Class	Register Number	Name

南洋女子中学校
NANYANG GIRLS' HIGH SCHOOL

End-of-Year Examination 2019
Secondary 3

INTEGRATED MATHEMATICS 2

2 hours

Tuesday **8 October 2019** **0845 - 1045**

READ THESE INSTRUCTIONS FIRST

1. Write your name, register number and class on all the work you hand in.
2. Answer **all** questions.
3. Write your answers and working on the separate writing paper provided, unless otherwise stated.
4. Write in dark blue or black ink on both sides of the paper.
5. You may use an HB pencil for any diagrams or graphs.
6. Do not use staples, paper clips, glue or correction tape/fluid.
7. Omission of essential working will result in loss of marks.
8. The use of an electronic calculator is expected, where appropriate.
9. If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degree to one decimal place.

   For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.
10. At the end of the examination, fasten all your work securely together.
11. The number of marks is given in brackets [ ] at the end of each question or part question.
12. The total number of marks for this paper is 80.

Setter: CP	This document consists of **6** printed pages, including this cover page. **NANYANG GIRLS' HIGH SCHOOL**	**[Turn over**

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} .$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 Find the range of values of k for which the equation $kx^2 + 6x + \frac{3k}{2} = 0$ has real and distinct roots. [3]

2 Solve the inequality $|x^2 - 5x + 2| \leq 2$. [4]

3 Without using a calculator, find x in the form of $\frac{a+\sqrt{b}}{7}$, where a and b are integers, such that $x\sqrt{40} = x\sqrt{5} + \sqrt{10}$. [5]

4 **Answer the whole of this question on the <u>INSERT</u> provided.**

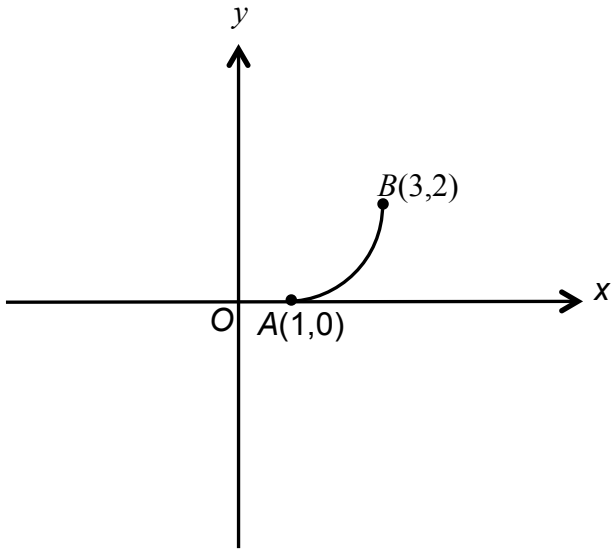
The diagram shows the graph of $y = \mathrm{f}(x)$ for $1 \leq x \leq 3$. On <u>separate</u> axes, sketch the graphs of

(a) $y = -2\mathrm{f}(x)$, [2]

(b) $y = \mathrm{f}(\frac{1}{2}x)$, [2]

(c) $y = f^{-1}(x)$, [2]

labelling clearly the images of the point A and B as A' and B' respectively under the respective transformations.



5 The roots of the quadratic equation $2x^2 + 3x + 8 = 0$ are α and β.

(a) Show that $\frac{\alpha^2+\beta^2}{\alpha\beta} = -\frac{23}{16}$. [4]

(b) Given that the roots of $x^2 + ax + b = 0$ are $\frac{\alpha}{\beta} - 3$ and $\frac{\beta}{\alpha} - 3$, find the value of a and of b where a and b are constants. [4]

6 Given that $\sec\theta = p$, where θ is an acute angle, find in terms of p,

(i) $\sin(-\theta)$, [2]

(ii) $\tan\left(\frac{\pi}{2} - \theta\right)$, [1]

(iii) $\sin(\pi + \theta)$. [1]

