Paper E Sec 3 IP Solutions for students

Qn	Solutions
1(a)	$\frac{8x^3+1}{(2x-3)(2x-1)} \div \left(2x-\frac{1}{1-2x}\right)$ $= \frac{(2x+1)(4x^2-2x+1)}{(2x-3)(2x-1)} \div \left(\frac{2x(1-2x)-1}{1-2x}\right)$ $= \frac{(2x+1)(4x^2-2x+1)}{(2x-3)(2x-1)} \times \frac{2x-1}{4x^2-2x+1}$ $= \frac{2x+1}{2x-3}$
1(b)	$-1<4x+5\le 3$ $4x+5>-1$ and $4x+5\le 3$ $-\frac{3}{2}<x\le -\frac{1}{2}$ ---------(1) Alternatively, $-1<4x+5\le 3$ $-6<4x\le -2$ $-\frac{3}{2}<4x\le -\frac{1}{2}$ $(x+4)(5-x)\ge 18$ $-x^2+x+20\ge 18$ $x^2-x-2\le 0$ $(x+1)(x-2)\le 0$ $-1\le x\le 2$ -----------(2) Combining (1) & (2), $-1\le x\le -\frac{1}{2}$
2(a)	$8^{2x}=\frac{2^{x-2}}{2^3}$ $8^{2x+1}=2^{x-2}$ $2^{6x+3}=2^{x-2}$ $6x+3=x-2$ $5x=-5$ $x=-1$
2(b)	$4^{x+\frac{1}{2}}=7(2^x)-5$

	$\left(2^x\right)^2 \cdot 2 = 7\left(2^x\right) - 5$ let $a = 2^x$ $2a^2 - 7a + 5 = 0$ $(2a-5)(a-1) = 0$ $a = \frac{5}{2}$ or $a = 1$ $2^x = \frac{5}{2}$ or $2^x = 1$ $x = \frac{\ln\frac{5}{2}}{\ln 2}$ or $x = 0$ $= 1.32(3s.f)$ or $x = 0$ [Accept $\log_2 \frac{5}{2}$]
3(a)	$\log_{\sqrt{5}} x + 2\log_x \sqrt{5} = 3$ $\log_{\sqrt{5}} x + 2\left(\frac{\log_{\sqrt{5}} \sqrt{5}}{\log_{\sqrt{5}} x}\right) = 3$ $\left(\log_{\sqrt{5}} x\right)^2 + 2 = 3\left(\log_{\sqrt{5}} x\right)$ Let $a = \left(\log_{\sqrt{5}} x\right)$ $\therefore a^2 - 3a + 2 = 0$ $(a-1)(a-2) = 0$ $a = 1$ or $a = 2$ $\log_{\sqrt{5}} x = 1$ or $\log_{\sqrt{5}} x = 2$ $x = \sqrt{5},$ or $x = \left(\sqrt{5}\right)^2$ $x = 5$ [Accept 2.24 (3 s.f.)]
3(b)	$\sqrt{9-4x} = 2 + \sqrt{5-x}$ $\sqrt{9-4x} - \sqrt{5-x} = 2$ $(9-4x) - 2\sqrt{9-4x}\sqrt{5-x} + (5-x) = 4$ $10 - 5x = 2\sqrt{9-4x}\sqrt{5-x}$ $100 - 100x + 25x^2 = 4(9-4x)(5-x)$ $9x^2 + 16x - 80 = 0$ $(9x-20)(x+4) = 0$ $x = -4$ or $2\frac{2}{9}(\text{NA})$

