12. Temperature & Ideal Gases

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Learning Objectives

12.1 Temperature Scales

(a) Show an understanding that a thermodynamic scale of temperature has an absolute zero and is independent of the property of any particular substance.

(b) Convert temperatures measured in degrees Celsius to kelvin: $T/K = \theta/°C$ + 273.15.

12.2 Equation of State

(c) Recall and use the equation of state for an ideal gas expressed as $pV = NkT$, where N is the number of particles.

(d) State that one mole of any substance contains 6.02×10^{23} particles, and use the Avogadro constant $N_A = 6.02 \times 10^{23}$ mol^{-1} as well as the relationship $Nk = nR$ between the Boltzmann constant and the molar gas constant (n is the amount of substance in moles).

12.3 Kinetic theory of gases

(e) State the basic assumptions of the kinetic theory of gases.

(f) Explain how the random motion of gas particles exerts mechanical pressure and hence derive, using the definition of pressure as force per unit area, the relationship, $pV = \frac{1}{3}Nm\langle c^2\rangle$ (a simple model considering one-dimensional collisions and then extending to three dimensions using $\langle {c_x}^2\rangle = \frac{1}{3}\langle c^2\rangle$ is sufficient).

12.4 Kinetic energy of a molecule

(g) Recall and use the relationship that the mean translational kinetic energy of a particle of an ideal gas is (directly) proportional to the thermodynamic temperature (i.e. $\frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT$) to solve problems.

12.1 TEMPERATURE SCALES

12.1.1 Absolute scale of temperature

(a)	*Show an understanding that a thermodynamic scale of temperature has an absolute zero and is independent of the property of any particular substance.*
(b)	*Convert temperatures measured in degrees Celsius to kelvin:* $T/K = \theta/°C + 273.15$.

The thermodynamic scale is based on the idea that the average kinetic energy of a gas molecule is proportional to the thermodynamic temperature, and it is the same for all substances at a particular thermodynamic temperature. Therefore, the temperature scale is independent of the property of the substance. We say that the thermodynamic temperature scale is an absolute scale.

An **absolute temperature scale** is not defined in terms of a property of any particular substance.

You need to explain this

The thermodynamic scale has two fixed points:

- **an absolute zero, which is defined as zero kelvin or 0 K**

  Absolute zero is the temperature at which all substances have the **minimum internal energy**[1].

  You need to explain this

- **triple point of water**[2], the temperature at which ice, water and water vapour can co-exist, which is defined as 273.16 K (equal to 0.01 °C).

The gap between absolute zero and the triple point of water is divided into 273.16 equal divisions. Each division is 1 K. The scale is defined so that the scale divisions on the thermodynamic scale are equal in size to the divisions on the Celsius scale. The temperature in Celsius is defined as the temperature in Kelvin minus 273.15 so

$$T / \text{K} = T / \text{°C} + 273.15$$

Recall & apply in problems

(e.g. pure water freezes at 273.15 K or 0 °C and boils at 373.15 K or 100 °C at standard atmospheric pressure, the difference is both 100 units.)

Self-study resources		
Video explanation of the absolute zero temperature.	youtu.be/iMrWN99XHvQ	

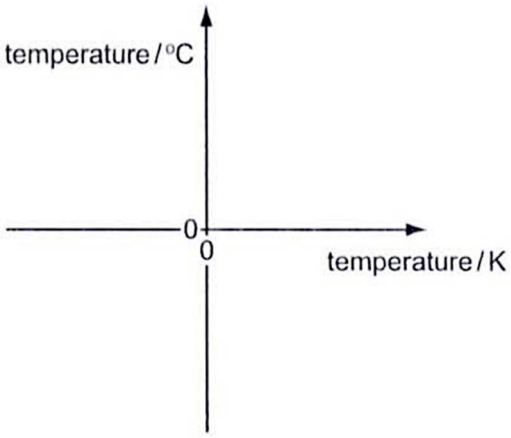
[1] In classical physics, the internal energy of a substance is zero at absolute zero. In quantum physics, it is impossible for any molecule or atom to be stationary due to Heisenberg Uncertainty Principle. Hence, absolute zero is defined as the temperature for which substances have the lowest internal energy.

