RIVER VALLEY HIGH SCHOOL
2026 JC2 Preliminary Examination
Higher 2

NAME	

CLASS	2	5	J		

INDEX NUMBER	

MATHEMATICS **9758/01**

Paper 1 **16 September 2026**

3 hours

Additional Materials: Printed Answer Booklet
List of Formulae (MF27)

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This document consists of **6** printed pages and **2** blank pages.

1 It is given that $y=\tan^{-1}(px)$, where p is a constant and $-\frac{\pi}{2}<\tan^{-1}(px)<\frac{\pi}{2}$.

(a) Show that $\left(1+p^2x^2\right)\frac{dy}{dx}=p$. [2]

(b) By further differentiation of the result in part **(a)**, find the first two non-zero terms in the Maclaurin series for y. [4]

2 **(a)** Solve the inequality $\left|e^x-2\right|>\frac{1}{x^2-1}$ using a graphical approach. [3]

(b) Without the use of a graphing calculator, solve the inequality $\frac{x+2}{x-1}\geq\frac{2}{x-3}$. [3]

3 A curve C has equation $y-2\sqrt{3}x+3e^{-y}=0$, for $x>0$. The normal to C at a point P has positive gradient and makes an angle of 30° with the x-axis. Find the exact equation of this normal. [6]

4 With reference to the origin O, the points A and B are such that $\overrightarrow{OA}=\mathbf{a}$ and $\overrightarrow{OB}=\mathbf{b}$, where **a** and **b** are both non-zero and non-parallel.
The point X lies on OA such that $OX:XA=2:1$. The point Y lies on OB such that $OY:YB=3:2$. The lines AY and BX intersect at the point P.

(a) Express $\overrightarrow{OP}$ in terms of **a** and **b**. [5]

(b) Point Z lies on AB such that $4AZ=3BZ$. Express $\overrightarrow{OZ}$ in terms of **a** and **b**. [1]

(c) Show that O, P and Z are collinear. [2]

5 **(a)** By writing $\cos 3x$ as $\cos(x+2x)$ and applying addition formula, show that

$$\cos 3x=\cos x\left(4\cos^2 x-3\right).$$ [2]

(b) By using integration by parts, or otherwise, find $\int \tan x \sin 3x \, dx$. [5]

6 A colony of bacteria is cultivated in a tank under experimental conditions. The amount of bacteria is y (in grams), at time t (in days) after the start of the experiment. The total growth rate of the bacteria is determined by the following two factors.

- The bacteria reproduce naturally and grow a rate directly proportional to the amount of bacteria present.
- An automated feeder continuously supplies a liquid nutrient solution to the tank. This nutrient solution stimulates an additional growth rate which is directly proportional to the time taken from the start of the experiment.

Initially, there is 3 grams of bacteria, and the total growth rate of the bacteria is 6 grams per day. After 1 day, the additional growth rate, due to the liquid nutrient, is 3 grams per day.

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}t} = 2y + 3t$. [2]

(b) By substituting $u = 2y + 3t$, show that the differential equation can be written as $\frac{\mathrm{d}u}{\mathrm{d}t} = 2u + 3$. [2]

(c) Find u in terms of t and hence y in terms of t. [5]

7 Let a and b be fixed positive real numbers. A variable line C passing through (a, b) with varying gradient m has equation $y - b = m(x - a)$, where $m < 0$. The line C cuts the x-axis and y-axis at the points P $(p, 0)$ and Q $(0, q)$ respectively.

(a) **(i)** Show that A, the area of triangle OPQ, where O is the origin, is given by

$$A = \frac{1}{2}\left(2ab - a^2 m - \frac{b^2}{m}\right) \text{ units}^2.$$ [2]

(ii) Using differentiation, find the value of m that gives the minimum A. [3]

(b) As m varies, the line C rotates about point (a, b) causing the points P and Q to move along the x- and y- axes, respectively. At the instant when $m = -1$, point P is moving to the right, where the rate of change of p is 1 unit per second and point Q is moving downwards. At this instant, find the rate of change of q, in units per second, leaving your answer in terms of a and b. [4]

8 **(a)** A sequence $u_1, u_2, u_3, ...$ is such that $u_1 = 5$ and $u_{n+1} = 3u_n + 2n$.

It is known that the nth term of this sequence is given by

$$u_n = a(3^n) + bn + c,$$

where a, b and c are constants.

Find the constants a, b and c. [4]

(b) **(i)** State the value of k such that $\sum_{r=1}^{2n} \frac{1}{r} = \sum_{r=1}^{n} \left(\frac{1}{2r+k} + \frac{1}{2r} \right)$. [1]

(ii) Hence, show that $\sum_{r=n+1}^{2n} \frac{1}{r} = \sum_{r=1}^{n} \left(\frac{1}{2r-1} - \frac{1}{2r} \right)$. [2]

(iii) Write down the value of $\lim_{n \to \infty} \sum_{r=n+1}^{2n} \frac{1}{r}$. [1]

9 **(a)** The function f is defined by $\mathrm{f} : x \mapsto 4 + \mathrm{e}^{-x}$, where $0 < x \le 4$.

