Candidate Name: *Answer Scheme.	Class:	Index No.:

FUHUA SECONDARY SCHOOL
Secondary 4 Express/ 5 Normal (Academic)
PRELIMINARY EXAMINATION 2025

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ADDITIONAL MATHEMATICS **4049/01**

Paper 1

DATE **26 August 2025**
TIME **1050 – 1305**
DURATION **2 hours 15 minutes**

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **90**.

FOR EXAMINER'S USE			PARENT'S SIGNATURE
Statements/Workings		/ 90	
Accuracy (incl. Units)			
Presentation			

Setter: **Ms Gao Conger** **Vetter:** **Ms Lim Fen Niu**

This document consists of 24 printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

SWAP (presentation) { ① missing essential brackets
For all questions | ② table without borders

1 Express $\dfrac{x^3-4x^2-24x+41}{x^2-4x-21}$ in partial fractions. [5]

$$
\begin{array}{r}
x \\
x^2-4x-21 \enclose{longdiv}{x^3-4x^2-24x+41} \\
-(x^3-4x^2-21x) \\
\hline
-3x+41
\end{array}
$$

$$\therefore \frac{x^3-4x^2-24x+41}{x^2-24x-21} = x + \frac{-3x+41}{x^2-4x-21}$$

$$= x + \frac{-3x+41}{(x-7)(x+3)}$$

let $\dfrac{-3x+41}{(x-7)(x+3)} = \dfrac{A}{x-7} + \dfrac{B}{x+3}$

$$-3x+41 = A(x+3) + B(x-7)$$

let $x=-3$, $-3(-3)+41 = B(-3-7)$

$$50 = -10B$$

$$-5 = B$$

let $x=7$, $-3(7)+41 = A(7+3)$

$$20 = 10A$$

$$2 = A$$

$$\therefore \frac{x^3-4x^2-24x+41}{x^2-4x-21} = x + \frac{2}{x-7} - \frac{5}{x+3}$$

5m

2 (a) In a quiz question, the quadratic equation $k\left(x^2+1\right)-8x-6x^2=0$ is given.

Find the range of values of the constant k, for which the equation has no real roots. [4]

$kx^2+k-8x-6x^2=0$

$kx^2-6x^2-8x+k=0$

$a=k-6,\ b=-8,\ c=k$

$b^2-4ac<0$

$(-8)^2-4(k-6)(k)<0$

$64-4k^2+24k<0$

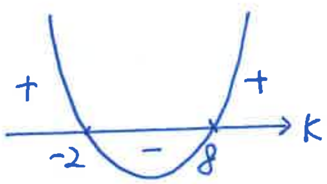
$\times(-1)$

$4k^2-24k-64>0$

$k^2-6k-16>0$

$(k-8)(k+2)>0$

$\boxed{\begin{array}{l}\text{For } (k-8)(k+2)=0 \\ k=8 \text{ or } k=-2\end{array}}$ (optional working)



$\therefore\ k<-2$ or $k>8$

SWAP (presentation): "and" used in final answer

4m

(b) Another question in the quiz is shown below.

> Find the range of values of the constant k, for which the curve
> $y=(k-6)x^2-2x$ lies completely above the line $y=6x-k$.
> $-①$ $\qquad$ $-②$

Jeremy stated that the range of values of k found in part **(b)** is the same as that in part **(a)**. Is Jeremy correct? Explain your answer clearly. [2]

Sub. ① into ②,

$(k-6)x^2-2x=6x-k$

$(k-6)x^2-2x-6x+k=0$

$(k-6)x^2-8x+k=0$

$b^2-4ac<0$

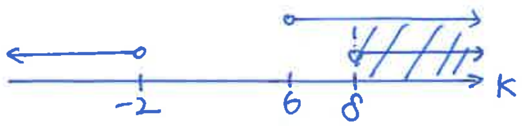
$(-8)^2-4(k-6)(k)<0$

From (a), $k<-2$ or $k>8$

However, the curve should be a U-shaped curve
(or the curve should open upwards)
(or coefficient of x^2 should be positive)

$\therefore k-6>0$

$k>6$



$\therefore$ the solution for (b) is only $k>8$

(or: the solution for (b) cannot include $k<-2$)

$\therefore$ (a) and (b) have different ranges of values of k

$\therefore$ No, Jeremy is not correct.

2m

3 Solve the simultaneous equations. [4]

$$2x + y = 10 \quad -① $$
$$\frac{3}{y} + \frac{2}{x} = 2 \quad -② $$

From ①, $y = 10 - 2x$ $\quad -③$

② $\times xy$, $3x + 2y = 2xy$ $\quad -④$

Sub. ③ into ④,

$3x + 2(10-2x) = 2x(10-2x)$

$3x + 20 - 4x = 20x - 4x^2$

$-x + 20 - 20x + 4x^2 = 0$

$4x^2 - 21x + 20 = 0$

$(x-4)(4x-5) = 0$

$x = 4$ or $x = \frac{5}{4}$

Sub. $x = 4$ into ③, $y = 10 - 2(4)$
$= 2$

Sub. $x = \frac{5}{4}$ into ③, $y = 10 - 2(\frac{5}{4})$
$= \frac{15}{2}$

$\therefore x = 4, y = 2$ or $x = \frac{5}{4}, y = \frac{15}{2}$

Method 2

Sub. ③ into ②.

