1 Solve the simultaneous equations

$$\frac{z^*}{z}+1=-\mathrm{i}\left(w^*-3\right) \text{ and } w=-2z+\mathrm{i},$$

where z and w are non-zero complex numbers. Leave your answer in the form $a+\mathrm{i}b$ where a and b are real numbers. [5]

2 A curve is defined by the parametric equations

$$x=\frac{1}{t}, \qquad y=t^2, \qquad \text{for } 0<t<1.$$

Show that the equation of the normal to the curve at the point $P\left(\frac{1}{p},p^2\right)$ is

$$2p^4y-px=2p^6-1.$$

Hence show that the normal at P cuts the curve exactly once. [6]

3 Andy and his fiancée signed up for a new 4-room flat in Boon Keng. They take up a housing loan of \$450,000 provided by BEST bank for the purchase. The couple pay a fixed monthly instalment of \$$A$ on the first day of each month. Interest is charged on the last day of each year at a fixed rate of 1.6% of the remaining loan amount at the beginning of that year. If the first instalment is paid in January 2026,

(i) Show that the amount the couple owe the bank at the end of 2027 is $\$\left[464515.2-24.192A\right]$. [1]

(ii) Given that A is 1500, find the date and amount of the final repayment to the nearest cent. [5]

4 Without using a calculator, solve the inequality $\dfrac{2x+4}{(x-2)^2}\geq\dfrac{1}{x+1}$. [4]

Hence, solve $\dfrac{2\mathrm{e}^{-x}+4}{\left(\mathrm{e}^{-x}-2\right)^2}\geq\dfrac{\mathrm{e}^x}{1+\mathrm{e}^x}$. [3]

5 The rate at which the number of people infected with Middle East Respiratory Syndrome (MERS), x, is varying at any time t, is proportional to the difference between the number of infected people and the number of deaths due to MERS. It is also known that the number of deaths due to MERS is proportional to the square of the number of people infected with MERS. Initially there were 10 people infected and the number infected remains constant when it reaches 100.

Show that $\dfrac{\mathrm{d}x}{\mathrm{d}t}=\dfrac{k}{100}\left(100x-x^2\right)$, where k is a constant. Hence find x in terms of k and t in the form $x=\dfrac{p}{1+q\mathrm{e}^{-kt}}$ where p and q are constants to be determined. [7]

6 The functions f and g are defined as follows:

$$\mathrm{f}: x \mapsto -\left|x^2+2x\right|, \quad a < x \le 0,$$

$$\mathrm{g}: x \mapsto -\sqrt{x+1}, \quad x > -1.$$

(i) State the least value of a for the inverse function of f to exist. Hence, find f^{-1} in similar form. [4]

For the following parts, use the value of a found in part **(i)**.

(ii) Write down ff^{-1} in similar form. [1]

(iii) Find the rule for gf in the form $bx + c$, where $b, c \in \mathbb{R}$. State its range. [3]

(i) The curve with equation $\frac{y^2}{9} - x^2 = 1$ undergoes a two-step transformations to become curve C with equation $\frac{y^2}{9} - \frac{(x-1)^2}{4} = 1$.

State the two transformations involved for the curve $\frac{y^2}{9} - x^2 = 1$. [2]

(ii) Draw a sketch of the curve C, labelling clearly the equation(s) of its asymptote(s), intersection with the axes and the coordinates of any turning points. [3]

(iii) Show that the point $(-1, -3)$ lies on the line $y = mx + m - 3$ for all real values of m. [1]

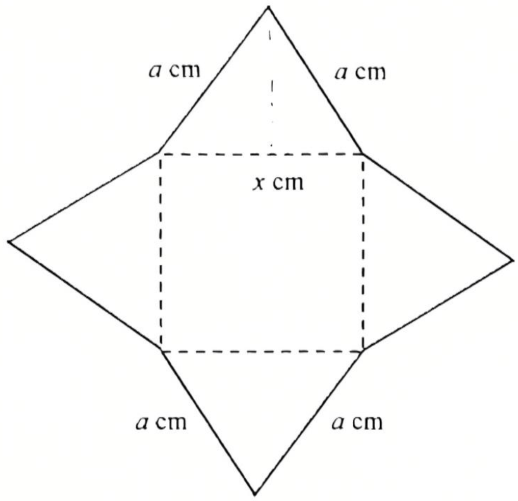
(iv) Hence using the diagram drawn in **(ii)**, find the range of values of k such that the equation $\frac{(kx+k-3)^2}{9} - \frac{(x-1)^2}{4} = 1$ has 2 negative real roots. [2]