(i) Show that f^{-1} exists. [1]

(ii) Find the $\mathrm{f}^{-1}(x)$ and the domain of f^{-1}. [3]

(b) The function g is defined by

$$\mathrm{g} : x \mapsto \mathrm{f}(x), \text{ for } 0 < x \le 4,$$

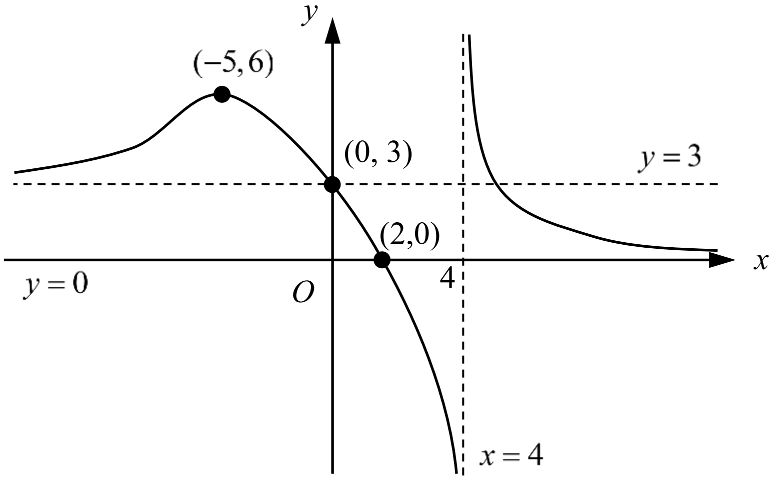
and that $\mathrm{g}(x) = \mathrm{g}(x+4)$ for all real values of x.

(i) Explain why the composite function g^2 exists. [1]

(ii) Find the exact value of $\mathrm{g}^2(2)$. [2]

(iii) State the range of the composite function g^2. [2]

10 **(a)** The diagram shows the graph of $y = \mathrm{f}(x)$. The graph intersects the axes at (2,0) and (0,3). It has a turning point at (–5, 6) and asymptotes with the equations $x = 4$, $y = 0$ and $y = 3$.



On separate diagrams, sketch each of the following graphs. For each graph, state the equation of any asymptotes, the coordinates of any points where the curve crosses the axes and the coordinates of any turning points.

(i) $y = \mathrm{f}(|x|)$, [2]

(ii) $y = \dfrac{1}{\mathrm{f}(x)}$. [3]

(b) Express $\dfrac{9x+5}{3x+2}$ in the form $a - \dfrac{b}{3x+2}$ where a and b are positive constants.

Hence, describe a sequence of transformations which transform the graph of $y = \dfrac{9x+5}{3x+2}$ to the graph of $y = \dfrac{1}{x}$. [5]

11 Do not use a calculator in answering this question.

(a) The function f is defined by $\mathrm{f}(z) = (2-\mathrm{i})z^2 + 10z + 20 + 10\mathrm{i}$.

(i) Find the roots of $\mathrm{f}(z) = 0$. [3]

The function p is defined by $\mathrm{p}(z) = z^4 + az^3 + bz^2 + cz + d$, where a, b, c and d are real numbers. It is given that $\mathrm{f}(z)$ is a factor of $\mathrm{p}(z)$.

(ii) Without any further calculations, state all the roots of $\mathrm{p}(z) = 0$, explaining your answer clearly. [2]

(b) Solve the simultaneous equations, giving z and w in the form $a + b\mathrm{i}$, where a and b are real numbers.

$$z^* w = 8 - 20\mathrm{i},$$
$$w - 2z = 10.$$

[5]

12 Underwater sonar buoys are deployed, in the ocean, to detect and monitor threats in the ocean and on the ocean surface. Two underwater sonar buoys, buoy A and buoy B, are located at points A and B respectively. The underwater sonar buoys are in the water body.

Points are defined relative to an origin $(0,0,0)$ on the ocean bed, with units in metres.

The ocean surface is modelled as the plane with equation $3y + 4z = 240$. A submarine moves along a straight path with equation $\frac{x}{2} = y - 20 = \frac{-z+13}{2}$. A and B have coordinates $(-10,31,5)$ and $(4,17,19)$ respectively.

(a) Find a cartesian equation of the plane containing the path of the submarine and buoy B. [3]

(b) Determine the acute angle between the path of the submarine and the ocean surface. [2]

(c) An unidentified boat on the ocean surface is detected by buoys A and B. The boat is found to be equidistant from both buoys A and B. Describe geometrically the set of all possible positions of the location of the unidentified boat. [5]

(d) At low tide, the ocean surface falls below the current ocean surface at a perpendicular distance of 0.8 metres. The ocean surface at low tide is also modelled as a plane. Find the cartesian equation of this plane. [2]

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