[2] In thermodynamics, the triple point of a substance is the temperature and pressure at which the three phases (gas, liquid, and solid) of that substance coexist in thermodynamic equilibrium.

Exercise 1: Thermal Equilibrium and Temperature Scale

You should take about 15 minutes to complete this exercise.

1.1 A student draws a linear graph on the axes shown in order to convert temperatures in Kelvin to temperatures in degrees Celsius.



What is the intercept on the vertical axis and the gradient of the line?

	intercept	gradient
A	0	1/273
B	–273	1
C	+273	1
D	0	273

(2005 P1 Q13)

1.2 **(a)** State

(i) in what way the absolute scale of temperature differs from other temperature scales, [1]

(ii) what is meant by the absolute zero of temperature. [1]

(b) The temperature of a water bath increases from 50.00°C to 80.00°C.

Determine, in kelvin and to an appropriate number of significant figures,

(i) the temperature 50.00°C, [1]

(ii) the change in temperature of the water bath. [1]

Answer key

1.1 B **1.2 (b)(i)** 323.15 K **1.2 (b)(ii)** 30.00 K

12.2 EQUATION OF STATE

(c)	*Recall and use the equation of state for an ideal gas expressed as $pV = NkT$, where N is the number of particles.*
(d)	*State that one mole of any substance contains 6.02×10^{23} particles and use the Avogadro constant $N_A = 6.02 \times 10^{23}$ mol^{-1} as well as the relationship $Nk = nR$ between the Boltzmann constant and the molar gas constant (n is the amount of substance in moles).*

12.2.1 Concept of the Mole

Self-study resources		
This video covers the term "mole", explains "Avogadro number", and runs through examples of the sort of calculations you might have to do with moles.	youtu.be/wPGVQu3UXpw	

Amount of gas is measured in moles.

One mole of any substance contains 6.02×10^{23} particles

The Avogadro number N_A is

$N_A = 6.02 \times 10^{23}$ mol^{-1} ← Refer to "data and formulae"

The number of moles n of a gas can be calculated by

$$n = \frac{\text{mass of substance}}{\text{molar mass of gas}} \quad \text{or} \quad n = \frac{\text{number of gas molecules}}{\text{Avogadro number } N_A}$$

← Recall & apply in problems

We can obtain the approximate molar mass of a substance if its mass number is known

The molar mass of the substance is **approximately** equal to the mass number expressed in grammes ← Recall & apply in problems

For example: The mass number of iron–56 atom ($^{56}_{26}Fe$) is 56. So, the molar mass is ≈ 56 g.

Checkpoint. What is the mass of 0.150 mol of ammonium sulfate $(NH_4)_2SO_4$?

[Answer: 19.8 g]

12.2.2 Equation of State

Recall the gas laws for a fixed amount of gas you have learnt in secondary school:

Boyle's law	**Charles' law**	**Gay-Lussac's law**
For constant temperature T, $pV = \text{constant}$ or $p \propto \frac{1}{V}$	For constant pressure p, $\frac{V}{T} = \text{constant}$ or $V \propto T$	For constant volume V, $\frac{P}{T} = \text{constant}$ or $P \propto T$
video on Boyle's law	video on Charles' law	Video on Guy-Lussac's law

These laws combine to describe an **ideal gas**.

An ideal gas obeys the equation

$$pV = \text{constant} \times T \quad \text{or} \quad pV \propto T$$

where p is the pressure, V is the volume and T is the thermodynamic temperature of the gas.

You need to explain this

The state of an amount of ideal gas (with N particles) is determined by its state variables – thermodynamic temperature (T), pressure (p) and volume (V) – through the **equation of state**

$$pV = NkT$$

Recall & apply in problems

where p is in Pascal (Pa), V in m^3 and T in Kelvin (K). The Boltzmann constant k is

$$\text{Boltzmann constant } k = 1.38 \times 10^{-23} \text{ J K}^{-1}$$

Refer to "data and formulae"

The Boltzmann constant k is related to the molar gas constant R by

$$Nk = nR$$

Recall & apply in problems

where

$$R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$$

Refer to "data and formulae"

Combining $pV = NkT$ and $Nk = nR$, the equation of state can also be expressed as

$$pV = nRT$$

Example 1

A container of air, initial volume 2.4 m^3 and pressure 100 kPa, is cooled from 57 °C to –23 °C. After the cooling, its final pressure is 90 kPa and final volume is *V*. Determine *V*.