7 The function f is defined by $\mathrm{f} : x \longmapsto 3x^2 - 6x - 1$, for $0 \leq x \leq 3$.

(a) **(i)** Express $\mathrm{f}(x)$ in the form $a(x-b)^2 + c$, where a, b and c are constants. [2]

(ii) Find the range of f. [2]

(b) The function g is defined by $\mathrm{g} : x \longmapsto 3x^2 - 6x - 1$, for $x \geq k$. Given that g has an inverse,

(i) state the smallest value of k, [1]

(ii) find in similar form an expression for g^{-1} and state its domain, [3]

(iii) find $\mathrm{g}^2(3)$. [2]

8 **(i)** Sketch the graph of $y = e^{-\frac{1}{2}x} - 1$, labelling the equation of asymptote and intercept(s) clearly. [2]

(ii) Find the equation of the straight line to be added onto the same axes so as to solve the equation $2\ln(2-x) + x = 0$. Add this line to your sketch in **(i)** to determine the number of solutions to the equation $2\ln(2-x) + x = 0$. [3]

9 The function $\mathrm{f}(x) = 4x^3 + hx^2 - 11x + k$, where h and k are constants, is exactly divisible by $x - 3$. It leaves a remainder of -4 when divided by $x + 1$.

(i) Calculate the value of h and of k. [4]

(ii) Factorise $\mathrm{f}(x)$ completely. [3]

(iii) Hence, express $\frac{5+\mathrm{f}(x)}{\mathrm{f}(x)}$ in partial fractions. [5]

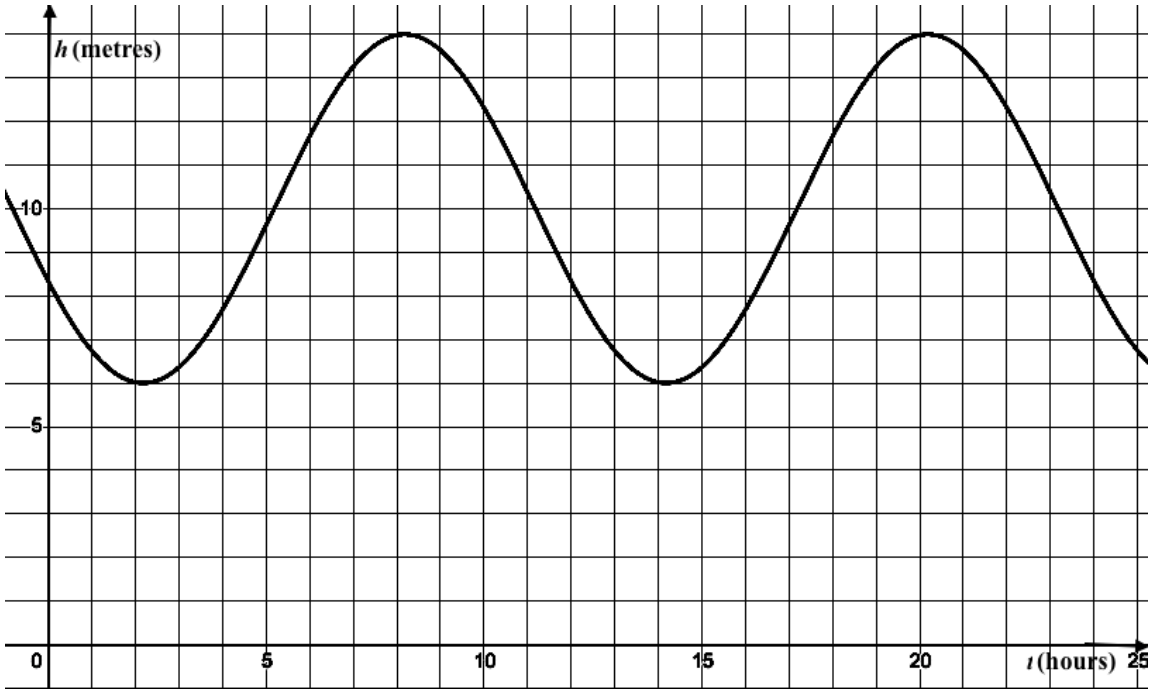
10 **(a)** Solve the equation $4^{x+0.5} = 4 - 7(2^x)$. [4]

