RAFFLES INSTITUTION
H2 Mathematics (9758)
2025 Year 5

Tutorial 7A: Sequences and Series

Section A (Basic Questions)

1 Write down the series given by the summation notation, showing the first three terms and the last two terms whenever possible.

(a) $\displaystyle\sum_{r=1}^{8}\left(\frac{4}{r}+\frac{r}{4}\right)$ **(b)** $\displaystyle\sum_{r=4}^{30}(5-3r)$ **(c)** $\displaystyle\sum_{j=5}^{15}(m+3j)$, where m is a constant

2 For each of the following series, write down the rth term and express the series using the summation notation.

(a) $3+6+9+\ldots+369$ **(b)** $(-4)+(-1)+(2)+\ldots+(53)$

(c) $4+2+1+\frac{1}{2}+\frac{1}{4}+\ldots$ **(d)** $\frac{1}{2^2}-\frac{2}{3^2}+\frac{3}{4^2}-\frac{4}{5^2}+\ldots+\frac{7}{8^2}-\frac{8}{9^2}$

3 Justify whether each of the following statements is true or false.

(a) $\displaystyle\sum_{r=1}^{2n}r^4=\sum_{r=0}^{2n-1}(r+1)^4$ **(b)** $\displaystyle\sum_{r=1}^{n}1=\frac{1}{2}n(n+1)$

(c) $\displaystyle\sum_{r=1}^{n}r^2=\left(\sum_{r=0}^{n}r\right)^2$ **(d)** $\displaystyle\sum_{r=1}^{3n}r^5=\sum_{r=0}^{3n}r^5$

4 It is given that $\displaystyle\sum_{r=1}^{n}u_r=5n+2^n$, where u_r is an expression in terms of r. Find the value of

(i) $\displaystyle\sum_{r=1}^{10}u_r$ **(ii)** $\displaystyle\sum_{r=5}^{10}u_r$ **(iii)** u_{10}

Section B (Discussion Questions)

1 Use the following standard results

$$\sum_{r=1}^{n} a = na \text{, where } a \text{ is a constant,} \qquad \sum_{r=1}^{n} r = \frac{1}{2}n(n+1),$$

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1) \quad \text{and} \quad \sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

to evaluate each of the following sum.

(a) $\displaystyle\sum_{r=0}^{15}(r^2-4)$

(b) $\displaystyle\sum_{r=4}^{11}r^2(r-3)$

(c) $\displaystyle\sum_{r=1}^{10}r(2-r)$

For each part, use the graphing calculator to verify that your answer is correct.

2 A sequence is such that $u_1 = p$, where p is a constant, and $u_{n+1} = 2u_n - 5$, for $n > 0$.

(i) Describe how the sequence behaves when

(A) $p = 7$, [1]

(B) $p = 5$. [1]

(ii) Find the value of p for which $u_5 = 101$. [2]

[(ii) 11**]**

3 It is given that $\displaystyle\sum_{r=1}^{n}\frac{1}{r(r+1)} = 1 - \frac{1}{n+1}$.

(a) Find $\displaystyle\sum_{r=5}^{n+1}\frac{1}{r(r+1)}$.

(b) Give a reason why the series in part **(a)** is convergent and state the value of $\displaystyle\sum_{r=5}^{\infty}\frac{1}{r(r+1)}$.

[**(a)** $\frac{1}{5} - \frac{1}{n+2}$ **(b)** $\frac{1}{5}$]

4 A sequence $u_1, u_2, u_3, \ldots$ is such that $u_1 = 2$, $u_{n+1} = \frac{u_n}{5} - 3$ for all $n \geq 1$, where k is a constant.

(i) Write down the values of u_2, u_3 and u_4. [2]

(ii) Given that the sequence converges to l, find the value of l. [1]

[**(i)** $u_2 = -2.6, u_3 = -3.52, u_4 = -3.704$, **(ii)** $l = -3.75$]

5 The sum, S_n, of the first n terms of a sequence $u_1,\ u_2, u_3, \ldots$ is given by $S_n = n^2\left(An+B\right)$, where A and B are non-zero constants.

(i) Find an expression for u_n in terms of A, B and n, simplifying your answer. [3]

(ii) It is also given that the second term is 6 and the sixth term is 218. Find the values of A and B. [2]

(iii) Determine if the sum to infinity exists. [1]

[**(i)** $u_n = A\left(3n^2-3n+1\right)+B\left(2n-1\right)$, **(ii)** $A=3$, $B=-5$, **(iii)** No]

6 For this question, you may use the results

$$\sum_{r=1}^{n} r^2 = \frac{n\left(n+1\right)\left(2n+1\right)}{6} \quad \text{and} \quad \sum_{r=1}^{n} r^3 = \frac{n^2\left(n+1\right)^2}{4}.$$

(i) Find $\sum_{r=1}^{n} r^2\left(2r-1\right)$ in terms of n. [2]

(ii) Find $\sum_{r=1}^{n} r^2\left(r-1\right)$ in terms of n. Hence find $\sum_{r=2}^{n-1} r\left(r+1\right)^2$ in terms of n. [5]

(iii) Without using a graphing calculator, find the sum of the series
$4\left(25\right)-5\left(36\right)+6(49)-7(64)+\cdots-59\left(3600\right)$. [3]

[**(i)** $\frac{1}{6}n\left(n+1\right)\left(3n^2+n-1\right)$

(ii) $\frac{1}{12}n\left(n+1\right)\left(3n+2\right)\left(n-1\right)$, $\frac{1}{12}n\left(n+1\right)\left(n-1\right)\left(3n+2\right)-4$

(iii) -108836]

7 It is given that $\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2\left(n+1\right)^2$.

(a) Find $\sum_{r=n}^{2n}\left(r+1\right)^3$ where $n>1$. Express your answer in the form

$$\frac{1}{a}\left(n+1\right)^2\left[a\left(2n+1\right)^2-n^2\right],$$

where a is a constant to be determined. [4]

(b) Find $1^3+3^3+5^3+\ldots+(2k-3)^3+(2k-1)^3$, where k is a positive integer. [2]

[**(a)** $a=4$ **(b)** $2k^4-k^2$]

THE END