Hwa Chong Institution 华侨中学
Mathematics (SMTP)

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MA304.2.3 – Trigonometric Ratios of Any Angle

At the end of this unit, you should be able to
- **understand that trigonometric ratio of an angle is closely related to its basic angle**
- **understand and use ASTC to determine sign of trigonometric ratios of angles**
- **use ASTC to reduce trigonometric ratios of any angle in terms of its basic angle**
- **simplify trigonometric ratios of negative angles**

E1. Given $-360^\circ < A < 720^\circ$ and that A has a basic angle $\alpha = 80^\circ$, find all the values of A if A is in the

(a) 2^{nd} quadrant

$A_1 = 180^\circ - 80^\circ$
$= 100^\circ$

$A_2 = 100^\circ + 360^\circ$
$= 460^\circ$

$A_3 = 100^\circ - 360^\circ$
$= -260^\circ$

(b) 3^{rd} quadrant

$A = -100^\circ, 260^\circ, 620^\circ$

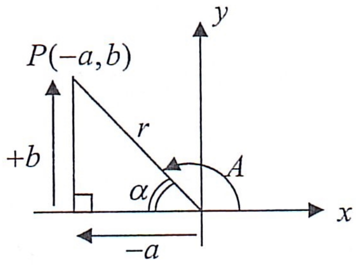
(c) 1[st] quadrant

$A = -280°, 80°, 440°$

So far, we are able to find trigonometric ratios of acute angles. What about angles which are not acute? That is when our basic angle comes into play – to evaluate the trigonometric ratio of an angle A which is not acute, it is *almost* the same as evaluating the trigonometric ratio of its basic angle, except for an important difference. Read on to find out.

Trigonometric ratio of *any* angle

Consider angle A which sits in the 2[nd] quadrant. In this case, To move from the origin to a point on the line, $P(-a,b)$, where $a,b>0$, we have to go along the *negative* x-axis for a units before moving along the *positive* y-axis for b units.



r, the hypotenuse, is given by $r=\sqrt{a^2+b^2}$ and is always positive.

Hence for this quadrant, we define, with the 'help' of the basic angle α,

$$\sin A = \frac{b}{r} \text{ (since opposite side is positive)}$$

$$\cos A = \frac{-a}{r} = -\frac{a}{r} \text{ (since adjacent side is negative)}$$

and $$\tan A = \frac{b}{-a} = -\frac{b}{a}$$

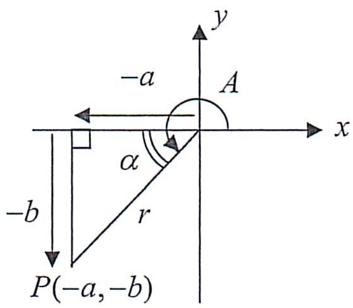
So in this quadrant, **only the sine ratio is positive**. The cosine and tangent ratios are negative.

P1. Now consider an angle A which sits in the 3[rd] quadrant. Investigate and define the trigonometric ratios of angle A in that quadrant. The diagram is drawn for you here.

$\sin A = -\frac{b}{r}$

$\cos A = -\frac{a}{r}$

$\tan A = \frac{b}{a}$



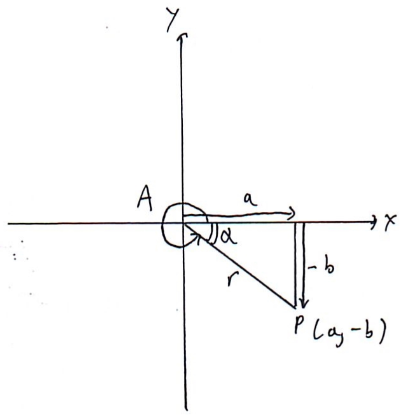
In this quadrant, **only the** tangent **ratio is positive.**

P2. Do the same for the above for an angle A which sits in the 4[th] quadrant.

$\sin A = -\frac{b}{r}$

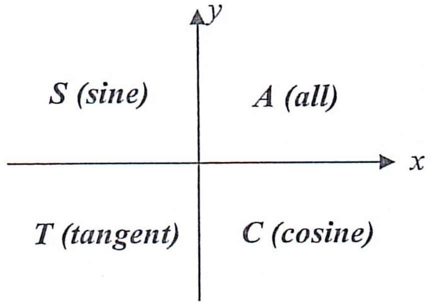
$\cos A = \frac{a}{r}$

$\tan A = -\frac{b}{a}$



In this quadrant, **only the** cosine **ratio is positive.**

Summarising, we can see that the trigonometric ratios take different signs over different quadrants, and the **positive** ones are illustrated as below:



ASTC

How would you plan to memorise this order?

E2. Are the following trigonometric ratios positive or negative?