(b) Solve $\log_9(5x-1) - \log_3(x-1) = 1$. [6]

(c) Solve the equation $4\sin 2x \cos x = 3\cos x$ for $0° \leq x \leq 360°$. [6]

11 The captain of a shipping vessel has to consider the tides of the day very carefully when entering a port because the depth of water at the port varies throughout the day.

The figure below shows the depth of the water, h, in metres, at various times of a certain day at a particular port. It is given that the depth of water at the port is modelled by the function $h = a\cos(bt + 2) + c$, where t is the number of hours after midnight, and a, b and c are constants. Daylight hours are assumed to last from 7am to 7pm.

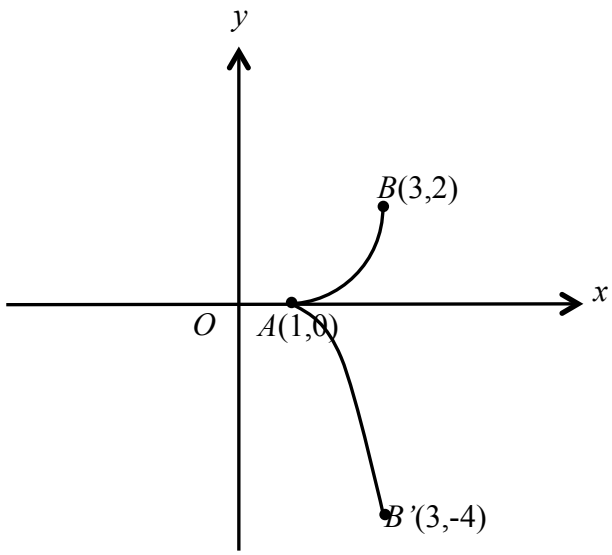
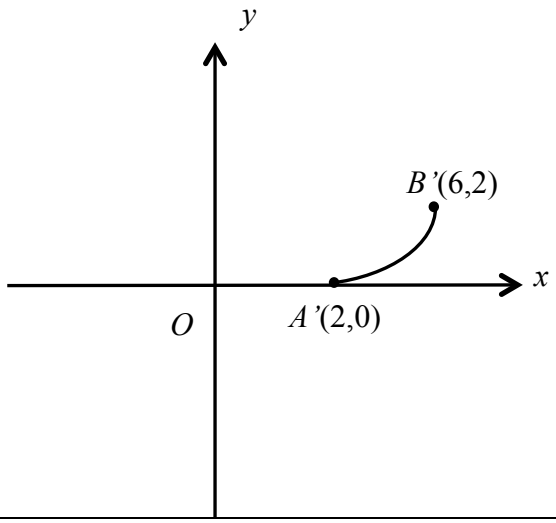


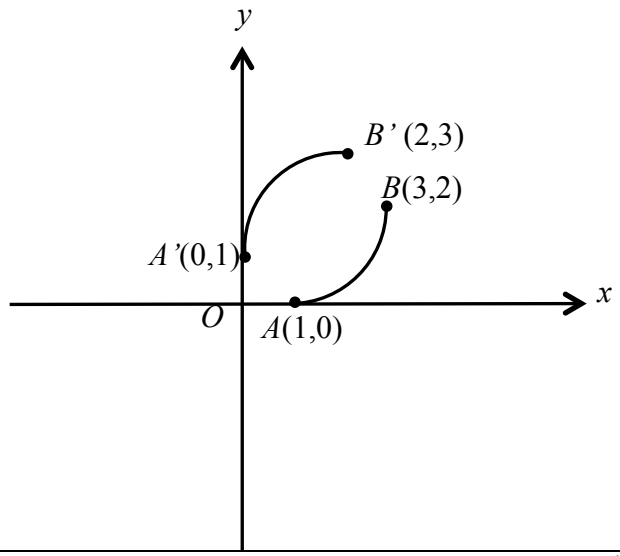
(a) Find the value of a, b and c. [3]