$\frac{3}{10-2x} + \frac{2}{x} = 2$

$\frac{3x + 2(10-2x)}{x(10-2x)} = 2$

$\frac{3x + 20 - 4x}{10x - 2x^2} = \frac{2}{1}$

$-x + 20 = 20x - 4x^2$

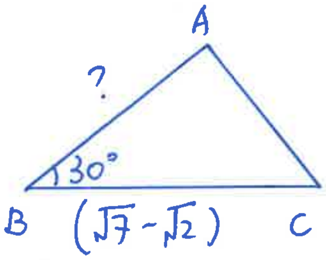
$4x^2 - 21x + 20 = 0$

4m

4 In triangle ABC, $BC = \left(\sqrt{7}-\sqrt{2}\right)$ cm and angle $ABC = 30°$.

The area of triangle ABC is $\left(4\sqrt{7}-\sqrt{2}\right)$ cm^2.

Without using a calculator, express the length of AB in the form $\left(p+q\sqrt{14}\right)$ cm, where p and q are constants, and state the value of p and of q. [4]



$$\text{area} = \frac{1}{2} \times AB \times BC \times \sin \angle ABC$$

$$\frac{1}{2} \times AB \times (\sqrt{7}-\sqrt{2}) \times \sin 30° = 4\sqrt{7}-\sqrt{2}$$

$$\frac{1}{2} \times AB \times (\sqrt{7}-\sqrt{2}) \times \frac{1}{2} = 4\sqrt{7}-\sqrt{2}$$

$$AB \times (\sqrt{7}-\sqrt{2}) = 16\sqrt{7}-4\sqrt{2}$$

$$AB = \frac{16\sqrt{7}-4\sqrt{2}}{\sqrt{7}-\sqrt{2}} \times \frac{\sqrt{7}+\sqrt{2}}{\sqrt{7}+\sqrt{2}}$$

$$= \frac{16(7)+16\sqrt{14}-4\sqrt{14}-4(2)}{7-2}$$

$$= \frac{104+12\sqrt{14}}{5}$$

$$= \left(\frac{104}{5} + \frac{12}{5}\sqrt{14}\right) \text{ cm}$$

$$p = \frac{104}{5}, \quad q = \frac{12}{5}$$

4m

5 A circle C_1, with centre C, has equation $x^2 - 8x + y^2 + 2y - 17 = 0$.

(a) Find the coordinates of C and the radius of the circle. [3]

$(x-4)^2 - 16 + (y+1)^2 - 1 - 17 = 0$

$(x-4)^2 + (y+1)^2 = 34$

centre $C\ (4, -1)$

radius $= \sqrt{34}$ units

(OR 5.83 units (3 s.f.)

SWAP (accuracy): missing unit / degree of accuracy.

Method 2:

compare with

$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = -8 \quad 2f = 2 \quad c = -17$

$g = -4 \quad f = 1$

Centre $C\ (4, -1)$

radius $= \sqrt{1^2 + (-4)^2 - (-17)}$

$= \sqrt{34}$ units

(b) Determine whether the point $S(-1, -5)$ lies inside, outside or on the circle. Show your working clearly. [2]

$CS = \sqrt{[4-(-1)]^2 + [-1-(-5)]^2}$

$= \sqrt{41}$

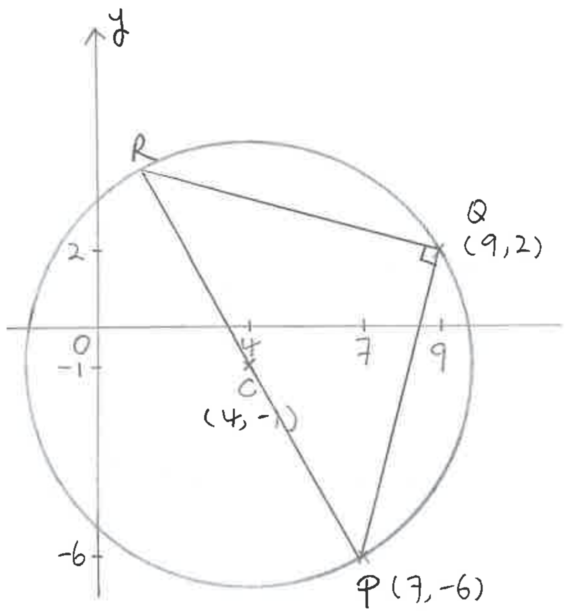
$= 6.40$ units (3 s.f.) > 5.83 units

The distance of S from centre of circle is larger than the radius of circle.

$\therefore$ S lies outside the circle.

5m

(c) Given that $P(7,-6)$, $Q(9,2)$ and R are three points on the circle, and PQ is perpendicular to QR, find the coordinates of R. [3]



Method 1

$\because PQ \perp QR$

$\therefore \angle PQR = 90^\circ$

$\therefore$ PR is a diameter of circle (rt. $\angle$ in a semicircle)

Let R be (x, y)

$\left(\frac{x+7}{2}, \frac{y+(-6)}{2}\right) = (4, -1)$

$x+7=8$ $\quad$ $y-6=-2$

$x=1$ $\quad$ $y=4$ $\quad$ $\therefore R(1,4)$

Method 2

gradient of PQ $= \frac{2-(-6)}{9-7}$

$= 4$

gradient of QR $= \frac{-1}{4}$

$\therefore$ QR : $y = -\frac{1}{4}x + c$

Sub. $(9,2)$ $\quad 2 = -\frac{1}{4}(9) + c$

$\frac{17}{4} = c$

$\therefore$ Equation of QR: $y = -\frac{1}{4}x + \frac{17}{4}$ $\quad$ -①

circle: $(x-4)^2 + (y+1)^2 = 34$ $\quad$ -②

s. ① into ②,

$(x-4)^2 + \left(-\frac{1}{4}x + \frac{17}{4} + 1\right)^2 = 34$

$(x-4)^2 + \left(-\frac{1}{4}x + \frac{21}{4}\right)^2 = 34$

$x^2 - 8x + 16 + \frac{1}{16}x^2 - \frac{21}{8}x + \frac{441}{16} = 34$

$16x^2 - 128x + 256 + x^2 - 42x + 441 = 544$

$17x^2 - 170x + 153 = 0$

$x^2 - 10x + 9 = 0$

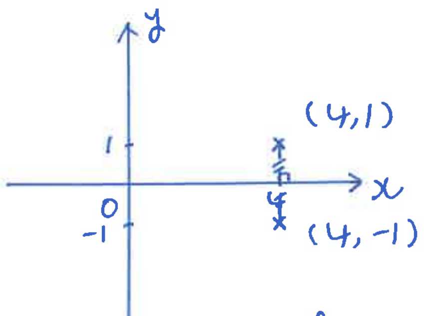
$(x-1)(x-9) = 0$

$x = 1$ or $x = 9$ (rej. point Q)