(a) $\tan\frac{5\pi}{3}$

negative

(b) $\cos 160^\circ$

negative

(c) $\sin(-100^\circ)$

negative

(d) $\cos(-545^\circ)$

negative

(e) $\sin\frac{7\pi}{3}$

positive

(f) $\tan\left(-\frac{7\pi}{6}\right)$

negative

P3. Evaluate, to three significant figures $\sin 10\pi^\circ - \frac{\pi}{5}$.

$\sin(10\pi^\circ) - \frac{\pi}{5} = -0.107$ (3s.f.)

E3. Given that A is obtuse and that $\tan A = -0.8$, find the value of $\cos A$ and of $\sin A$.

$\therefore$ A is in second quadrant

$-0.8 = -\frac{4}{5}$

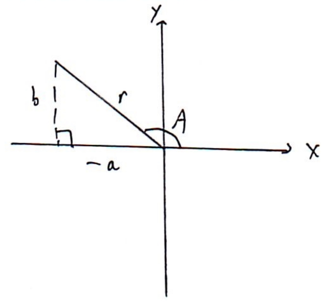
$b = 4$ units

$a = 5$ units

$r = \sqrt{41}$ units

$\sin A = \frac{4}{\sqrt{41}}$

$\cos A = -\frac{5}{\sqrt{41}}$



P4. If A is such that $\tan A = -1.4$ and $\cos A$ is positive, evaluate the exact values of $\sin A$ and $\cos A$. Hence show that $\sin^2 A + \cos^2 A = 1$.

A is in the 4th quadrant

$-1.4 = -\frac{7}{5}$

$\sin A = -\frac{7}{\sqrt{74}}$

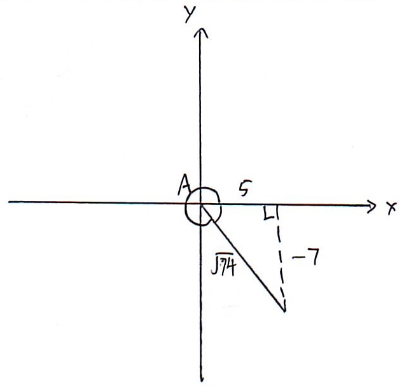
$\cos A = \frac{5}{\sqrt{74}}$

$\left(-\frac{7}{\sqrt{74}}\right)^2 + \left(\frac{5}{\sqrt{74}}\right)^2$

$= \frac{49}{74} + \frac{25}{74}$

$= \frac{74}{74}$

$= 1$ (shown)



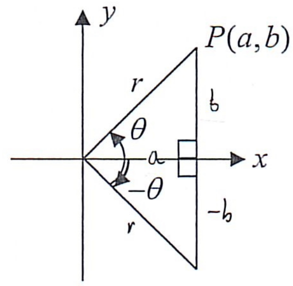
Negative angles

E4. Consider $\theta > 0$ and acute. Write down, in terms of a, b and r, the expressions for $\sin\theta$, $\cos\theta$ and $\tan\theta$.

$\sin\theta = \frac{b}{r}$

$\cos\theta = \frac{a}{r}$

$\tan\theta = \frac{b}{a}$



(a) Now express, using the triangle drawn above, $\sin(-\theta)$, $\cos(-\theta)$ and $\tan(-\theta)$ in terms of a, b and r.

$\sin(-\theta) = -\frac{b}{r}$

$\cos(-\theta) = \frac{a}{r}$

$\tan(-\theta) = -\frac{b}{a}$

$\sin(-\theta) = -\sin(\theta)$

$\cos(-\theta) = \cos(\theta)$

$\tan(-\theta) = -\tan(\theta)$

(b) How are $\sin(-\theta)$ and $\sin\theta$ related?

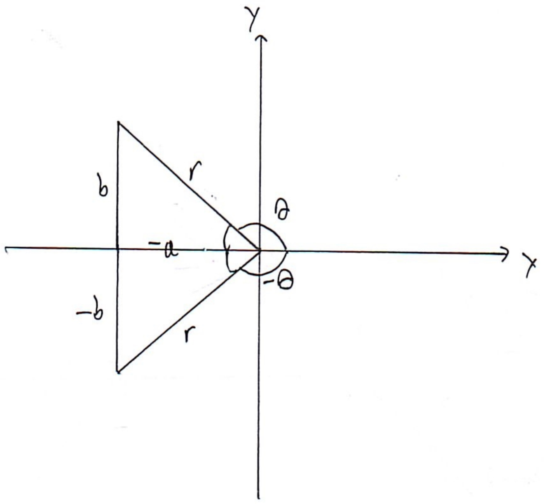
They are negative of each other.

(c) How are $\cos(-\theta)$ and $\cos\theta$ related?

They have the same value

(d) How are $\tan(-\theta)$ and $\tan\theta$ related?

They are negative of each other.



$\sin(-\theta) = -\sin(\theta)$

$\cos(-\theta) = \cos(\theta)$

$\tan(-\theta) = -\tan(\theta)$