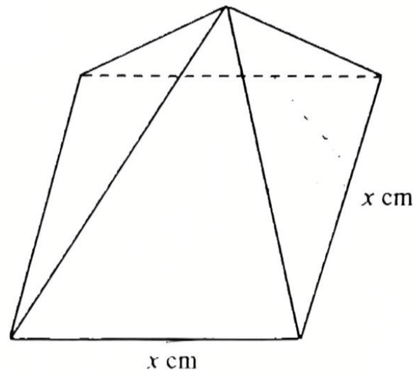
8 **(i)** Given that $y = \sqrt{4+\sin 2x}$, show that $y\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 + 2y^2 = 8$. [3]

(ii) By repeated differentiation of this result, find the Maclaurin's series for y, in ascending powers of x, up to and including the term in x^3. [3]

(iii) Deduce the Maclaurin's series expansion for $\sqrt{\frac{4+\sin 2x}{1-\sin 2x}}$ up to and including the term in x^2, given that x is small. [3]

9 A piece of metal with negligible thickness has been cut into a shape that is made up of four isosceles triangles each with base x cm and fixed sides a cm. Their bases frame to a form a square with sides of length x cm. A right pyramid is formed by folding along the dotted lines as shown in the diagram below.



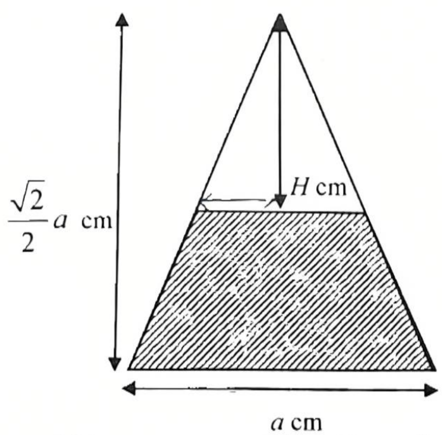


[Volume of a pyramid $= \frac{1}{3} \times$ base area $\times$ height]

(i) Show that the volume of the pyramid is $\frac{x^2}{3}\sqrt{a^2 - \frac{x^2}{2}}$ cm^3. [2]

(ii) Find the value of x, in terms of a, that will give maximum volume for the pyramid. [4]

(iii)



To make the pyramid into a paperweight with negligible thickness, a viscous fluid is pumped into the interior at a rate of 1 cm^3/s. Given that H cm is the perpendicular distance from the apex of the pyramid to the viscous fluid surface, $x = a$ and the height of the pyramid is $\frac{\sqrt{2}}{2}a$ cm, find the rate at which H is changing when $H = \frac{a}{2}$, giving your answer in terms of a. [3]

[The diagram above shows the cross sectional area of the pyramid.]

10 **(a)** The sequence of positive real numbers x_1, x_2, x_3,... satisfies the recurrence relation $x_{n+1} = \dfrac{6}{x_n + 1}$ for $n \geq 1$.

(i) If this sequence converges to l, determine the exact value of l. [2]

(ii) Determine the behaviour of the sequence when $x_1 = 3$. [1]

(iii) Show that $\dfrac{1}{x_{n+1}} - \dfrac{1}{l} = \dfrac{1}{6}\left(x_n - l\right)$.

Hence, show that if $x_n > l$, then $x_{n+1} < l < x_n$. [3]

(b) Given that $\displaystyle\sum_{n=2}^{N} \frac{n+1}{n^2\left(n+2\right)^2} = \frac{13}{144} - \frac{1}{4\left(N+1\right)^2} - \frac{1}{4\left(N+2\right)^2}$,

(i) find $\displaystyle\sum_{n=6}^{N} \frac{n}{\left(n^2-1\right)^2}$. [3]

(ii) Show that $\displaystyle\sum_{n=2}^{N} \frac{n}{\left(n+2\right)^4} < \frac{13}{144}$. [2]

11 **(a)** If $0 < a < b$, solve $\int_0^b x\left|a - x\right| \, dx$, leaving your answers in terms of a and b. [2]

(b)(i) Find $\dfrac{d}{dx}\left(\dfrac{3-x}{\sqrt{1-x}}\right)$. [1]

(ii) Find $\displaystyle\int \frac{3-x}{x^2-3x+2} \, dx$. [2]

(iii) Hence find $\displaystyle\int \frac{1+x}{\left(1-x\right)^{\frac{3}{2}}} \tan^{-1}\sqrt{1-x} \, dx$. [3]

(c) The region R is the finite region enclosed by the curve $\left(y-1\right)^2 = 1-x$ and the y-axis. The region S is the region in the 2[nd] quadrant enclosed by the curve $y = 2\tan\left(x + \dfrac{\pi}{4}\right)$ and the axes.

Find the total volume generated when region R and S is rotated through 2π radians about the x-axis, leaving your answers in exact form. [4]

12 A drone carrying a bomb departs from a point $A(-1,1,-3)$. It flies in a direction of $-a\mathbf{i}+\mathbf{j}+2a\mathbf{k}$, where $a>0$, across a lake to a target location. It is given that the surface of the lake is part of a plane with equation $x-z=2$.