Strategy

Since the air remains in the container, the number of particles *N* is constant during the cooling. You may assume, for similar questions, that the air is an ideal gas and use the equation of state for ideal gas.

Solution

Rearranging the equation of state $\frac{pV}{T} = NkT$ = constant, so $\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$

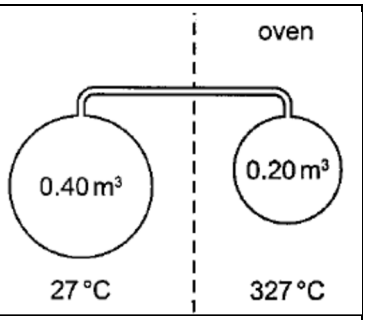
$$\frac{100 \times 10^3 \times 2.4}{(57+273.15)} = \frac{90 \times 10^3 \times V}{(-23+273.15)}$$

$$V = 2.0\ m^3$$

Example 2 (Equation of state – connected containers)

Two glass bulbs of constant volume 0.40 m^3 and 0.20 m^3 are connected by a fine tube of negligible volume. This sealed system contains a mass of ideal gas at atmospheric pressure 1.00×10^5 Pa at a temperature of 27 °C.

The smaller bulb is then placed in an oven and heated to 327 °C. Determine the new pressure *p* of the system.



Strategy

Both bulbs are connected, the pressures of the gas in the bulbs are always equal. The system is sealed, so the total number of gas particles remains constant.

Solution

initial: $1 \times 10^5 \times (0.40 + 0.20) = N(k)(27 + 273.15) \Rightarrow N = 1.4485 \times 10^{25}$

Let N_1 and N_2 be the number of gas particles in left and right bulbs respectively after heating.

Final: $N_1 = \frac{p \times 0.40}{k(27+273.15)}$ and $N_2 = \frac{p \times 0.20}{k(327+273.15)}$

Since the total number of gas particles remains constant i.e. $N = N_1 + N_2$

$$1.4485 \times 10^{25} = \frac{p \times 0.40}{k(27 + 273.15)} + \frac{p \times 0.20}{k(327 + 273.15)}$$

$$p = 1.2 \times 10^5\ Pa$$

Exercise 2: Equation of State

You should take about 20 minutes to complete this exercise.

2.1 What is the number of hydrogen atoms, in terms of the Avogadro constant N_A, in one mole of water (H_2O)? (2009 P1 Q20)

A $N_A / 3$ **B** $2N_A / 3$ **C** N_A **D** $2N_A$

2.2 Argon and neon are monatomic gases with relative atomic masses of 40 and 20 respectively. The ratio $\frac{\text{number of atoms of argon in one mole}}{\text{number of atoms of neon in one mole}}$ is

A always 1.

B always 2.

C 1 only if both gases are at the same temperature and pressure.

D 2 only if both gases are at the same temperature and pressure.

2.3 A cylinder, closed by a moveable piston, contains an ideal gas at 50 °C and atmospheric pressure 1.0×10^5 Pa. The piston is moved out until the volume of the gas doubles and the temperature falls to 25 °C.
What is the new pressure of the gas in the cylinder? (2012 P1 Q18)

A 1.6×10^4 Pa

B 2.5×10^4 Pa

C 4.6×10^4 Pa

D 1.9×10^5 Pa

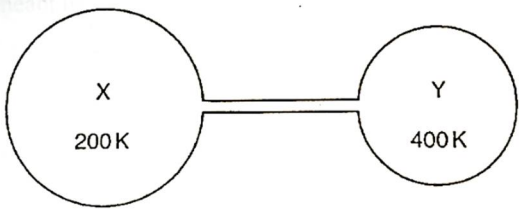
2.4 An ideal gas has a volume of 8.70 m^3 and contains 1.44×10^{26} molecules. Its pressure is 9.80×10^4 Pa. What is its temperature? (2012 P1 Q19)

A 156 K **B** 429 K **C** 4290 K **D** 29600 K

2.5 In the diagram the volume of bulb X is twice that of the bulb Y. The system is filled with an ideal gas and a steady state is established with the bulbs held at 200 K and 400 K.

