Secondary 3 Mathematics
WA3 Revision Time Trial — Bukit Merah (2023)

Duration: 65 minutes **Score: __________ / 43**

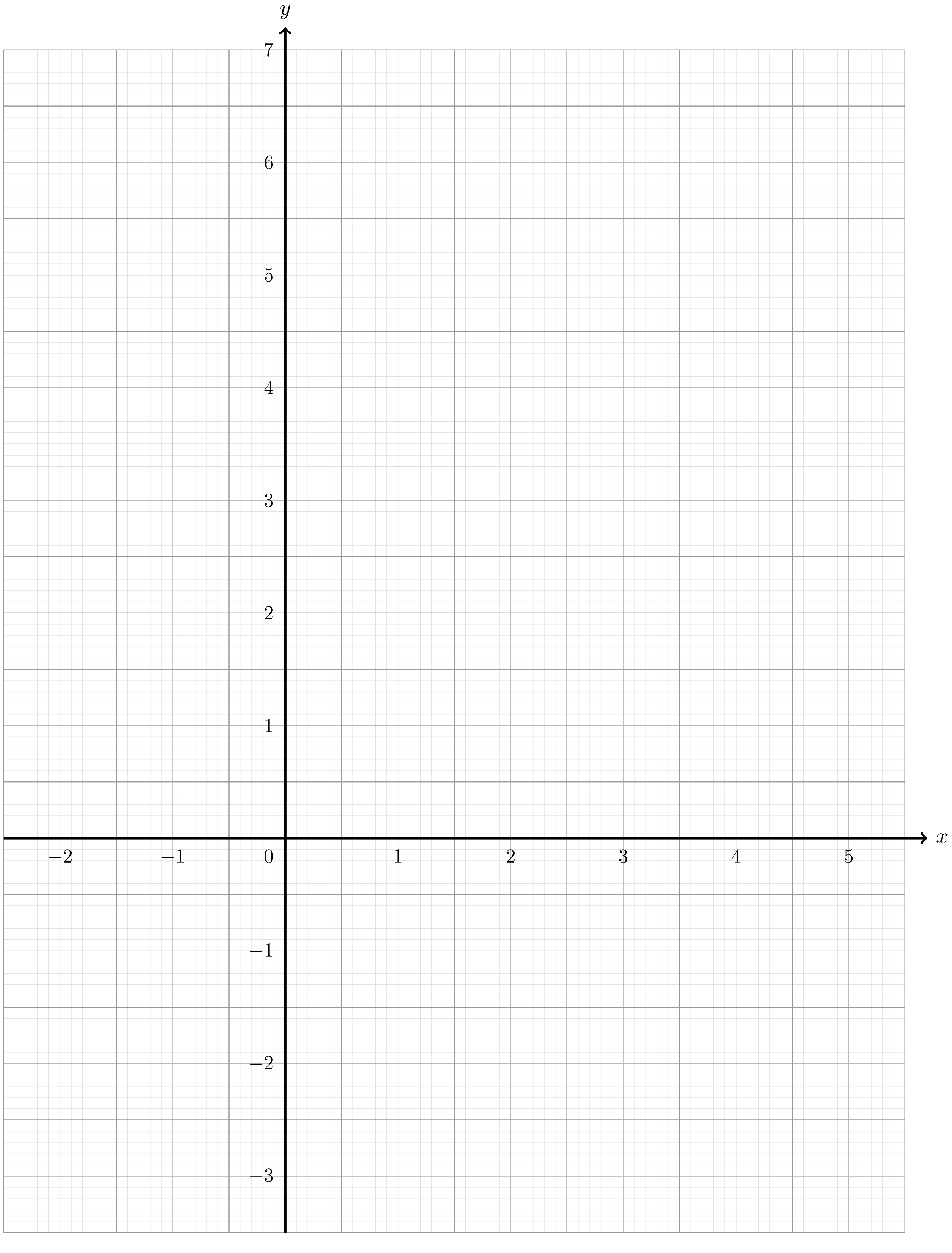
Topics tested: Graphs of Functions; Graphs in Practical Situations; Further Trigonometry; Applications of Trigonometry; Arc Length and Sector Area.