when $x=1$, $y = -\frac{1}{4}(1) + \frac{17}{4}$

$R(1,4)$

(d) Another circle C_2, is a reflection of C_1 in the x-axis. Write down the coordinates of the centre of C_2. [1]



centre of C_2 $(4, 1)$

4m

6 (a) Prove that $\dfrac{\sin x}{\sec x+1}+\dfrac{\sin x}{\sec x-1}=2\cot x$. [4]

LHS $= \dfrac{\sin x(\sec x-1)+\sin x(\sec x+1)}{(\sec x+1)(\sec x-1)}$

$= \dfrac{\sin x\sec x-\sin x+\sin x\sec x+\sin x}{\sec^2 x-1}$

$= \dfrac{2\sin x\sec x}{\tan^2 x}$

$= \dfrac{2\sin x\left(\frac{1}{\cos x}\right)}{\tan^2 x}$

$= \dfrac{2\tan x}{\tan^2 x}$

$= 2\left(\dfrac{1}{\tan x}\right)$

$= 2\cot x$

OR $= \dfrac{2\sin x\left(\frac{1}{\cos x}\right)}{\frac{1}{\cos^2 x}-1}\times\dfrac{\cos^2 x}{\cos^2 x}$

$= \dfrac{2\sin x\cos x}{1-\cos^2 x}$

$= \dfrac{2\sin x\cos x}{\sin^2 x}$

$= \dfrac{2\cos x}{\sin x}$

$= 2\cot x$

Method 2

LHS $= \dfrac{\sin x}{\frac{1}{\cos x}+1}+\dfrac{\sin x}{\frac{1}{\cos x}-1}$

$= \dfrac{\sin x}{\frac{1}{\cos x}+1}\times\dfrac{\cos x}{\cos x}+\dfrac{\sin x}{\frac{1}{\cos x}-1}\times\dfrac{\cos x}{\cos x}$

$= \dfrac{\sin x\cos x}{1+\cos x}+\dfrac{\sin x\cos x}{1-\cos x}$

$= \dfrac{\sin x\cos x(1-\cos x)+\sin x\cos x(1+\cos x)}{(1+\cos x)(1-\cos x)}$

$= \dfrac{\sin x\cos x-\sin x\cos^2 x+\sin x\cos x+\sin x\cos^2 x}{1-\cos^2 x}$

$= \dfrac{2\sin x\cos x}{\sin^2 x}$

$= \dfrac{2\cos x}{\sin x}$

$= 2\cot x$

OR $= \dfrac{\frac{\sin x}{\cos x}-\sin x+\frac{\sin x}{\cos x}+\sin x}{\frac{1}{\cos^2 x}-1}$

$= \dfrac{2\tan x}{\sec^2 x-1}$

$= \dfrac{2\tan x}{\tan^2 x}$

$= \dfrac{2}{\tan x}$

$= 2\cot x$

4m

(b) Hence, solve the equation $\dfrac{\sin(2\theta-50^\circ)}{\sec(2\theta-50^\circ)+1}+\dfrac{\sin(2\theta-50^\circ)}{\sec(2\theta-50^\circ)-1}=\sqrt{3}$,

for $-90^\circ \le \theta \le 90^\circ$. [4]

$x = 2\theta - 50^\circ$

$2\cot(2\theta-50^\circ) = \sqrt{3}$

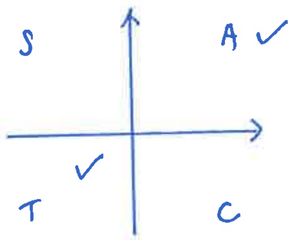
$\cot(2\theta-50^\circ) = \frac{\sqrt{3}}{2}$

$\tan(2\theta-50^\circ) = \frac{2}{\sqrt{3}}$

$-90^\circ \le \theta \le 90^\circ$

$-180^\circ \le 2\theta \le 180^\circ$

$-230^\circ \le 2\theta-50^\circ \le 130^\circ$



basic angle $\alpha = \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$

$= 49.1066^\circ$ (4 d.p.)

$2\theta - 50^\circ = 49.1066^\circ,\ 229.1066^\circ$ (rej.)

$\xrightarrow{-360^\circ} -130.8934^\circ$

$2\theta = 99.1066^\circ,\ -80.8934^\circ$

$\theta = 49.6^\circ$ or -40.4° (1 d.p.)

SWAP (accuracy) : missing degree of accuracy.