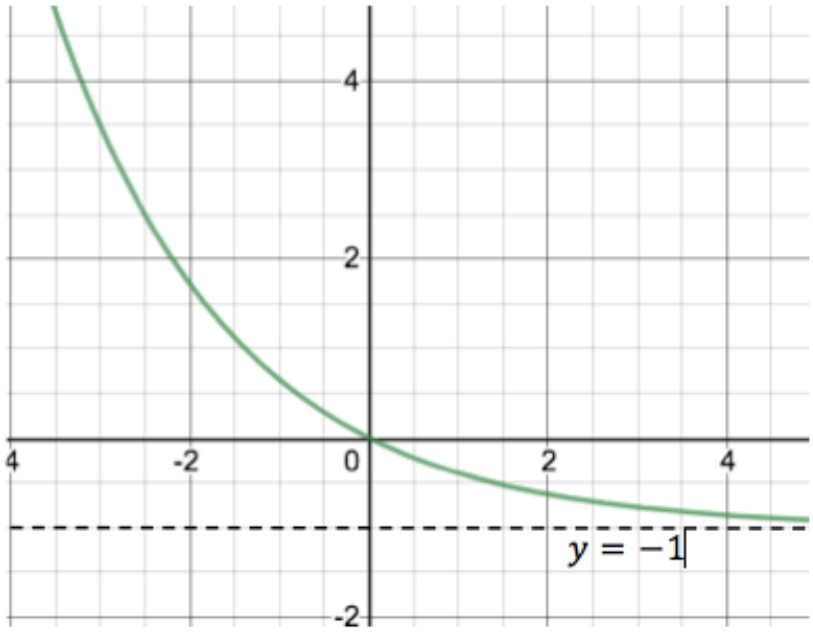
(b) The minimum depth of water at which the captain can safely navigate the particular shipping vessel is 8 metres, during daylight hours. What is the range of time in that day that the captain can safely navigate the shipping vessel into the port? Leave your answer to the nearest hour. [4]

End of Paper

Answers

1	$-\sqrt{6} < k < \sqrt{6}$
2	$0 \leq x \leq 1$ or $4 \leq x \leq 5$
3	$x = \frac{4+\sqrt{2}}{7}$
4a	
4b	

4c	
5b	$a = 7\frac{7}{16}\ , b = 14\frac{5}{16}$
6i	$-\frac{\sqrt{p^2-1}}{p}$
6ii	$\frac{1}{\sqrt{p^2-1}}$
6iii	$-\frac{\sqrt{p^2-1}}{p}$
7ai	$\mathrm{f}(x) = 3(x-1)^2 - 4$
7aii	$-4 \leq \mathrm{f}(x) \leq 8$
7bi	$k = 1$
7bii	$\mathrm{g}^{-1}: x \mapsto 1 + \sqrt{\frac{x+4}{3}}$ Domain of g is $x \geq -4$
7biii	$\mathrm{g}^2(3) = 143$

8i	
8ii	Draw $y = 1 - x$ Number of solutions = 2
9i	$h = -8, k = -3$
9ii	$\mathrm{f}(x) = (x - 3)(2x + 1)^2$
9iii	$1 + \frac{5}{49(x-3)} - \frac{10}{49(2x+1)} - \frac{10}{7(2x+1)^2}$
10a	$x = -1$
10b	$x = 2$ or $\frac{5}{9}$(rej)
10c	$x = 24.3°, 65.7°, 90°, 204.3°, 245.7°, 270°$
11a	$a = 4, b = \frac{\pi}{6}, c = 10$
11b	7am to 12pm, and 5pm to 7pm

1.