Section A: Graphs of Functions [11 marks]

1. (a) Complete the table of values for $y = \frac{1}{5}x^3 - \frac{4}{5}x^2$. *[1]*

x	-1.5	-1	0	1	2	3	4	5
y	-2.5	-1	0	-0.6		-1.8	0	5

(b) On the grid on the next page, draw the graph of $y = \frac{1}{5}x^3 - \frac{4}{5}x^2$ for $-1.5 \leq x \leq 5$. *[3]*



y
7
6
5
4
3
2
1
0
−1
−2
−3
x
−2
−1
1
2
3
4
5

(c) Explain how your graph shows that there is only one solution of the equation $\frac{1}{5}x^3 - \frac{4}{5}x^2 = 2$. *[1]*

Answer ....

....

(d) The equation $x^3 - 4x^2 + 10x - 20 = 0$ can be solved by finding the points of intersection of the straight line $y = ax + b$ and the curve $y = \frac{1}{5}x^3 - \frac{4}{5}x^2$.

(i) Find the value of a and the value of b. *[2]*

Answer $a =$ .... , $b =$ ....

(ii) By drawing the line $y = ax + b$, solve the equation $x^3 - 4x^2 + 10x - 20 = 0$. *[2]*

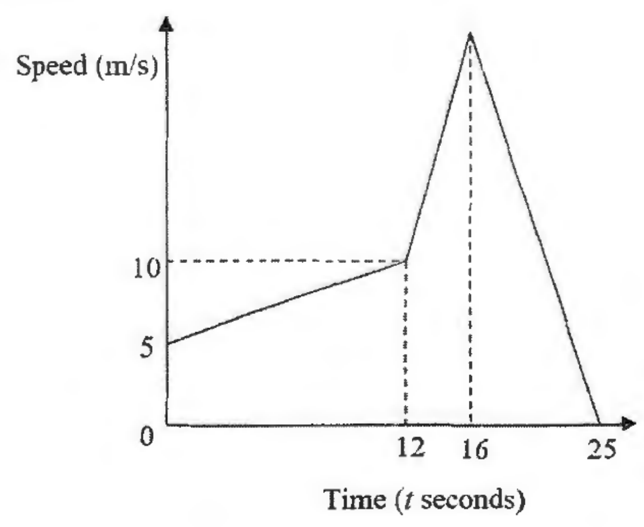
Answer $x =$ ....

(e) By drawing a tangent, find the gradient of the curve at $(-1, -1)$. *[2]*

Answer ....

Section B: Graphs in Practical Situations [7 marks]

2. The diagram shows the speed-time graph of an object moving in a straight line over a period of 25 seconds. The object travelled 90 m from $t = 12$ to $t = 16$.



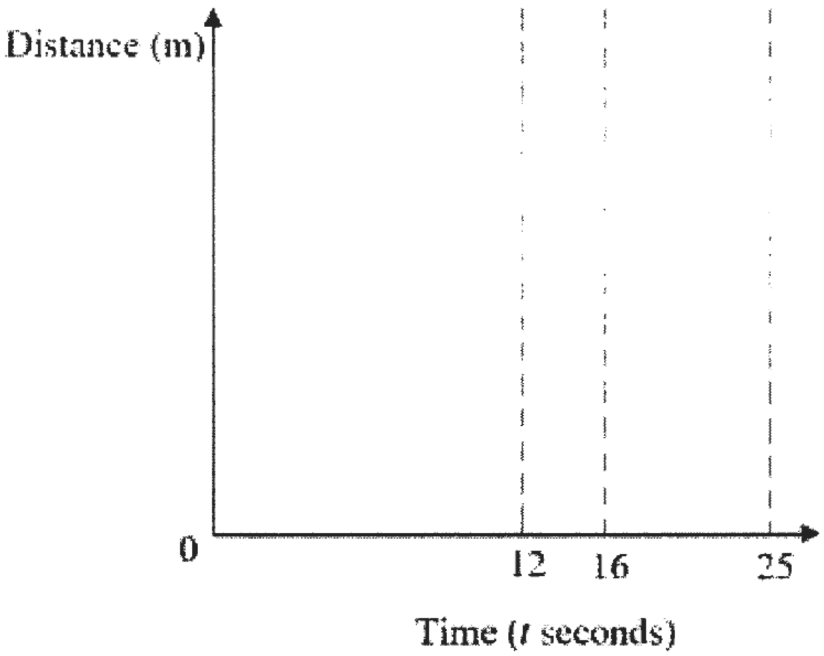
(a) Find the maximum speed of the object. *[2]*

