **right** of the x-intercept using the arrow keys. Press enter to confirm. 4. Upon prompt "Guess", press enter to confirm. (Your cursor should be within the 2 bounds) Points of intersection with x-axis ≡ (0.586,0), (3.41,0) ***Note that the answers obtained here is <u>NOT exact</u> coordinates***. *To find exact coordinates, solve algebraically using quadratic formula.*	
• To find maximum point: 1. Press [2nd] [TRACE] to select [CALC], then select 4: maximum. 2. Apply the same process for finding zero upon the prompts "Left Bound?", "Right Bound?" and "Guess" to move cursor to the left and right of the maximum point. *(Tip: In cases where there are more than one turning point in a graph, make sure the two lines bounds one turning point.)* Maximum point given is (2,2).	

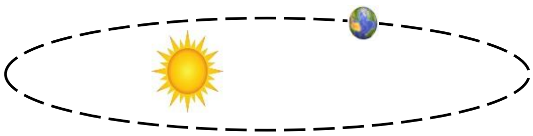
ANNEX 2: Standard Curves

Standard Curve	**Common Forms of Equation**	**Graph**	**Asymptote (s)**
Modulus Function	$y = \|x\|$		NIL
Circle	$(x-h)^2 + (y-k)^2 = r^2$ $Ax^2 + Ay^2 + Cx + Dy + E = 0$		NIL
Ellipse	$Ax^2 + By^2 + Cx + Dy + E = 0$ E1: $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$ E2: $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$	 E1: $a > b$ E2: $b > a$	NIL
Hyperbola	Rectangular hyperbola (Rational Function) $y = \dfrac{ax+b}{cx+d}$ $(c \neq 0)$ ⇓ Use Long division $y = A + \dfrac{B}{cx+d}$ $(B \neq 0)$	 Note: Graph under Functions mode!	$y = A,$ $x = -\dfrac{d}{c}$
	H1: $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ H2: $\dfrac{y^2}{b^2} - \dfrac{x^2}{a^2} = 1$	 H1 H2	H1: $y = \pm\dfrac{b}{a}x$ H2: $y = \pm\dfrac{b}{a}x$
Parabola	$y = ax^2 + bx + c$ $(a \neq 0)$ $y = a(x-h)^2 + k$ $(a \neq 0)$	$a > 0$  $a < 0$	NIL
	$x = ay^2 + by + c$ $(a \neq 0)$ $x = a(y-k)^2 + h$ $(a \neq 0)$	$a > 0$  $a < 0$	NIL

STEAM – Applications of Conic Sections in Real Life

There are many different types of curves that are studied in Mathematics and used in real life applications. Parabolic mirrors are reflective surfaces designed such that its shape is generated by a parabola revolving around its axis. These mirrors are used to collect or project energy such as light, sound, or radio waves at (or from) the focus of the parabola. Hyperbolic as well as parabolic mirrors and lenses are used in systems of telescopes. Conic sections are used in astronomy where the paths of the planets around the sun are ellipses with the sun at one focus.

Can you answer this question?



The furthest distance Earth is from the sun is 152 million km which is 5 million km more than Earth's closest distance to the sun. Find a Cartesian equation to describe the orbit of Earth about the sun.

References

Text(s): Bostock, L. & Chandler, F.S. (2002). *Pure Mathematics 1*. Pg. 354. Oxford University Press.

Bostock, L., Chandler, F. S. & Rourke, C (2014). *Further Pure Mathematics*. Oxford University Press.

Website(s): http://mathworld.wolfram.com/ConicSection.html

http://en.wikipedia.org/wiki/Conic_section

http://mathinsite.bmth.ac.uk/applet

http://tutorial.math.lamar.edu/classes/alg/graphing_functions.aspx (description of standard curves)

My Notes