



NANYANG JUNIOR COLLEGE

DEPARTMENT OF MATHEMATICS

2026 J2 H2 FM

Linear Algebra (Set 1)

Instruction: Attempt the questions marked with *.

1* Do not use a calculator in answering this question.

The matrix $\mathbf{P}$ is given by $\mathbf{P} = \begin{pmatrix} 5 & 3 & -1 & 9 \\ 3 & 2 & -1 & 5 \\ 1 & 1 & 1 & -1 \end{pmatrix}$.

(a) Find a matrix $\mathbf{B}$ such that $\mathbf{BP} = \begin{pmatrix} 1 & 1 & 1 & -1 \\ 0 & 1 & a & b \\ 0 & 0 & 1 & c \end{pmatrix}$, where a , b and c are some constants to be determined. [3]

(b) Hence solve the equation $\mathbf{Px} = \begin{pmatrix} -4 \\ 1 \\ 2 \end{pmatrix}$. [3]

(c) The equations below represent three planes
 $5x + 3y - z = d$
 $3x + 2y - z = e$
 $x + y + z = f$

where d , e and f are constants.

(d) What possible geometrical relationship(s) can you say about the three planes? Justify your answer. [2]

2* The sets A and B are defined as follows:

$$A = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 : x + y - z = 0 \right\}, B = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 : 2x + 3y - 5z = 1 \right\}.$$

(a) Show that A is a subspace of $\mathbb{R}^3$ but B is not a subspace of $\mathbb{R}^3$. [3]

(b) Determine if $A \cup B$ is a subspace of $\mathbb{R}^3$. [2]

- 3* (a) Given that $\mathbf{A}$ is an invertible (non-singular) 3×3 matrix, show that the first column of

$$\mathbf{A}^{-1} \text{ is the solution of the equation } \mathbf{Ax} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}. \quad [1]$$

- (b) Give the equations whose solutions are the second and third columns of $\mathbf{A}^{-1}$ respectively. [1]

- (c) Using parts (a) and (b), find the inverse of $\mathbf{P}$, where $\mathbf{P} = \begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix}$ for unknown constants a, b and c . [3]

- (d) Using the result in part (c), find the inverse of the matrix $\mathbf{Q}$ where $\mathbf{Q} = \begin{pmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{pmatrix}$. [1]

4 **Unsupported answers from a graphing calculator are not allowed in this question.**

In response to the government's effort to promote food source diversification and local production, a local entrepreneur set up Apollo Poultry Farm. The farm has a total poultry rearing area of 200 m^2 . The entrepreneur plans to rear c chickens, d ducks, g geese and q quails to be sold every three months. The rearing space needed for each chicken, duck, goose and quail is 0.5 m^2 , 1.5 m^2 , 2 m^2 and 0.5 m^2 respectively. Each chicken, duck, goose and quail consumes 10 kg, 15 kg, 20 kg and 10 kg of poultry feed in three months. The labour cost of caring for each chicken, duck, goose and quail for three months is \$5, \$10, \$15 and \$10 respectively, which will be paid to a farm hand as salary. Every month, the entrepreneur has available 1000 kg of poultry feed and a budget of \$800 to pay the farm hand.

- (a) Express the constraints in rearing space, feed and labour cost as inequalities in terms of c, d, g and q . [3]

For the following parts, assume that the resources in rearing space, feed and labour cost are all fully utilised.

- (b) Apply elementary row operations to solve the system of linear equations, expressing c, d and g in terms of q . [4]
- (c) For every chicken, duck, goose and quail sold, the entrepreneur expects to earn profits of \$5, \$12, \$20 and \$10 respectively. Find the maximum profit that he can make in three months, and state the corresponding values of c, d, g and q . [4]
- (d) Based on the answer obtained, what advice would you give to the entrepreneur? [1]