There are *x* moles of gas in X.

How many moles of gas are in Y?



A $x / 4$ **B** $x / 2$ **C** x **D** $2x$

Answer key

2.1 D **2.2 A** **2.3 C** **2.4 B** **2.5 A**

12.3 KINETIC THEORY OF GASES

(e) State the basic assumptions of the kinetic theory of gases.
(f) Explain how the random motion of gas particles exerts mechanical pressure and hence derive, using the definition of pressure as force per unit area, the relationship $pV = \frac{1}{3}Nm\langle c^2\rangle$ *(a simple model considering one-dimensional collisions and then extending to three dimensions using* $\langle {c_x}^2\rangle = \frac{1}{3}\langle c^2\rangle$ *is sufficient).*

12.3.1 Assumptions of kinetic theory of gases

The kinetic theory of gases describes a gas as a large number of identical sub-microscopic particles or molecules in constant, rapid and random motion. The pressure of gases can be explained by considering the particles bombarding the walls of the container and average force exerted by the gas on the wall equals the total change in momentum per unit time of the particles.

The following are keys assumptions made:

Assumption 1	Perfectly elastic collisions between the gas molecules and the walls of the container.
Assumption 2	Total volume of the gas molecules is negligible compared to the volume occupied by the gas.
Assumption 3	The duration of a collision is negligible compared with the time between collisions.
Assumption 4	There is a large number of molecules in constant random motion.
Assumption 5	No intermolecular forces between molecules except during collisions.

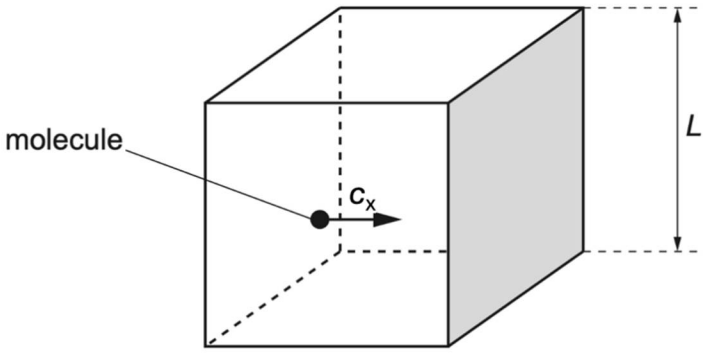
MUST MEMORISE!

Other assumptions include
- the molecules are identical (same mass, size, etc)
- they obey Newton's laws of motion.

How do the kinetic theory assumptions lead to the expression $pV = \frac{1}{3}Nm\langle c^2 \rangle$**?**

Step 1: Force exerted by a molecule on a face

A molecule of an ideal gas has mass *m* and is contained in a cubic box of side length *L*. The molecule is moving with a component velocity c_x towards the face of the box that is shaded perpendicularly.



Quantity	**Approach**
Change in momentum	Before impact: momentum of the molecule = mc_x After impact: momentum reverses with same speed c_x because collision is elastic momentum of the molecule = $-mc_x$ So, the magnitude of change in momentum Δp $= \lvert -mc_x - mc_x \rvert = 2mc_x$
Time between consecutive collisions with shaded face	Distance moved to the opposite face and back to X is $2L$. Time taken between consecutive collisions of the molecule with the shaded face $\Delta t = 2L / c_x$
Average force exerted on shaded face	Average force exerted on the molecule by the shaded face $= \frac{\Delta p}{\Delta t} = \frac{2mc_x}{2L/c_x} = \frac{mc_x^2}{L}$ Magnitude of average force F on the shaded face is $F = \frac{mc_x^2}{L}$ (Newton's third law)

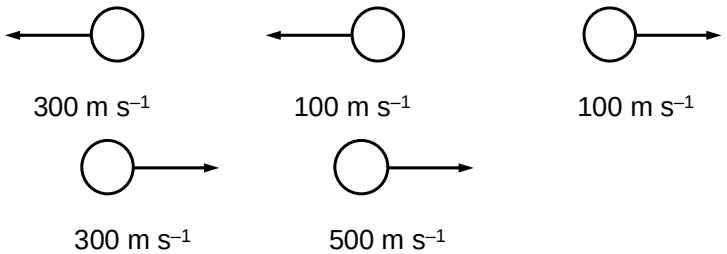
Step 2: Pressure on a face

The ideal gas contains N molecules.