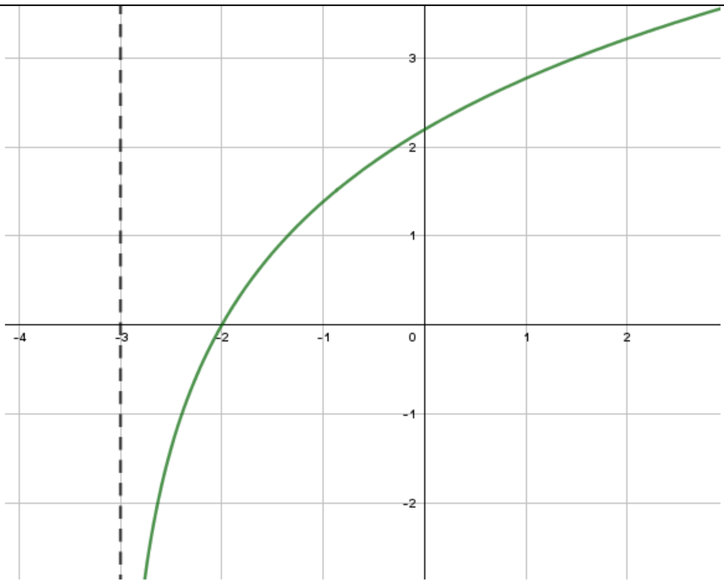
	Or alternatively, $9-4x=4+5-x+4\sqrt{5-x}$ $(-3x)^2=16(5-x)$ $9x^2+16x-80=0$ $(9x-20)(x+4)=0$ $x=-4$ or $2\frac{2}{9}$ (NA)
4(a)	$2x^2-6x+13$ $=2\left[x^2-3x+\frac{13}{2}\right]$ $=2\left[\left(x-\frac{3}{2}\right)^2+\frac{17}{4}\right]$ $=2\left(x-\frac{3}{2}\right)^2+\frac{17}{2}$
4(b)	Let $x^2+x(2-k)+k^2=0$ discriminant, b^2 - $4ac<0$ $(2-k)^2-4(1)(k^2)<0$ $4-4k-3k^2<0$ $3k^2+4k-4>0$ $(3k-2)(k+2)>0$ $k<-2$ or $k>\frac{2}{3}$
5(i)	$5y^2-20y+125=50x-5x^2$ $5y^2-20y+125-50x+5x^2=0$ $x^2+y^2-10x-4y+25=0$ Centre = $\left(\frac{-10}{-2},\frac{-4}{-2}\right)=(5,2)$ Radius = $\sqrt{5^2+2^2-25}=2$ units [Copy qn wrongly max 1m]
5(ii)	Radius of circle $C_2=2+\lvert 7\rvert=9$ Equation of circle C_2 is $(x-5)^2+(y-2)^2=81$ [PP if answer is $(x-5)^2+(y-2)^2=9^2$]

6(i)	$\mathrm{f}(x) = 2(x+3)(x-3)(x+k)$ $\mathrm{f}(-1) = 8$ Let $2(-1+3)(-1-3)(-1+k) = 8$ $-16(k-1) = 8$ $k = \frac{1}{2}$
6(ii)	$\therefore \mathrm{f}(x) = 2(x+3)(x-3)\left(x+\frac{1}{2}\right)$ $= (x+3)(x-3)(2x+1)$ Remainder $= \mathrm{f}(5)$ $= (8)(2)(11)$ $= 176$ [PP if students did not mention 'remainder' in answer]
7(a)	$\mathbf{P} = 3\begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix} + \begin{pmatrix} 5 & -4 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 5 & -4 \\ 1 & 1 \end{pmatrix}$ $= \begin{pmatrix} 6 & -3 \\ 3 & 9 \end{pmatrix} + \begin{pmatrix} 21 & -24 \\ 6 & -3 \end{pmatrix}$ $= \begin{pmatrix} 27 & -27 \\ 9 & 6 \end{pmatrix}$
7(b)	$\mathbf{Q} = \begin{pmatrix} 2 & 5 & 3 \end{pmatrix}$ $\mathbf{QC} = \begin{pmatrix} 2 & 5 & 3 \end{pmatrix}\begin{pmatrix} 3.30 & 3.20 \\ 1.90 & 2.00 \\ 3.40 & 3.45 \end{pmatrix}$ $= \begin{pmatrix} 6.60+9.50+10.20 & 6.40+10.00+10.35 \end{pmatrix}$ $= \begin{pmatrix} 26.30 & 26.75 \end{pmatrix}$ [accept $\begin{pmatrix} 26.3 & 26.75 \end{pmatrix}$**]** Conclusion: it is cheaper to shop at mini-market A
8(i)	$g^{-1}(x) = f(3)$ $= 6-7$ $= -1$ Let $\mathrm{g}(-1) = x$ $\frac{3(-1)}{-1-2} = x$ $x = 1$ Alternatively, $g^{-1}(x) = \frac{2x}{x-3}$ $\Rightarrow \frac{2x}{x-3} = f(3) = -1$ $x = 1$
8(ii)	$\lvert 2x-7 \rvert = \lvert x-3 \rvert$