Answer .... m/s

(b) Find the speed of the object at $t = 6$. *[2]*

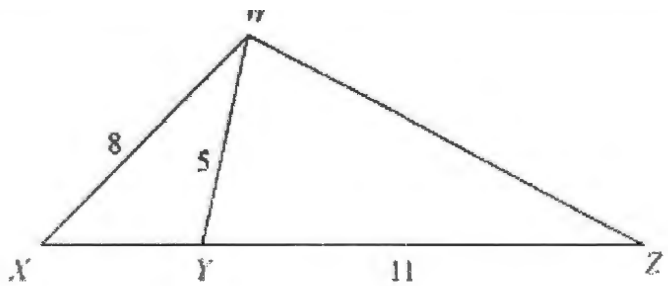
Answer .... m/s

(c) Sketch the distance-time graph for $0 \leq t \leq 25$. *[3]*



Section C: Further Trigonometry [4 marks]

3. In triangle WXZ, $WX = 8$ cm. Y is a point on XZ such that $WY = 5$ cm, $YZ = 11$ cm and $\sin \angle WYZ = \dfrac{4}{5}$.



(a) Find the area of triangle WYZ. *[2]*

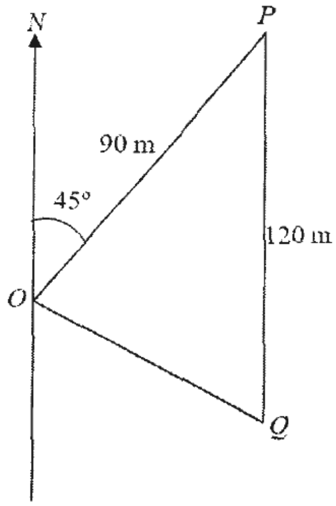
Answer .... cm^2

(b) Find the size of angle WXY. *[2]*

Answer ....

Section D: Applications of Trigonometry [15 marks]

4. The diagram shows three points O, P and Q. The bearing of P from O is $045°$ and P is due north of Q. $OP = 90$ m and $PQ = 120$ m.



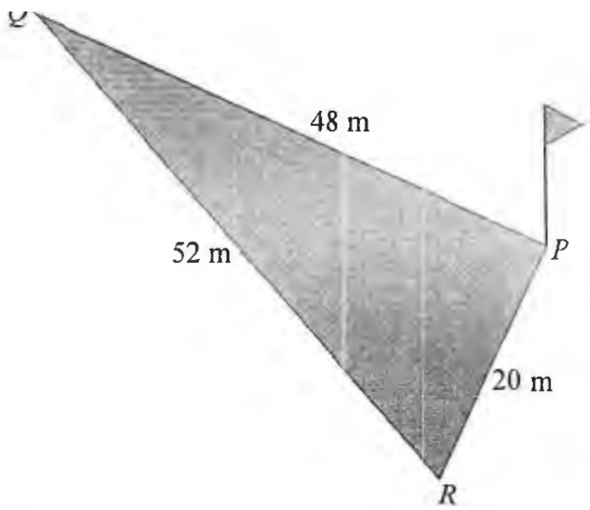
(a) Find the distance of OQ. *[3]*

Answer .... m

(b) Calculate the bearing of Q from O. *[3]*

Answer ....