Quantity	Approach
Total force exerted on shaded face	Magnitude of the force F_{total} exerted by N molecules on the shaded face is $$F_{\text{total}} = NF = \frac{Nmc_x^2}{L}$$
Pressure on the shaded face	Since the pressure on a surface is the force acting perpendicular to the surface per unit area, pressure P on the shaded face $$P = \frac{\text{force}}{\text{area}} = \frac{F_{\text{total}}}{L^2} = \frac{Nmc_x^2}{L}/L^2 = \frac{Nmc_x^2}{L^3}$$ Therefore, $$P = \frac{Nmc_x^2}{V} \quad \ldots (1)$$ where $V = L^3$

Step 3: Pressure on a container

The pressure in eq. (1) only consider the force of the molecules moving in the x–direction. The motion of the N molecules are in three dimensions and they are moving randomly at different speeds and directions.

Quantity	Approach
Mean square speed	Each molecule moves in the x–direction with different speeds and directions. We define the mean square speed in the x–direction by taking the average of the square speeds of N molecule $$\langle c_x^2 \rangle = \frac{(c_x^2)_{\text{molecule 1}} + (c_x^2)_{\text{molecule 2}} + \cdots + (c_x^2)_{\text{molecule } N}}{N}$$ Example: Five molecules are moving with the speeds and directions shown.  The mean square speed is $$\langle c^2 \rangle = \frac{300^2 + 100^2 + 100^2 + 300^2 + 500^2}{N} = 90000 \text{ m}^2 \text{ s}^{-2}$$

Pressure on a face	Using the mean square speed, eq. (1) becomes $\therefore P = \frac{Nm\langle c_x^2\rangle}{V} \ldots (2)$
Velocity of a molecule moving in three dimensional space	The velocity of the molecule can be expressed has the sum of the square of the velocity components $c^2 = c_x^2 + c_y^2 + c_z^2 \ldots (3)$
Mean square velocity	With no other external factors affecting the motion of the molecules, the mean square value of the component of the velocity in each direction is equal and independent of each other $\langle c_x^2\rangle = \langle c_y^2\rangle = \langle c_z^2\rangle$ Therefore, the mean value of the square velocity in eq. (3) is $\langle c^2\rangle = \langle c_x^2\rangle + \langle c_y^2\rangle + \langle c_z^2\rangle$ $\langle c^2\rangle = 3\langle c_x^2\rangle$ $\therefore \langle c_x^2\rangle = \frac{1}{3}\langle c^2\rangle \ldots (4)$
Pressure of the gas	Substitute eq. (4) into eq. (2), we obtain $P = \frac{1}{3}\frac{Nm}{V}\langle c^2\rangle$ ⟸ Refer to "data and formulae"

Example 3

A surface is bombarded by particles, each of mass m, which has velocity v normal to the surface. On average, N particles strike unit area of the surface each second and rebound elastically. What is the pressure on the surface?

A Nmv **B** $2Nmv$ **C** $mv^2 / 3$ **D** Nmv^2

Solution

Consider each particle,

before impact: momentum of particle is mv

after impact: momentum reverses and is $-mv$

$\therefore$ magnitude of momentum change on impact = $mv - (-mv) = 2mv$

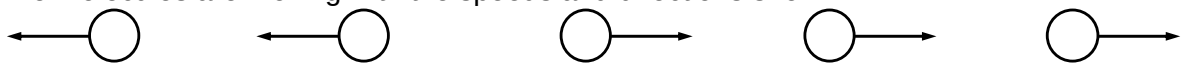
In one second, N particles hit the surface so total change in momentum $2\,Nmv$

$$\therefore \text{force on particles} = \frac{\text{change in momentum}}{\text{time}} = \frac{2Nmv}{1} = 2Nmv$$

By Newton's third law, force exerted by particles on the surface = $2\,Nmv$

So, pressure $= \frac{\text{force}}{\text{area}} = \frac{2Nmv}{1} = 2Nmv$

Example 4 (Calculating mean square and root mean square speed)
Five molecules are moving with the speeds and directions shown.