	$(2x-7)^2=(x-3)^2$ $3x^2-22x+40=0$ $(3x-10)(x-4)=0$ $x=\frac{10}{3}$ or $x=4$
9(i)	$2x^2+7x+14=0$ $\alpha+\beta=\frac{-7}{2}$ and $\alpha\beta=\frac{14}{2}=7$
9(ii)	$\alpha+\beta=\frac{-7}{2}$ $\Rightarrow(\alpha+\beta)^2=\frac{49}{4}$ $\alpha^2+\beta^2=\frac{49}{4}-2\alpha\beta$ $=\frac{49}{4}-14$ $=\frac{-7}{4}$ $\frac{2\alpha}{\beta}+\frac{2\beta}{\alpha}$ $=\frac{2\alpha^2+2\beta^2}{\alpha\beta}$ $=\frac{2\left(\frac{-7}{4}\right)}{7}$ $=\frac{-1}{2}$ Product $\frac{2\alpha}{\beta}\cdot\frac{2\beta}{\alpha}=4$ The new equation is $x^2+\frac{1}{2}x+4=0$ [or equivalent equation. A0 if an expression is given.]
10	$a=\frac{6}{2}=3$ $\frac{360^\circ}{b}=180^\circ$ $\therefore b=2$ $c=1$ $\because$ graph is shifted 1 unit upwards

11(i)	Draw a line from O to T. □ AOT is a right-angled triangle. $\tan \angle AOT = \dfrac{l}{8}$ $\therefore \angle AOT = \tan^{-1}\left(\dfrac{l}{8}\right)$
11(ii)	Area of shaded part = Area Quad. AOBT – area of sector AOPB $S = 2\left(\dfrac{1}{2}8l\right) - 2\left(\dfrac{1}{2}\cdot 8^2 \cdot \tan^{-1}\left(\dfrac{l}{8}\right)\right)$ $S = 8l - 64\tan^{-1}\left(\dfrac{l}{8}\right)$
11(iii)	Perimeter = arc AOB + $2l$ $= \left[8\cdot 2\left(\tan^{-1}\left(\dfrac{l}{8}\right)\right) + 2l\right]$ m $= \left[16\tan^{-1}\left(\dfrac{l}{8}\right) + 2l\right]$ m
11(iv)	$(OT)^2 = 8^2 + 8^2$ $= 128$ $OT = \sqrt{128}$ $= 8\sqrt{2}$ Let top of pole be V. $\therefore \tan \angle VTO = \dfrac{12}{8\sqrt{2}} = \dfrac{3\sqrt{2}}{4}$ Angle of elevation, $\angle VTO =$ 46.7° (1 d.p.)
12(a)	$\dfrac{1+\tan x}{1-\tan x} = \sqrt{3}$ $1+\tan x = \sqrt{3} - \sqrt{3}\tan x$ $\left(1+\sqrt{3}\right)\tan x = \sqrt{3} - 1$ $\tan x = \dfrac{\sqrt{3}-1}{\left(1+\sqrt{3}\right)} \cdot \dfrac{\left(\sqrt{3}-1\right)}{\left(\sqrt{3}-1\right)}$ $= 2 - \sqrt{3}$
12(b) (i)	$4\sin 2x = -1$ Basic α = 14.48° $\therefore 2x = 194.48°$ or $345.52°$ $x = 97.2°, 172.8°$ (1d.p)

12(ii) (b)	$3\cos^2 x = 4\sin x\cos x$ $\cos x[3\cos x - 4\sin x] = 0$ $\cos x = 0, \quad 3\cos x - 4\sin x = 0$ $x = \frac{\pi}{2}, \frac{3\pi}{2}$ rad or $\quad 4\sin x = 3\cos x$ $\tan x = \frac{3}{4}$ $x =$ 0.644, 3.79 rad [If students divide the equation by cos *x*, max 2/3: M1 – Using $\tan x = \frac{\sin x}{\cos x}$. A1 – Correct values for solving $\tan x = \frac{3}{4}$] [Minus 1 mark if students give the <u>all correct answers</u> in degree for <u>this question</u>.]
13(i)	$\angle RSQ = 180^\circ - 56^\circ$ $= 124^\circ$ $QR = \sqrt{6^2 + 10^2 - 2\times 6\times 10\cos 124^\circ}$ $= 14.251\ 426\ 19$ $= 14.3$ km (3s.f)
13(ii)	$\frac{\sin\angle SQR}{10} = \frac{\sin 124°}{14.25142619}$ $\sin\angle SQR = \frac{10\sin 124°}{14.25142619}$ $\angle SQR = 35.57178777°$ $= 35.6°$ (1d.p)
13(iii)	Let N_2 be north of R $\angle N_2RS = \angle QPS$ (alt.∠ *s*) $= 38°$ $\therefore$ bearing of *S* from *R* is $038°$.
13(iv)	$\frac{\text{shortest distance}}{6} = \sin\angle SQR$ shortest distance $= 6\sin 35.57178777°$ $= 3.490335191$ $= 3.49$ km
14(i)	Step1:Horizontal translation of 3 units to the left Step2:a stretch with scale factor 2 parallel to *y*-axis
14(ii)	Diagram



