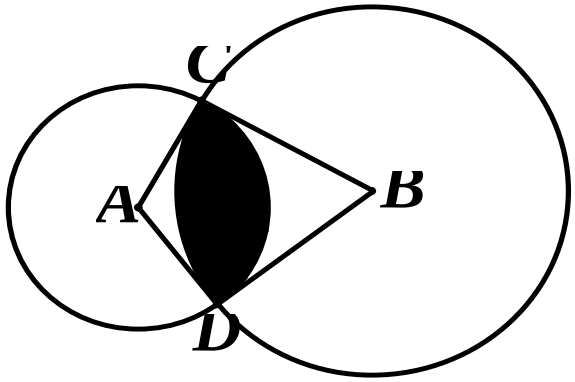
Topical Revision Paper 13

Term 3 Unit 13: Circular Measure

Question 1:

The diagram shows two circles with centres *A* and *B*, intersecting at points *C* and *D*. The radii of the circles are 8 cm and 12 cm respectively, and angle *ACB* = angle *ADB* = $\frac{\pi}{2}$ radians.



(i) Show that angle *CBD* is approximately 1.18 radians. [2]

The shaded region is bounded by minor arcs *CD* with centres *A* and *B* respectively.

(ii) Find the perimeter of the shaded region. [3]

(iii) Find the area of the shaded region. [3]

Solution:

(*i*) $\angle CBD = 2\tan^{-1}\left(\frac{8}{12}\right) \approx 1.176 \approx 1.18 \ rad$

(*ii*) *arc CD centre A* + *arc CD centre B*

$= 8(\pi - 1.18) + 12(1.18)$

$\approx 29.9 \ cm$

(*iii*) Area of segment given by $\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$.

(iii)

area of segment centre A

$$= \frac{1}{2}8^2\left[(\pi - 1.18) - \sin(\pi - 1.18)\right] \approx 33.184$$

[Accept 33.360 if students use 1.176]

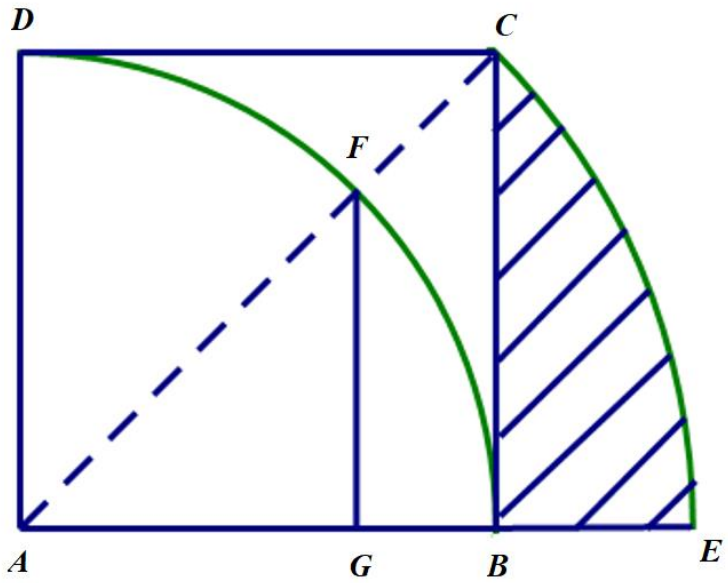
area of segment centre B

$$= \frac{1}{2}12^2\left[1.18 - \sin 1.18\right] \approx 18.388$$

Area of shaded region = 51.6 cm^2.

Question 2:

The diagram shows a quadrant inscribed in the square *ABCD* of sides 6 cm. *ACE* is a sector centred at *A*. *F* is the point of intersection of the diagonal *AC* and the arc *DB*. *FG* is perpendicular to *AE*.



Find

(i) the area of the shaded region *BCE*, [3]

(ii) the perimeter of *FGBECF*. [4]

Solution:

(i)

Area of shaded region *BCE*

$= \frac{1}{2}(AC)^2 (\frac{\pi}{4}) - \frac{1}{2}(6)^2$

$= \frac{\pi}{8}(6^2 + 6^2) - 18$

$= 9\pi - 18 \approx 10.3 \text{ cm}^2$

(ii)