4m

7 (a) Find $\frac{d}{dx}\left(\frac{\ln 4x}{x}\right)$. [2]

SWAP (presentation): incorrect notation for differentiation and integration (including inappropriate use of arbitrary constants)

$u = \ln 4x \qquad v = x$

$\frac{du}{dx} = \frac{4}{4x} \qquad \frac{dv}{dx} = 1$

$= \frac{1}{x}$

$\frac{d}{dx}\left(\frac{\ln 4x}{x}\right) = \frac{x \times \frac{1}{x} - \ln 4x \times 1}{x^2}$

$= \frac{1 - \ln 4x}{x^2}$

(b) Hence, find $\int \frac{\ln 4x}{x^2}\,dx$. [3]

Method 1

$\int \frac{1-\ln 4x}{x^2}\,dx = \frac{\ln 4x}{x} + C$

$\int x^{-2}\,dx - \int \frac{\ln 4x}{x^2}\,dx = \frac{\ln 4x}{x} + C$

$\frac{x^{-1}}{-1} + C_1 - \int \frac{\ln 4x}{x^2}\,dx = \frac{\ln 4x}{x} + C$

$-\frac{1}{x} + C_1 - \frac{\ln 4x}{x} - C = \int \frac{\ln 4x}{x^2}\,dx$

$\int \frac{\ln 4x}{x^2}\,dx = -\frac{1}{x} - \frac{\ln 4x}{x} + C_2$

(C, C_1, C_2: arbitrary constants)

Method 2

$\int \frac{\ln 4x}{x^2}\,dx$

$= \int -\frac{1-\ln 4x}{x^2} + \frac{1}{x^2}\,dx$

$= -\int \frac{1-\ln 4x}{x^2}\,dx + \int x^{-2}\,dx$

$= -\frac{\ln 4x}{x} + C + \frac{x^{-1}}{-1} + C_1 = -\frac{\ln 4x}{x} - \frac{1}{x} + C_2$

5m

(c) Find the range of values of x for which $f(x) = \frac{\ln 4x}{x}$ is increasing. [3]

$f'(x) = \frac{1 - \ln 4x}{x^2}$

$\because$ $f(x)$ is increasing

$\therefore f'(x) > 0$

$\frac{1 - \ln 4x}{x^2} > 0$

$\because$ $\ln 4x$ is defined $\therefore 4x > 0$, $x > 0$

$\therefore x^2 > 0$

$\therefore 1 - \ln 4x > 0$

$1 > \ln 4x$

$\ln 4x < 1$

$4x < e^1$

$x < \frac{1}{4}e$

$\therefore 0 < x < \frac{1}{4}e$

3m

8 The equation of a curve is $y=(1-3x)^6(x^2+2)$.

(a) Find $\frac{dy}{dx}$ in the form $\cancel{x}\ \frac{dy}{dx}=2(1-3x)^p(qx^2+kx-18)$, where p, q and k are constants. [3]

$u=(1-3x)^6 \qquad v=x^2+2$

$\frac{du}{dx}=6(1-3x)^5(-3) \qquad \frac{dv}{dx}=2x$

$=-18(1-3x)^5$

$\frac{dy}{dx}=(1-3x)^6\times 2x+(x^2+2)\times[-18(1-3x)^5]$

$=2x(1-3x)^6-18(1-3x)^5(x^2+2)$

$=2(1-3x)^5[x(1-3x)-9(x^2+2)]$

$=2(1-3x)^5[x-3x^2-9x^2-18] \quad =2(1-3x)^5(-12x^2+x-18)$

(b) Explain why the curve has only one stationary point and find the x-coordinate of the stationary point. [4]

For stationary point, $\frac{dy}{dx}=0$

$2(1-3x)^5(-12x^2+x-18)=0$

$1-3x=0$ or $-12x^2+x-18=0$

$x=\frac{1}{3}$

$b^2-4ac=1^2-4(-12)(-18)$

$=-863<0$

$\therefore$ the quadratic equation has no real roots.

$\therefore$ the equation $\frac{dy}{dx}=0$ has only 1 real solution

$\therefore$ The curve has only 1 stationary point

its x-coordinate $=\frac{1}{3}$

Method 2

$-12x^2+x-18$

$=-12(x^2-\frac{1}{12}x)-18$

$=-12[(x-\frac{1}{24})^2-\frac{1}{576}]-18$

$=-12(x-\frac{1}{24})^2+\frac{1}{48}-18$

$=-12(x-\frac{1}{24})^2-\frac{863}{48}$

$\because (x-\frac{1}{24})^2\geq 0$

$\therefore -12(x-\frac{1}{24})^2\leq 0$

$-12(x-\frac{1}{24})^2-\frac{863}{48}\leq-\frac{863}{48}$

$\therefore -12x^2+x-18<0$

$-12x^2+x-18$ cannot be 0

$-12x^2+x-18=0$ has no real roots.

SWAP (presentation): incorrect notation for differentiation

7m

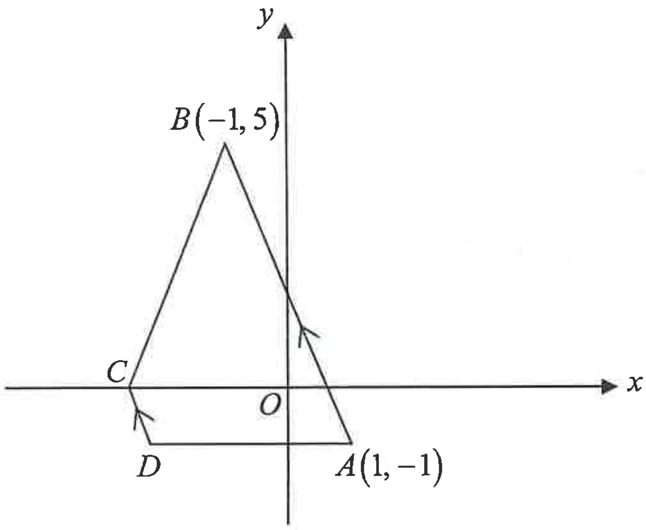
Continuation of working space for question **8(b)**.