14(iii)

$$e^2 = (x+3)e^{\frac{x}{2}+1}$$

$$\frac{e^2}{e^{\frac{x}{2}+1}} = x+3$$

$$e^{1-\frac{x}{2}} = x+3$$

$$\ln(x+3) = 1-\frac{x}{2}$$

$$2\ln(x+3) = 2-x$$

Hence the graph of $y = 2-x$ is to be drawn.

The <u>x-coordinate</u> of the point of intersection of these two graphs is the solution to the equation.

15(i)

At B: $2(0) = x-4$

$\therefore x = 4 \quad \Rightarrow B(4,0)$

Eqn $BC: 2y = x-4$

$$\therefore y = \frac{x}{2}-2$$

Let $C\left(x, \frac{x}{2}-2\right)$

$AB = AC$

$$\sqrt{(6-x)^2 + \left(6-\left(\frac{x}{2}-2\right)\right)^2} = \sqrt{(6-0)^2 + (6-4)^2}$$

	$(6-x)^2+\left(8-\frac{x}{2}\right)^2=40$ $x^2-16x+48=0$ $(x-12)(x-4)=0$ $x=12$ or $x=4$ $(given)$ $\therefore\ x=12$ and $y=\frac{12}{2}-2=4$ Hence, $C(12,4)$ $M=\left(\frac{12+4}{2},\frac{4+0}{2}\right)=(8,2)$
15(ii)	grad $BC=\frac{1}{2}$ $\therefore$ grad $\perp$ bisector of $BC=-2$ Let (x,y) be a point on $\perp$ bisector of BC $\frac{y-2}{x-8}=-2$ $\therefore y=-2x+18$
15(iii)	Reason 1: grad $BC\times$ grad $AD=\frac{1}{2}\times\frac{6+2}{6-10}=-1$ $\therefore$ diagonal $BC\perp$ diagonal AD Reason 2: mid-pt $AD=\left(\frac{6+10}{2},\frac{6+(-2)}{2}\right)$ $=(8,2)$ = mid-pt BC $\therefore$ diagonal AD bisects diagonal BC Diagonals are $\perp$ bisectors. Reason 3: AD = BC = $\sqrt{80}$ Hence $ABDC$ is a square Alternatively, Reason 1: AC = DC = $\sqrt{40}$ => Rhombus Reason 2: D lies on perpendicular bisector of BC (can be checked) or another pair of equal sides Reason 3: $AB\perp AC$ => $ABDC$ is a square

15(iv)	Area $= \frac{1}{2}\begin{vmatrix} 4 & 10 & 12 & 6 & 4 \\ 0 & -2 & 4 & 6 & 0 \end{vmatrix}$ $= \frac{1}{2}\left[\left(-8+40+72+0\right)-\left(0-24+24+24\right)\right]$ $= \frac{1}{2}\left[104-24\right]$ $= 40 \text{ units}^2$
16(i)	[See Graph Paper]
16(ii)	[See Graph Paper] The gradient = $\frac{b}{2}$, so $b \approx 1.00$ (accept $0.90 \le b \le 1.10$) The vertical intercept = $\frac{1}{2}\ln a$, so $a \approx 4.95$ (accept $4.48 \le b \le 5.47$)
16(iii)	$\ln y \approx 1.675 \Rightarrow y \approx 5.34$ (accept $5.21 \le y \le 5.75$)