5. The diagram shows a triangular court PQR. $PR = 20$ m, $PQ = 48$ m and $RQ = 52$ m.



(a) Prove that QPR is a right angle. *[2]*

Answer ....

....

....

(b) Hence, or otherwise, calculate angle PRQ. *[2]*

Answer $\angle PRQ =$ ....

(c) Find the shortest distance of P from the line RQ. *[2]*

Answer .... m

(d) If a flag post of height 4.5 m is hoisted up at P and a bird was perched on top of the flag post, calculate

(i) the greatest angle of elevation of the bird from a wild chicken walking from Q to R, *[2]*

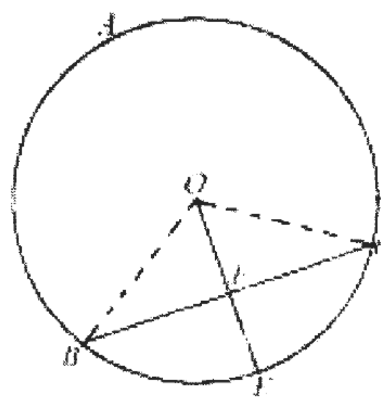
Answer ....

(ii) the greatest angle of depression of the wild chicken from the bird. *[1]*

Answer ....

Section E: Arc Length and Sector Area [6 marks]

6. The diagram below shows a circle of radius 7 cm, centre O.
OE is perpendicular to the chord BC at the point F, $OF = 4$ cm.



(a) Find the angle BOC, in radians. *[2]*

Answer .... radians

(b) Express angle BOC, in degrees. *[1]*

Answer ....

(c) Find the area of the major segment BAC. *[3]*

Answer .... cm^2

— END OF PAPER —

Answer Key
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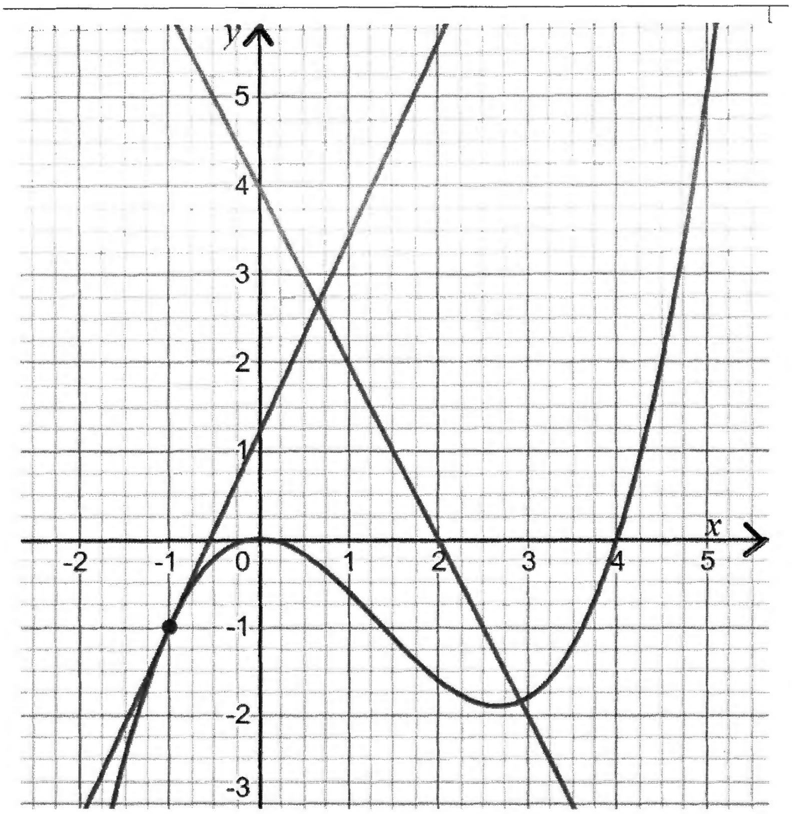
Section A: Graphs of Functions [11 marks]