(c) Determine the nature of this stationary point. [2]

x	0.2	$\frac{1}{3}$	0.4
$\frac{dy}{dx}$	≈ -0.374 $-$	0	≈ 0.0125 $+$
sketch of tangent	\	—	/

$\therefore$ the stationary point at $x = \frac{1}{3}$ is a minimum point.

9 The diagram shows a trapezium with vertices $A(1,-1)$, $B(-1,5)$, C and D. AB and DC are parallel, and AD is parallel to the x-axis. The perpendicular bisector of AB crosses the x-axis at C.



(a) Find the coordinates of C. [4]

Method 1

gradient of $AB = \frac{5-(-1)}{-1-1}$

$= -3$

gradient of $\perp$ bisector of $AB = \frac{-1}{-3}$

$= \frac{1}{3}$

$\therefore \perp$ bisector: $y = \frac{1}{3}x + c$

midpoint of AB $\left(\frac{-1+1}{2}, \frac{5+(-1)}{2}\right)$

$(0, 2)$

$2 = \frac{1}{3}(0) + c$

$2 = c$

$\therefore$ equation of $\perp$ bisector of AB: $y = \frac{1}{3}x + 2$

Sub. $y = 0$, $0 = \frac{1}{3}x + 2$

$-2 = \frac{1}{3}x$

$-6 = x$

$\therefore C(-6, 0)$

Method 2 Let C be $(x, 0)$

$CA = CB$

$\sqrt{[x-(-1)]^2 + (0-5)^2} = \sqrt{(x-1)^2 + [0-(-1)]^2}$

$\sqrt{(x+1)^2 + 25} = \sqrt{(x-1)^2 + 1}$

$x^2 + 2x + 1 + 25 = x^2 - 2x + 1 + 1$

$4x = -24$

$x = -6$

$\therefore C(-6, 0)$

4m

(b) Find the area of the trapezium *ABCD*. [4]

gradient of CD = gradient of AB

$= -3$

(OR : $\frac{-1}{\frac{1}{3}} = -3$)

$\therefore$ CD: $y = -3x + c$

Sub. $(-6, 0)$ $\quad 0 = -3(-6) + c$

$0 = 18 + c$

$-18 = c$

$\therefore$ equation of CD: $y = -3x - 18$

$\because$ DA // x-axis $\quad \therefore$ y-coordinate of D $= -1$

Sub. into eqn of CD: $\quad -1 = -3x - 18$

$3x = -17$

$x = -\frac{17}{3}$

$\therefore D(-\frac{17}{3}, -1)$

area of ABCD $= \frac{1}{2}\begin{vmatrix} -6 & -\frac{17}{3} & 1 & -1 & -6 \\ 0 & -1 & -1 & 5 & 0 \end{vmatrix}$

$= \frac{1}{2}\left|6 + \frac{17}{3} + 5 + 0 - 0 - (-1) - 1 - (-30)\right|$

$= \frac{1}{2}\left|\frac{140}{3}\right|$

$= \frac{70}{3}$ units^2

SWAP (presentation) : missing unit.

4m

10 **(a)** By expressing $-2x^2+4x+\frac{7}{4}$ in the form $a(x+b)^2+c$, where a, b and c are constants, explain why the value of $-2x^2+4x+\frac{7}{4}$ cannot be 4. [4]

$-2x^2+4x+\frac{7}{4}$

$=-2(x^2-2x)+\frac{7}{4}$

$=-2[(x-1)^2-1]+\frac{7}{4}$

$=-2(x-1)^2+2+\frac{7}{4}$

$=-2(x-1)^2+\frac{15}{4}$

OR $=-2(x^2-2x-\frac{7}{8})$

$=-2[(x-1)^2-1-\frac{7}{8}]$

$=-2[(x-1)^2-\frac{15}{8}]$

$=-2(x-1)^2+\frac{15}{4}$

$\because (x-1)^2 \geq 0$

$\therefore -2(x-1)^2 \leq 0$

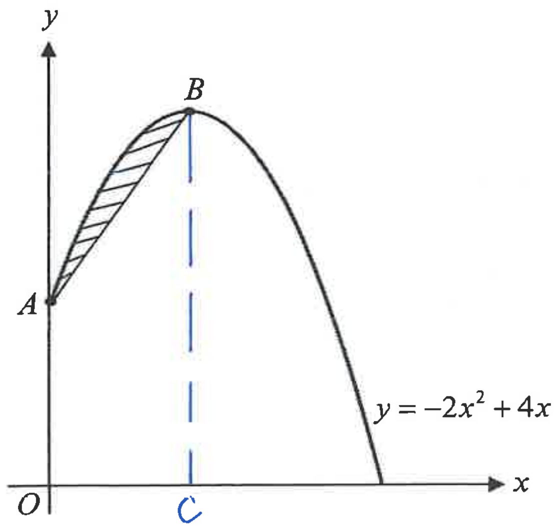
$-2(x-1)^2+\frac{15}{4} \leq \frac{15}{4}$

maximum value of $-2x^2+4x+\frac{7}{4}$ is $3\frac{3}{4}$

4 is larger than the maximum value of $3\frac{3}{4}$

$\therefore$ the value of $-2x^2+4x+\frac{7}{4}$ cannot be 4.

The diagram shows part of the curve $y=-2x^2+4x+\frac{7}{4}$. The curve intersects the y-axis at A, and the highest point of the curve is B.