300 m s^{-1} 100 m s^{-1} 100 m s^{-1} 300 m s^{-1} 500 m s^{-1}

Calculate the mean speed, mean velocity, mean square speed and root mean square speed.

Solution

mean speed: direction does not matter in calculating mean speed

$$\text{mean speed} = \frac{300+100+100+300+500}{5} = 260 \text{ m s}^{-1}$$

mean velocity: consider velocity to the right as positive

$$\text{mean velocity} = \frac{(-300)+(-100)+100+300+500}{5} = 100 \text{ m s}^{-1}$$

mean square speed:

$$\text{mean square speed} = \frac{300^2+100^2+100^2+300^2+500^2}{5} = 90000 \text{ m}^2 \text{ s}^{-2}$$

r.m.s. speed:

$$\text{r.m.s. speed} = \sqrt{\text{mean square speed}} = 300 \text{ m s}^{-1}$$

Further exploration (beyond A-level)
The mean velocity of a gas with many particles in random motion is zero and therefore is not helpful in understanding the gas. This video (https://youtu.be/hon05fziwAY) explains on why this is so.
Caution: The content contains advance statistics.

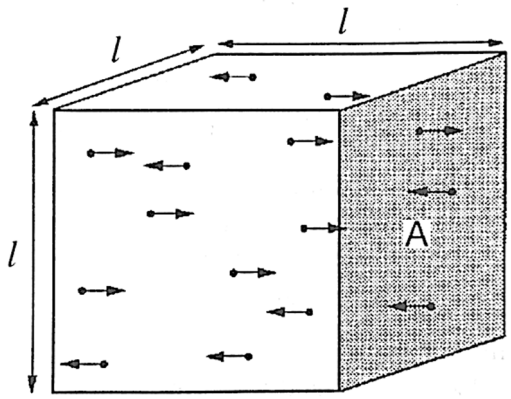
Further exploration (beyond A-level)
This video (https://youtu.be/hxFbfd4WLeQ) shows the Maxwell-Boltzmann distribution for the molecular speed and the difference between mean speed and root mean square speed.
Caution: The content contains advance statistics.

Further exploration
Why do we use root mean square speed and not mean speed to describe the motion the gas molecules? This video (https://youtu.be/jUz5Zfj7sGw) explains why this is so.

Further exploration (beyond A-level)
This video (https:// youtu.be/Im8HIgJNrC8) explains how the temperature of the gas changes the Maxwell-Boltzmann distribution.
Caution: The content contains advance statistics.

Example 5

Consider N molecules of mass m moving in the x–direction colliding with the wall A of the container, as shown below.



We can deduce that the pressure on the wall is $p = \frac{Nm}{V}\langle c_x^2 \rangle$ (where $\langle c_x^2 \rangle$ is the mean square speed in the x–direction).

A better model for the gas is to consider molecules moving in three dimensional space and we obtain $p = \frac{1}{3}\frac{Nm}{V}\langle c^2 \rangle$ (where $\langle c^2 \rangle$ is the mean square speed of the molecules).

Explain why is there a factor of $\frac{1}{3}$ in the formula.