1(a) At $x = 2$: $y = \dfrac{8}{5} - \dfrac{16}{5} = -\mathbf{1.6}$. *[1]*

1(b) All eight points plotted and joined with a smooth curve: local maximum at $(0,\,0)$, dipping to a local minimum near $(2.7,\,-1.9)$, then rising to $(5,\,5)$. *[3]*



(Marking scheme graph shown, including the line $y = -2x + 4$ for (d) and the tangent at $(-1,\,-1)$ for (e).)

1(c) The horizontal line $y = 2$ meets the curve at exactly **one** point of intersection, so the equation has only one solution. *[1]*

1(d)(i) $x^3 - 4x^2 + 10x - 20 = 0 \;\Rightarrow\; \dfrac{1}{5}x^3 - \dfrac{4}{5}x^2 + 2x - 4 = 0$
$\Rightarrow\; \dfrac{1}{5}x^3 - \dfrac{4}{5}x^2 = -2x + 4$. $\therefore$ **a** $= -\mathbf{2}$, **b** $= \mathbf{4}$. *[2]*

1(d)(ii) Line $y = -2x + 4$ drawn; single intersection with the curve: **x** $\approx$ **2.9** (accept ± 0.1). *[2]*

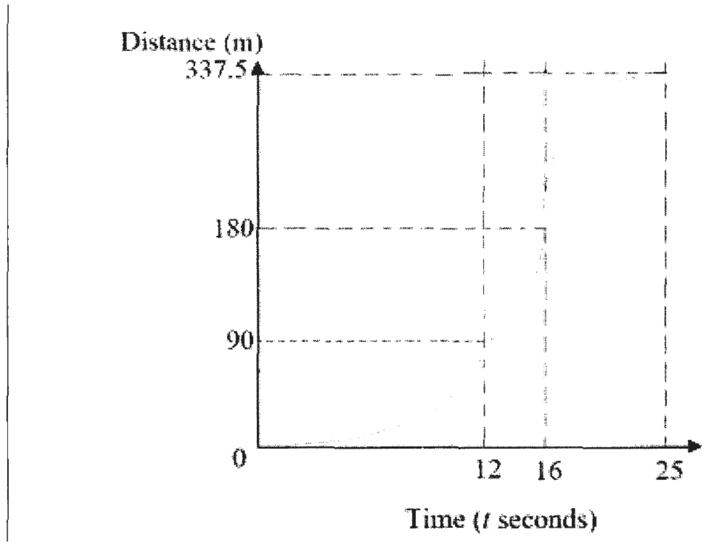
1(e) Tangent drawn at $(-1,\,-1)$.
Exact gradient: $\dfrac{dy}{dx} = \dfrac{3}{5}x^2 - \dfrac{8}{5}x = \dfrac{3}{5} + \dfrac{8}{5} = \mathbf{2.2}$ (accept 1.98 to 2.42). *[2]*

Section B: Graphs in Practical Situations [7 marks]

2(a) Distance from $t = 12$ to $t = 16$ = area of trapezium $= \frac{1}{2}(4)(10 + v_{\text{max}}) = 90$
$10 + v_{\text{max}} = 45 \;\Rightarrow\; v_{\text{max}} = $ **35** m/s. *[2]*

2(b) From $t = 0$ to $t = 12$ the speed rises uniformly from 5 to 10 m/s: $a = \frac{5}{12}$ m/s^2.
Speed at $t = 6$: $5 + 6\left(\frac{5}{12}\right) = $ **7.5** m/s. *[2]*

2(c) Distances: at $t = 12$: $\frac{1}{2}(5 + 10)(12) = 90$ m; at $t = 16$: $90 + 90 = 180$ m; at $t = 25$: $180 + \frac{1}{2}(9)(35) = 337.5$ m. Curve with increasing gradient to $(12, 90)$, steeper curve to $(16, 180)$, then levelling off to $(25, 337.5)$. *[3]*



(Marking scheme sketch shown — the three key distances 90, 180 and 337.5 m.)