4m

SWAP { (presentation): incorrect notation for integration.
(accuracy): missing unit

(b) Find the area of the shaded region bounded by the curve and the line *AB*. [5]

Sub. $x=0$, $y=-2(0)^2+4(0)+\frac{7}{4}$

$=\frac{7}{4}$

$\therefore A(0,\frac{7}{4})$

$B(1,\frac{15}{4})$

Method 1

area of trapezium OABC

$=\frac{1}{2}\times(\frac{7}{4}+\frac{15}{4})\times 1$

$=\frac{11}{4}$

$\int_0^1(-2x^2+4x+\frac{7}{4})\,dx$

$=\left[\frac{-2x^3}{3}+\frac{4x^2}{2}+\frac{7}{4}x\right]_0^1$

$=\left[\frac{-2x^3}{3}+2x^2+\frac{7}{4}x\right]_0^1$

$=\left[\frac{-2(1)^3}{3}+2(1)^2+\frac{7}{4}(1)\right]-\left[\frac{-2(0)^3}{3}+2(0)^2+\frac{7}{4}(0)\right]$

$=\frac{37}{12}$

$\therefore$ area of shaded region

$=\frac{37}{12}-\frac{11}{4}$

$=\frac{1}{3}$ units^2

Method 2

gradient of AB $=\dfrac{\frac{15}{4}-\frac{7}{4}}{1-0}$

$=2$

$\therefore$ eqn of AB: $y=2x+\frac{7}{4}$

area of shaded region

$=\int_0^1\left[(-2x^2+4x+7x)-(2x+\frac{7}{4})\right]dx$

$=\int_0^1(-2x^2+2x)\,dx$

$=\left[\frac{-2x^3}{3}+\frac{2x^2}{2}\right]_0^1$

$=\left[-\frac{2(1)^3}{3}+\frac{2(1)^2}{2}\right]-\left[-\frac{2(0)^3}{3}+\frac{2(0)^2}{2}\right]$

$=\frac{1}{3}$ units^2

5m

11 The table shows the values of two variables x and y from an experiment.

The two variables are connected by the equation $\frac{\sqrt{x}}{y} = px + q\sqrt{x}$, where p and q are constants. The values of y are corrected to 2 decimal places.

x	5	15	25	30	40
y	0.37	0.30	0.26	0.25	0.23

(a) Draw, on the grid provided on the next page, a straight line graph of $\frac{1}{y}$ against $\sqrt{x}$. [3]

X	$\sqrt{x}$	2.24	3.87	5	5.48	6.32
Y	$\frac{1}{y}$	2.70	3.33	3.85	4	4.35

(2 d.p.)

(b) Use your graph to estimate the value of p and of q. [2]

$\frac{1}{y}$ - intercept ≈ 1.80

$q \approx 1.80$

(accept 1.65 to 1.90)

gradient $\approx \frac{4.50-1.80}{6.70-0}$

$p \approx 0.403$

(best: 0.407; accept 0.38 to 0.42)

(c) Use your graph to estimate the value of y when $x = 19$. [2]

$x = 19$ $\quad \sqrt{x} = 4.36$ (2 d.p.)

$\frac{1}{y} \approx 3.55$

$y \approx \frac{1}{3.55}$

$y \approx 0.282$ (3 s.f.)

or 0.28 (2 d.p.)