Solution

- Each molecule has component of velocity c in three perpendicular directions – c_x, c_y and c_z.
- Since there is a large number of molecules in random motion, the component of velocity along each direction must have the same mean square values i.e. $\langle c_x^2 \rangle = \langle c_y^2 \rangle = \langle c_z^2 \rangle$.
- From Pythagoras theorem, the square of the velocity $c^2 = c_x^2 + c_y^2 + c_z^2$ and the mean square speed $\langle c^2 \rangle = \langle c_x^2 \rangle + \langle c_y^2 \rangle + \langle c_z^2 \rangle = 3\langle c_x^2 \rangle$
- So, the mean square value of each component must then be equal to 1/3 of the mean square speed $\langle c_x^2 \rangle = \frac{1}{3}\langle c^2 \rangle$

12.3.3 Other forms of $p = \frac{1}{3}\frac{Nm}{V}\langle c^2 \rangle$

Density of gas

Since Nm equals to the total mass of the gas, density of the gas is $\rho = \frac{Nm}{V}$

So, $pV = \frac{1}{3}Nm\langle c^2 \rangle \Rightarrow p = \frac{1}{3}\frac{Nm}{V}\langle c^2 \rangle$

$$\therefore p = \frac{1}{3}\rho \langle c^2 \rangle$$

Number density

The number of molecules per unit volume $= \frac{N}{V}$

$$\therefore p = \frac{1}{3} \times (\text{number density}) \times m\langle c^2 \rangle$$

Example 6

The density of argon at 1.00×10^5 Pa and 300 K is 1.60 kg m^{-3}. What is the root mean square speed of argon molecules at this temperature?

A 216 m s^{-1} **B** 250 m s^{-1} **C** 306 m s^{-1} **D** 433 m s^{-1}

Strategy

Density of the argon is given, so $pV = \frac{1}{3}Nm\langle c^2 \rangle$ should be modified.

Solution

The product Nm is equal to the total mass of gas, so density of the $\rho = \frac{Nm}{V}$

So, $pV = \frac{1}{3}Nm\langle c^2 \rangle \Rightarrow p = \frac{1}{3}\frac{Nm}{V}\langle c^2 \rangle \Rightarrow p = \frac{1}{3}\rho\langle c^2 \rangle$

$1 \times 10^5 = \frac{1}{3}(1.6)\langle c^2 \rangle$

$\therefore$ root mean square (r.m.s.) speed $\sqrt{\langle c^2 \rangle} = \sqrt{\frac{3 \times 10^5}{1.6}} = 433$ m s^{-1}

Exercise 3: Kinetic Theory of Gases

You should take about 10 minutes to complete this exercise.

Exercises

3.1 A kinetic theory formula relating the pressure p and the volume V of a gas to the root mean-square speed of its molecules is

$$p = \frac{1}{3}\frac{Nm}{V}\langle c^2 \rangle$$

In this formula, what does the product Nm represent?

A the mass of gas present in volume V

B the number of molecules in unit volume of the gas

C the total number of molecules in one mole of gas

D the total number of molecules present in volume V

3.2 The simple kinetic theory of gases relate the pressure p to the density ρ of a gas.

$$p = \frac{1}{3}\rho\langle c^2 \rangle$$

In this expression, what does $\langle c^2 \rangle$ represent?

A the average of the squares of the speeds of the gas molecules

B the root-mean-square speed of the gas molecules

C the square of the average speed of the gas molecules

D the sum of the squares of the speeds of the gas molecules

3.3 Four gas molecules have the speeds shown.

speed / 10^2 m s^{-1}	1.0	3.0	5.0	7.0

What is their root-mean-square speed?

A 200 m s^{-1} **B** 230 m s^{-1} **C** 400 m s^{-1} **D** 460 m s^{-1}

Answer key

3.1 A **3.2 A** **3.3 D**

12.4 KINETIC ENERGY OF A MOLECULE

(g) Recall and use the relationship that the mean translational kinetic energy of a particle of an ideal gas is (directly) proportional to the thermodynamic temperature (i.e. $\frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT$) to solve problems.

Self-study resources		
This video explains how the mean translational kinetic energy of the molecule is obtained.	youtu.be/2XiVpWrU8Eo	

In this chapter, we have introduced two key formulae:

Equation of state relating the macroscopic state variables derived from empirical results	Formula relating pressure and volume of a gas to the mean square speed of the molecules
$pV = NkT$	$pV = \frac{1}{3}Nm\langle c^2\rangle$

Combining both equations, we have

$$\frac{1}{3}Nm\langle c^2\rangle = NkT$$

$$\therefore \frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT$$

Recall & apply in problems

where $\frac{1}{2}m\langle c^2\rangle$ is the mean translational kinetic energy of each molecule arising from the random motion of the molecules within the gas and is dependent on the temperature only.