(i) Determine the value of a if the path of the drone makes an angle of $\frac{\pi}{3}$ radians with the surface of the lake. [3]

It is given that $a=2$.

(ii) A launcher is launched to intercept the drone. It moves in a path with equation $\frac{10-x}{2}=-z,\ y=m$, where m is a real constant. Given that the launcher is successful in intercepting the drone, find the point of interception. [3]

(iii) At the point of interception, the drone falls perpendicular to the lake and meets the surface of the lake at N. Find the position vector of N. [3]

(iv) Denoting the line that contains the drone's path as L_1 and $L_1{}'$ being the reflection of L_1 in the surface of the lake, find a vector equation of $L_1{}'$. [3]

End of Paper

MF 27

Binomial expansion:

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \ldots + b^n \text{, where } n \text{ is a positive integer}$$

and $\binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

Maclaurin expansion:

$$\mathrm{f}(x) = \mathrm{f}(0) + x\mathrm{f}'(0) + \frac{x^2}{2!}\mathrm{f}''(0) + \ldots + \frac{x^n}{n!}\mathrm{f}^{(n)}(0) + \ldots$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad (|x|<1)$$

$$\mathrm{e}^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^r}{r!} + \ldots \quad \text{(all } x\text{)}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + \frac{(-1)^r x^{2r+1}}{(2r+1)!} + \ldots \quad \text{(all } x\text{)}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + \frac{(-1)^r x^{2r}}{(2r)!} + \ldots \quad \text{(all } x\text{)}$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + \frac{(-1)^{r+1}x^r}{r} + \ldots \quad (-1 < x \leqslant 1)$$

Partial fractions decomposition

Non-repeated linear factors:

$$\frac{px+q}{(ax+b)(cx+d)}=\frac{A}{(ax+b)}+\frac{B}{(cx+d)}$$

Repeated linear factors:

$$\frac{px^2+qx+r}{(ax+b)(cx+d)^2}=\frac{A}{(ax+b)}+\frac{B}{(cx+d)}+\frac{C}{(cx+d)^2}$$

Non-repeated quadratic factor:

$$\frac{px^2+qx+r}{(ax+b)(x^2+c^2)}=\frac{A}{(ax+b)}+\frac{Bx+C}{(x^2+c^2)}$$

Trigonometry

$$\sin(A\pm B)\equiv\sin A\cos B\pm\cos A\sin B$$
$$\cos(A\pm B)\equiv\cos A\cos B\mp\sin A\sin B$$
$$\tan(A\pm B)\equiv\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$
$$\sin 2A\equiv 2\sin A\cos A$$
$$\cos 2A\equiv\cos^2 A-\sin^2 A\equiv 2\cos^2 A-1\equiv 1-2\sin^2 A$$
$$\tan 2A\equiv\frac{2\tan A}{1-\tan^2 A}$$

Principal values:

$$-\tfrac{1}{2}\pi\leqslant\sin^{-1}x\leqslant\tfrac{1}{2}\pi\qquad(|x|\leqslant 1)$$
$$0\leqslant\cos^{-1}x\leqslant\pi\qquad(|x|\leqslant 1)$$
$$-\tfrac{1}{2}\pi<\tan^{-1}x<\tfrac{1}{2}\pi$$

Derivatives

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\sin^{-1}x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1}x$	$-\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1}x$	$\frac{1}{1+x^2}$
$\operatorname{cosec}x$	$-\operatorname{cosec}x\cot x$
$\sec x$	$\sec x\tan x$

[Turn Over

Integrals

(Arbitrary constants are omitted; a denotes a positive constant.)

$\mathrm{f}(x)$	$\int \mathrm{f}(x)\,\mathrm{d}x$	
$\dfrac{1}{x^2+a^2}$	$\dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right)$	
$\dfrac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\dfrac{x}{a}\right)$	$(\lvert x\rvert < a)$
$\dfrac{1}{x^2-a^2}$	$\dfrac{1}{2a}\ln\left(\dfrac{x-a}{x+a}\right)$	$(x > a)$
$\dfrac{1}{a^2-x^2}$	$\dfrac{1}{2a}\ln\left(\dfrac{a+x}{a-x}\right)$	$(\lvert x\rvert < a)$
$\tan x$	$\ln(\sec x)$	$(\lvert x\rvert < \frac{1}{2}\pi)$
$\cot x$	$\ln(\sin x)$	$(0 < x < \pi)$
$\operatorname{cosec} x$	$-\ln(\operatorname{cosec} x + \cot x)$	$(0 < x < \pi)$
$\sec x$	$\ln(\sec x + \tan x)$	$(\lvert x\rvert < \frac{1}{2}\pi)$

Vectors

The point dividing AB in the ratio $\lambda : \mu$ has position vector $\dfrac{\mu\mathbf{a}+\lambda\mathbf{b}}{\lambda+\mu}$

Vector product:

$$\mathbf{a}\times\mathbf{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$$