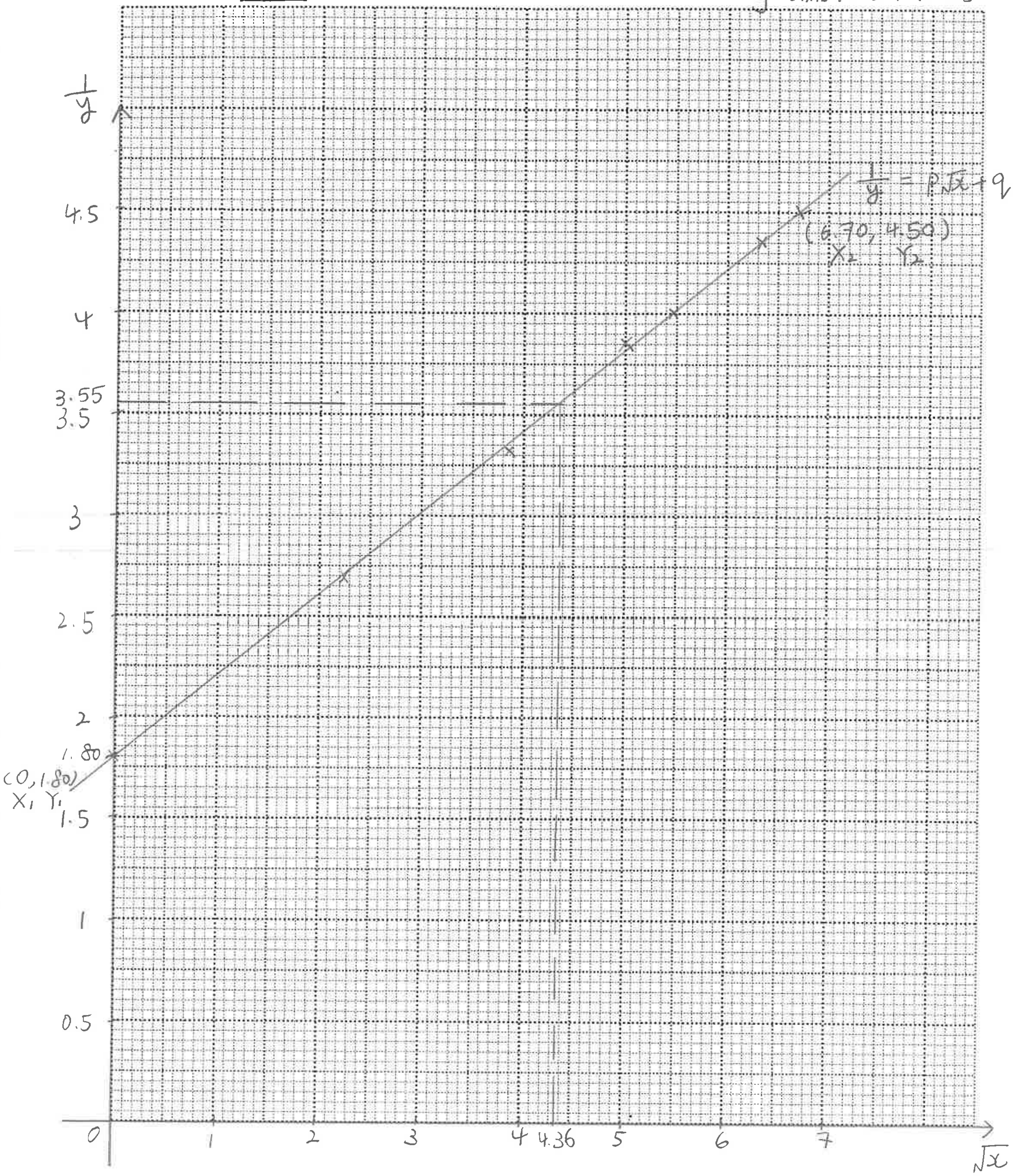
7m

Version 1

Scale: $\sqrt{x}$ - axis: 2cm to 1 unit

$\frac{1}{y}$ - axis: 2cm to 0.5 units

Qn. Number: 11.

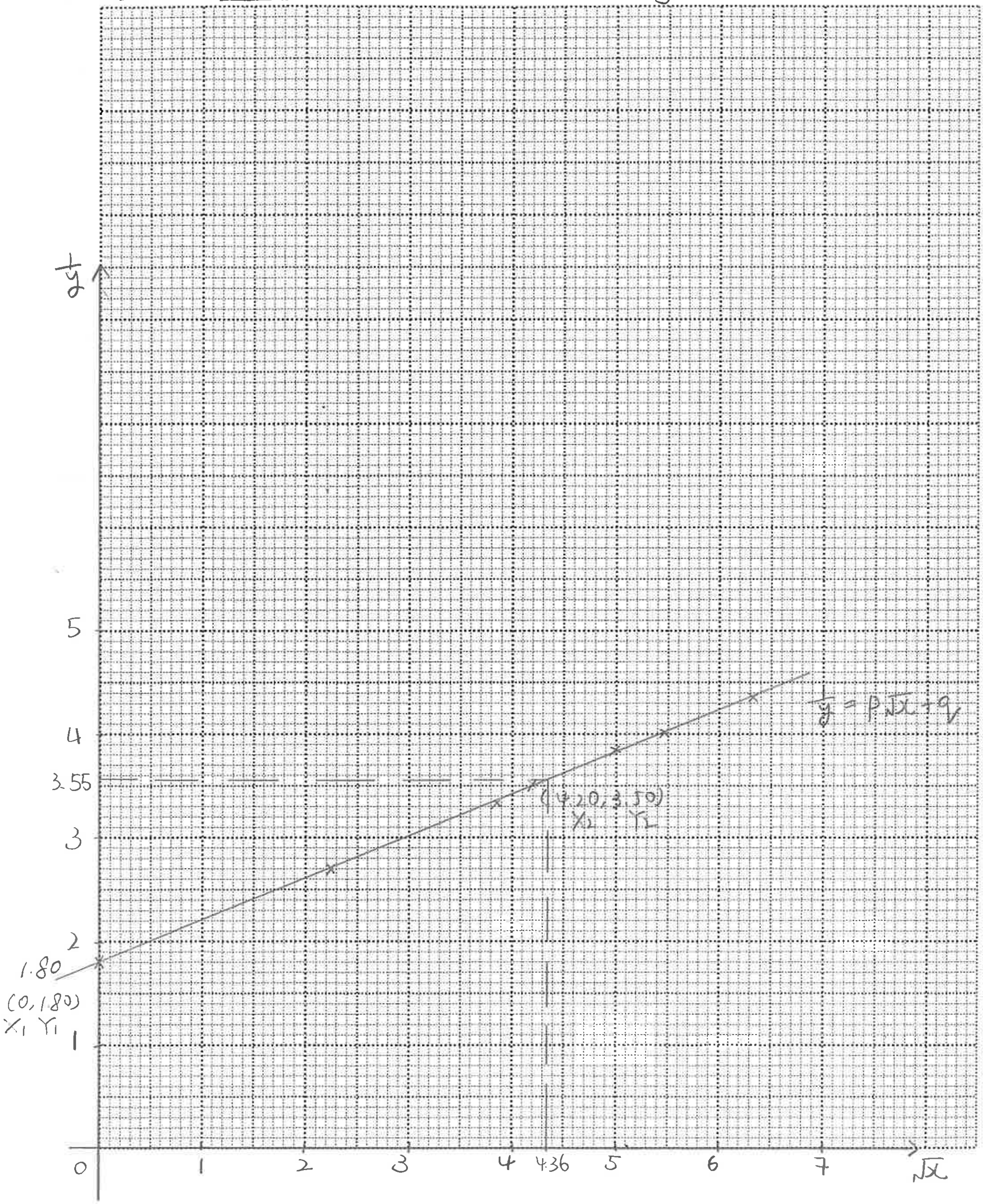


version 2

Scale: $\sqrt{x}$-axis: 2cm to 1 unit

$\frac{1}{y}$-axis: 2cm to 1 unit

Qn. Number: 11



(b) gradient $\approx \frac{3.50 - 1.80}{4.20 - 0}$

$p \approx 0.405$ (3 s.f.)

SWAP (presentation): incorrect notation for differentiation and integration.