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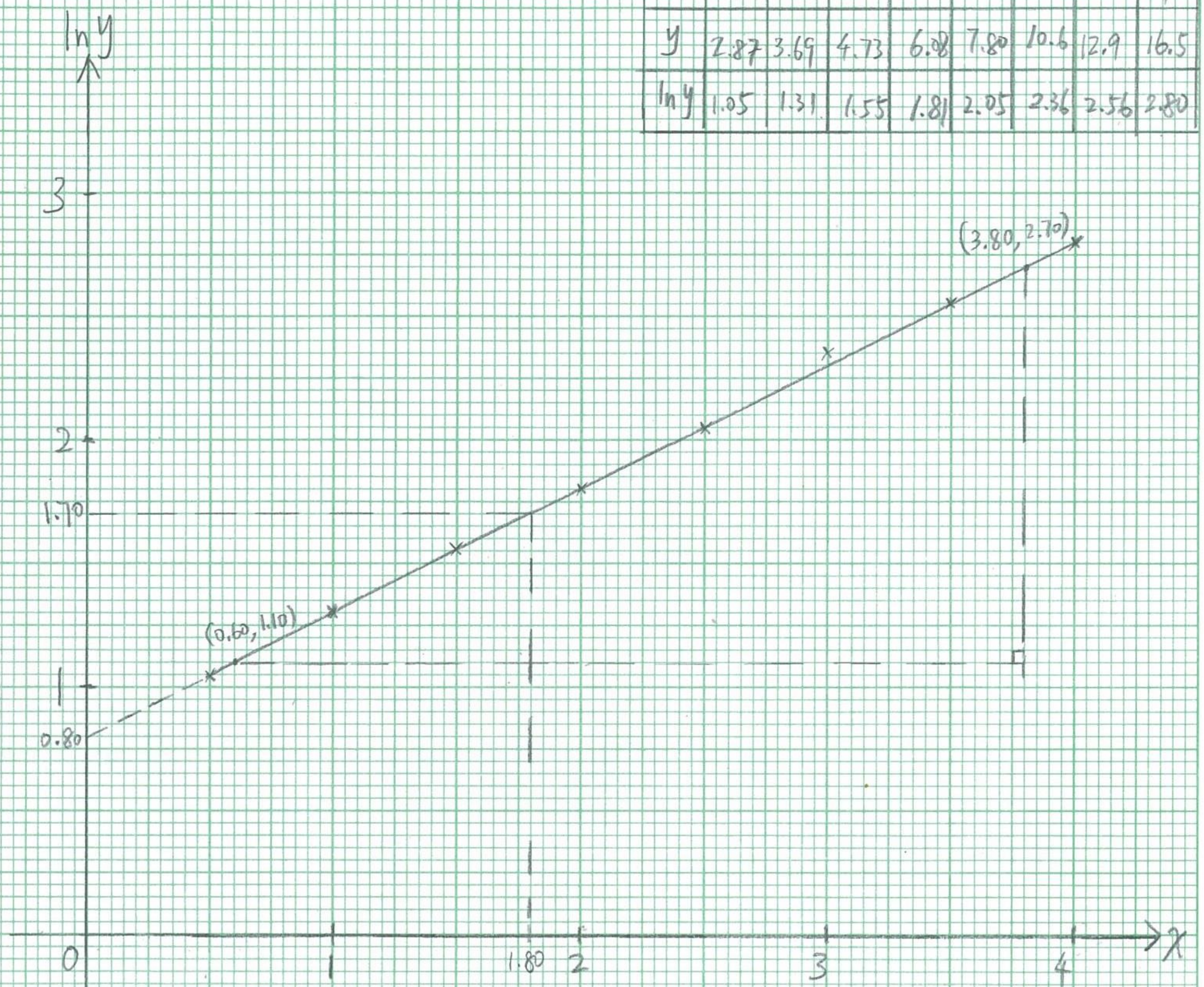
x	0.5	1	1.5	2	2.5	3	3.5	4
y	2.87	3.69	4.73	6.08	7.80	10.6	12.9	16.5
$\ln y$	1.05	1.31	1.55	1.81	2.05	2.36	2.56	2.80



(ii) $2\ln y = bx + \ln a$

$\ln y = \frac{b}{2}x + \frac{1}{2}\ln a$

From graph, $\frac{1}{2}\ln a = 0.80 \pm 0.05$

$a \approx 4.95$ (accept $4.48 \leq a \leq 5.47$)

$\frac{b}{2} = \frac{2.70 - 1.10}{3.80 - 0.60}$

$b \approx 1.00 \pm 0.1$

(iii) From graph, when $x = 1.8$

$\ln y = 1.70 \pm 0.05$

$y \approx 5.47$

(accept $5.21 \leq y \leq 5.75$)