Section C: Further Trigonometry [4 marks]

3(a) Area of triangle $WYZ = \frac{1}{2}(5)(11)\sin\angle WYZ = \frac{1}{2}(5)(11)\left(\frac{4}{5}\right) = \mathbf{22}$ cm^2. *[2]*

3(b) $\sin\angle WYX = \sin(180^\circ - \angle WYZ) = \sin\angle WYZ = \frac{4}{5}$.

By the Sine Rule in triangle WXY: $\frac{\sin\angle WXY}{5} = \frac{\sin\angle WYX}{8}$

$\sin\angle WXY = \frac{5\times\frac{4}{5}}{8} = \frac{1}{2} \;\Rightarrow\; \angle WXY = \mathbf{30}^\circ$. *[2]*

Section D: Applications of Trigonometry [15 marks]

4(a) $\angle OPQ = 45^\circ$ (alternate $\angle$s, $PQ \parallel$ north line at O).
By the Cosine Rule: $OQ^2 = 90^2 + 120^2 - 2(90)(120)\cos 45^\circ = 7226.4\ldots$
$OQ = \mathbf{85.0}$ m (3 s.f.). *[3]*

4(b) By the Sine Rule: $\frac{\sin\angle POQ}{120} = \frac{\sin 45^\circ}{85.008\ldots} \;\Rightarrow\; \angle POQ = 86.529\ldots^\circ$
Bearing of Q from $O = 45^\circ + 86.529\ldots^\circ = \mathbf{131.5}^\circ$ (1 d.p.). *[3]*

5(a) $PR^2 + PQ^2 = 400 + 2304 = 2704$ and $RQ^2 = 52^2 = 2704$.
Since $PR^2 + PQ^2 = RQ^2$, by the **Converse of Pythagoras' Theorem**, $\angle QPR = 90^\circ$. *[2]*

5(b) $\tan\angle PRQ = \frac{48}{20} \;\Rightarrow\; \angle PRQ = 67.380\ldots = \mathbf{67.4}^\circ$ (1 d.p.). *[2]*

5(c) Shortest distance $= 20\sin\angle PRQ = 20\sin 67.380\ldots^\circ = 18.461\ldots = \mathbf{18.5}$ m (3 s.f.) (or $\frac{240}{13}$ m, using area of the triangle with RQ as base). *[2]*

5(d)(i) Greatest angle of elevation at the foot of the perpendicular from P to RQ:
$\tan\theta = \frac{4.5}{18.461\ldots} \;\Rightarrow\; \theta = \mathbf{13.7}^\circ$ (1 d.p.). *[2]*

5(d)(ii) Angle of depression $=$ angle of elevation (alternate $\angle$s) $= \mathbf{13.7}^\circ$. *[1]*

Section E: Arc Length and Sector Area [6 marks]

6(a) In right-angled triangle OBF: $\cos\angle BOF = \frac{OF}{OB} = \frac{4}{7}$
$\angle BOF = 0.96255\ldots$ rad, so $\angle BOC = 2(0.96255\ldots) = \mathbf{1.93}$ radians (3 s.f.). *[2]*

6(b) $\angle BOC = 1.9251\ldots \times \frac{180^\circ}{\pi} = \mathbf{110.3}^\circ$ (1 d.p.). *[1]*

6(c) Area of major segment $BAC =$ area of triangle $OBC +$ area of major sector
$= \frac{1}{2}(7)^2\sin 1.9251\ldots + \frac{1}{2}(2\pi - 1.9251\ldots)(7)^2$
$= 22.978 + 106.773 = \mathbf{130}$ cm^2 (3 s.f.). *[3]*