12 The gradient of a curve is given as $\frac{dy}{dx} = Ae^{3x} + e^{-x}$, where A is a constant.

(a) The curve passes through the point $(0, -1)$. Find the equation of the curve in terms of A. [3]

$y = \int (Ae^{3x} + e^{-x})\, dx$

$= A(\frac{1}{3})e^{3x} + (-1)e^{-x} + c$

$= \frac{1}{3}Ae^{3x} - e^{-x} + c$

Sub. $(0, -1)$ $\quad -1 = \frac{1}{3}Ae^{3(0)} - e^{-0} + c$

$-1 = \frac{1}{3}A - 1 + c$

$-\frac{1}{3}A = c$

$\therefore$ equation of curve: $y = \frac{1}{3}Ae^{3x} - e^{-x} - \frac{1}{3}A$

(b) Given that $9y + 8e^{-x} + 12 = \frac{d^2y}{dx^2}$, find the value of A. [2]

$\frac{d^2y}{dx^2} = A(3)e^{3x} + (-1)e^{-x}$

$= 3Ae^{3x} - e^{-x}$

$9(\frac{1}{3}Ae^{3x} - e^{-x} - \frac{1}{3}A) + 8e^{-x} + 12 = 3Ae^{3x} - e^{-x}$

$3Ae^{3x} - 9e^{-x} - 3A + 8e^{-x} + 12 = 3Ae^{3x} - e^{-x}$

$-3A + 12 = 0$

$-3A = -12$

$A = 4$

Method2 $\quad 9y + 8e^{-x} + 12 = 3Ae^{3x} - e^{-x}$

Sub. $(0, -1)$ $\quad 9(-1) + 8e^{-0} + 12 = 3Ae^{3(0)} - e^{-0}$

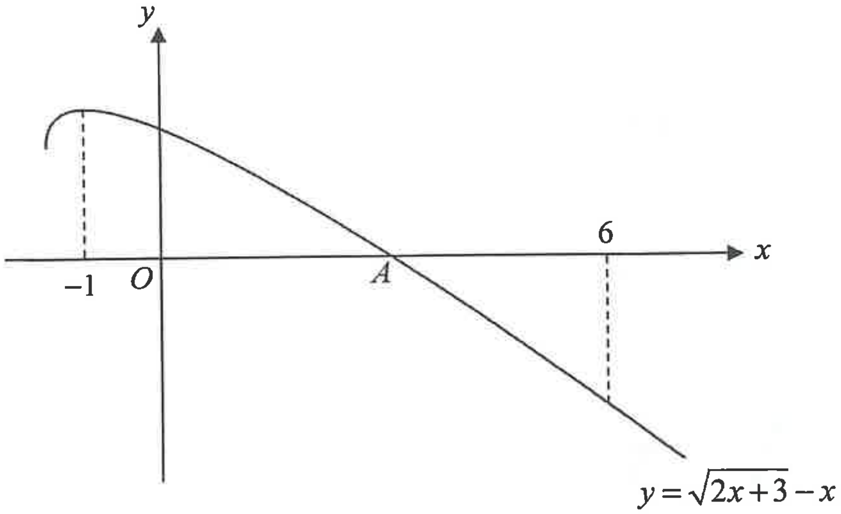
$-9 + 8 + 12 = 3A - 1$

$12 = 3A$

$A = 4$

5m

13 The diagram shows part of the curve $y=\sqrt{2x+3}-x$, and two vertical lines $x=-1$ and $x=6$.



(a) Explain why the curve intersects the x-axis at only one point, A.
Find the x-coordinate of point A. [3]

Sub. $y=0$, $0=\sqrt{2x+3}-x$

$x=\sqrt{2x+3}$

$x^2=2x+3$

$x^2-2x-3=0$

$(x-3)(x+1)=0$

$x=3$ or $x=-1$

when $x=-1$, RHS $=\sqrt{2(-1)+3}-(-1)$

$=2$

LHS $=0$

RHS $\neq$ LHS

$\therefore x=-1$ is rejected

$\therefore$ There is only 1 solution

$\therefore$ The curve intersects the x-axis at only one point A, and x-coordinate of $A=3$.

3m

(b) Point B is moving along the curve such that its y-coordinate is decreasing at a constant rate of 0.6 units per second. Find the rate of change of point B's x-coordinate at where the curve intersects the x-axis. [3]

$$\frac{dy}{dx} = \frac{2}{2\sqrt{2x+3}} - 1$$

$$= \frac{1}{\sqrt{2x+3}} - 1$$

at $x = 3$, $\frac{dy}{dx} = \frac{1}{\sqrt{2(3)+3}} - 1$

$$= -\frac{2}{3}$$

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

$$-0.6 = -\frac{2}{3} \times \frac{dx}{dt}$$

$$0.9 = \frac{dx}{dt}$$

$\therefore$ rate of change of x-coordinate = 0.9 units/s

SWAP (accuracy): missing unit

(c) Jenny claimed that $\int_{-1}^{6}\left(\sqrt{2x+3}-x\right)dx$ can be used to find the area bounded by the curve, the two vertical lines and x-axis.

Do you agree with her? Explain your answer clearly. [2]

Area from $x=3$ to $x=6$ bounded by the curve and the x-axis is below the x-axis.

$$\int_{-1}^{6}(\sqrt{2x+3}-x)\,dx = \int_{-1}^{3}(\sqrt{2x+3}-x)\,dx + \int_{3}^{6}(\sqrt{2x+3}-x)\,dx$$

$\int_{3}^{6}(\sqrt{2x+3}-x)\,dx$ is negative, but area cannot be negative

$\therefore \int_{-1}^{6}(\sqrt{2x+3}-x)\,dx$ cannot be used to find the required area

$\therefore$ No, I do not agree with her.

OR: $-\int_{3}^{6}(\sqrt{2x+3}-x)\,dx$ should be used to calculate the area from $x=3$ to $x=6$ instead of $\int_{3}^{6}(\sqrt{2x+3}-x)\,dx$.

5m