The total kinetic energy due to the random motion of a gas with N molecules is

$$\text{total kinetic energy } E_{K,random} = \frac{3}{2}NkT$$

Refer to "data and formulae"

Example 7 Two different gases A and B are contained in two identical vessels. If the ratio of their molecular masses and absolute temperatures are 8:1 and 2:1 respectively, what is the ratio of the r.m.s. molecular speeds?

Solution

Since mean kinetic energy of a molecule is proportional to the thermodynamic temperature,

$$\frac{1}{2}mc_{rms}^2 = \frac{3}{2}kT \Rightarrow c_{rms}^2 = \frac{3kT}{m} \Rightarrow c_{rms}^2 \propto \frac{T}{m}$$

So,

$$\left(\frac{c_{rms\ of\ A}}{c_{rms\ of\ B}}\right)^2 = \frac{T_A}{T_B} \times \frac{m_B}{m_A}$$

$$\frac{c_{rms\ of\ A}}{c_{rms\ of\ B}} = \sqrt{\frac{2}{1} \times \frac{1}{8}} = 0.50$$

Exercise 4: Kinetic Energy of a Molecule

You should take about 30 minutes to complete this exercise.

Exercises

4.1 In a mixture of two monatomic gases X and Y in thermal equilibrium, the molecules of Y have twice the mass of those of X. The mean translational kinetic energy of the molecules of Y is 6.0×10^{-21} J.

What is the mean translational kinetic energy of the molecules of X?

A 3.0×10^{-21} J **B** 4.2×10^{-21} J **C** 6.0×10^{-21} J **D** 12×10^{-21} J

4.2 Oxygen molecules in the Earth's atmosphere have a root mean square speed of about 500 m s^{-1}. If the relative molecular mass of oxygen and helium are 32 and 4 respectively, what is the best approximation to the root mean square speed of a helium molecules in the atmosphere?

A 180 m s^{-1} **C** 1400 m s^{-1} **E** 4000 m s^{-1}

B 1000 m s^{-1} **D** 2000 m s^{-1}

4.3 A mixture of two gases at constant temperature contains molecules of two kinds, the first of mass m_1 and r.m.s. speed c_1, the second of mass m_2 and r.m.s. speed c_2.

What is the value of $\frac{c_1}{c_2}$?

A $\frac{m_1}{m_2}$ **B** $\sqrt{\frac{m_1}{m_2}}$ **C** 1 **D** $\sqrt{\frac{m_2}{m_1}}$ **E** $\frac{m_2}{m_1}$

4.4 The molecules of an ideal gas at thermodynamics temperature T have a root-mean-square speed $c_{\text{r.m.s.}}$.

The gas is heated to temperature $2T$.

What is the new root-mean-square speed of the molecules?

A $\sqrt{2}\, c_{\text{r.m.s.}}$ **B** $2\, c_{\text{r.m.s.}}$ **C** $2\sqrt{2}\, c_{\text{r.m.s.}}$ **D** $4\, c_{\text{r.m.s.}}$

4.5 The temperature at which the r.m.s. speed of nitrogen molecules is twice as great as their r.m.s. speed at 300 K is

A 425 K **B** 600 K **C** 1146 K **D** 1200 K **E** 2292 K

4.6 The average kinetic energy of the molecules of an ideal gas in a closed, rigid container is increased by a factor of 4. What happens to the pressure of the gas?

A It remains the same.

B It increases by a factor of 2.

C It increases by a factor of 4.

D It increases by a factor of 8.

E It increases by a factor of 16.

4.7 At a certain temperature, the average speed of the molecules of an ideal gas is c. If the temperature of the gas is changed so that its pressure is halved while keeping its volume constant, then the average speed of the molecules becomes

A $c / 4$ **B** $c / 2$ **C** $c / \sqrt{2}$ **D** $c\sqrt{2}$ **E** $2c$

Answer key

4.1	**C**	**4.2**	**C**	**4.3**	**D**	**4.4**	**A**	**4.5**	**D**
4.6	**C**	**4.7**	**C**						