Perimeter of *FGBEC*

$= GE + CE + FC + FG$

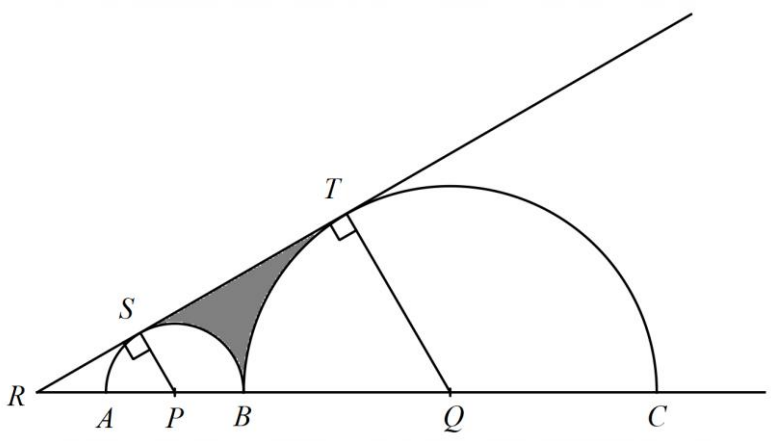
$= AC - AG + CE + FC + AG$

$= \sqrt{72} + \frac{\pi}{4}\sqrt{72} + \sqrt{72} - 6$

≈ 17.6 cm

Question 3:

The diagram below shows a tangent *RST* to two semi-circles with centre *P* and *Q*. The point *R, A, P, B, Q* and *C* lie on the same straight line. It is also given that *AB* = 2 cm and *AC* = 8 cm.



(i)	Show that *RA* = 1 cm.	[2]
(ii)	Find $\angle QRT$ in radian.	[2]
(iii)	Calculate the perimeter of the shaded region.	[4]
(iv)	Calculate the area of the shaded region.	[4]

Solution:

(i)

ΔRTQ and ΔRSP are similar

$\frac{RA+1}{RA+5} = \frac{1}{3}$

$3RA + 3 = RA + 5$

$2RA = 2$

$RA = 1$ cm

(ii)

$\sin \angle CRT = \frac{1}{2}$

$\angle CRT = \frac{\pi}{6}$

(iii)

Perimeter = Arc SB + Arc BT + Line segment ST

$= \frac{2}{3}\pi(1) + \frac{\pi}{3}(3) + \sqrt{4^2 - 2^2}$

$= \frac{2}{3}\pi + \pi + \sqrt{12}$

$= \frac{5}{3}\pi + \sqrt{12}$ *cm* or 8.70 cm

(iv)

Area of shaded region

$=$ Trapezium $PTQS -$ (Sector $PBS +$ Sector QBT)

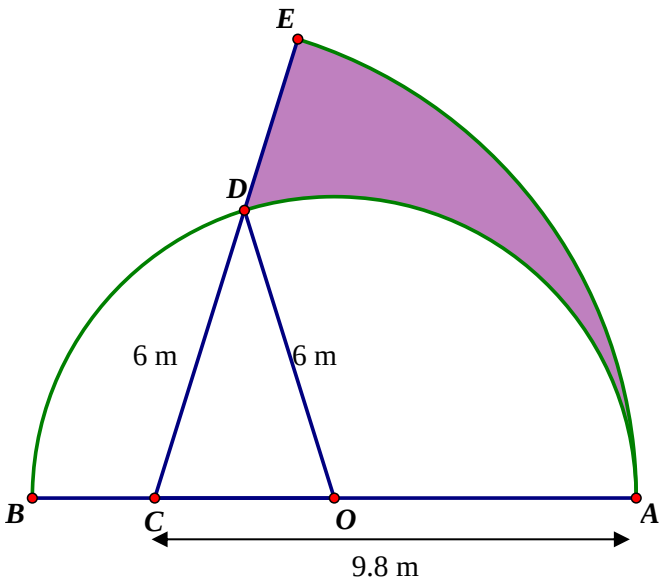
$= \frac{1}{3}(1+3)\sqrt{12} - [\frac{1}{2}(\frac{2}{3}\pi) + \frac{1}{2}(3^2)(\frac{\pi}{3})]$

$= 2\sqrt{12} - [\frac{\pi}{3} + \frac{3}{2}\pi]$

$= 2\sqrt{12} - \frac{11}{6}\pi$ *cm* or 1.17 cm^2

Question 4:

The diagram below shows a semicircle *ADB* with centre *O* and a radius of 6 m. *ACE* is a sector of a circle with centre *C* and a radius of 9.8 m. *CE* intersects the semicircle at *D* such that *CD* is 6 m.



(i) Show that $\angle AOD = 1.89$ radians. [2]

(ii) Calculate the area of the shaded region. [4]

Solution:

(i)

$$\cos \angle DOC = \frac{3.8 \div 2}{6}$$

$$\angle DOC = \cos^{-1}\left(\frac{1.9}{6}\right)$$

$$= 1.2486$$

$$\therefore \angle DOA = \pi - \angle DOC$$

$$= 1.8930$$

$$= 1.89 \quad (3\text{s.f.})$$

(ii) Shaded Area

= area sector ECA – area ΔDCO – area sector DOA

$$= \frac{1}{2}(9.8)^2(\angle DCO) - \frac{1}{2}(6)(3.8)\sin \angle DCO - \frac{1}{2}(6)^2 \angle DOA =$$

$$59.95696022 - 10.81332049 - 34.07417206$$

$$= 15.06946767$$

$$= 15.1 \text{ m}^2$$