1 Differentiate each of the following with respect to x.

(a) $\frac{1}{3}x^6 - x^{-2} + 3$ [1]

(b) $\ln\left(\frac{3x+1}{5-2x}\right)$ [2]

(c) $\frac{3\sin 2x}{\sqrt{5x^2-3}}$ [3]

2 The equation of a curve is $y = (2x-3)\sqrt{4x^2+1}$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{16x^2 - 12x + 2}{\sqrt{4x^2+1}}$. [2]

(b) Find the range of values of x for which $y = (2x-3)\sqrt{4x^2+1}$ is an increasing function. [4]

3 It is given that $y = P\mathrm{e}^{4x} + Q\mathrm{e}^{-3x}$, and that $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2\dfrac{\mathrm{d}y}{\mathrm{d}x} = 12\mathrm{e}^{4x} + 3\mathrm{e}^{-3x}$.

Find the value of each of the constants P and Q. [4]

4 The equation of a curve is $y = x + \dfrac{3x-1}{x-2}$, where $x > 0$.

(a) Find $\dfrac{dy}{dx}$. [2]

(b) Find the equation of the tangent to the curve at $x = 1$. [1]

(c) Find the equation of the normal to the curve at $x = 1$. [2]

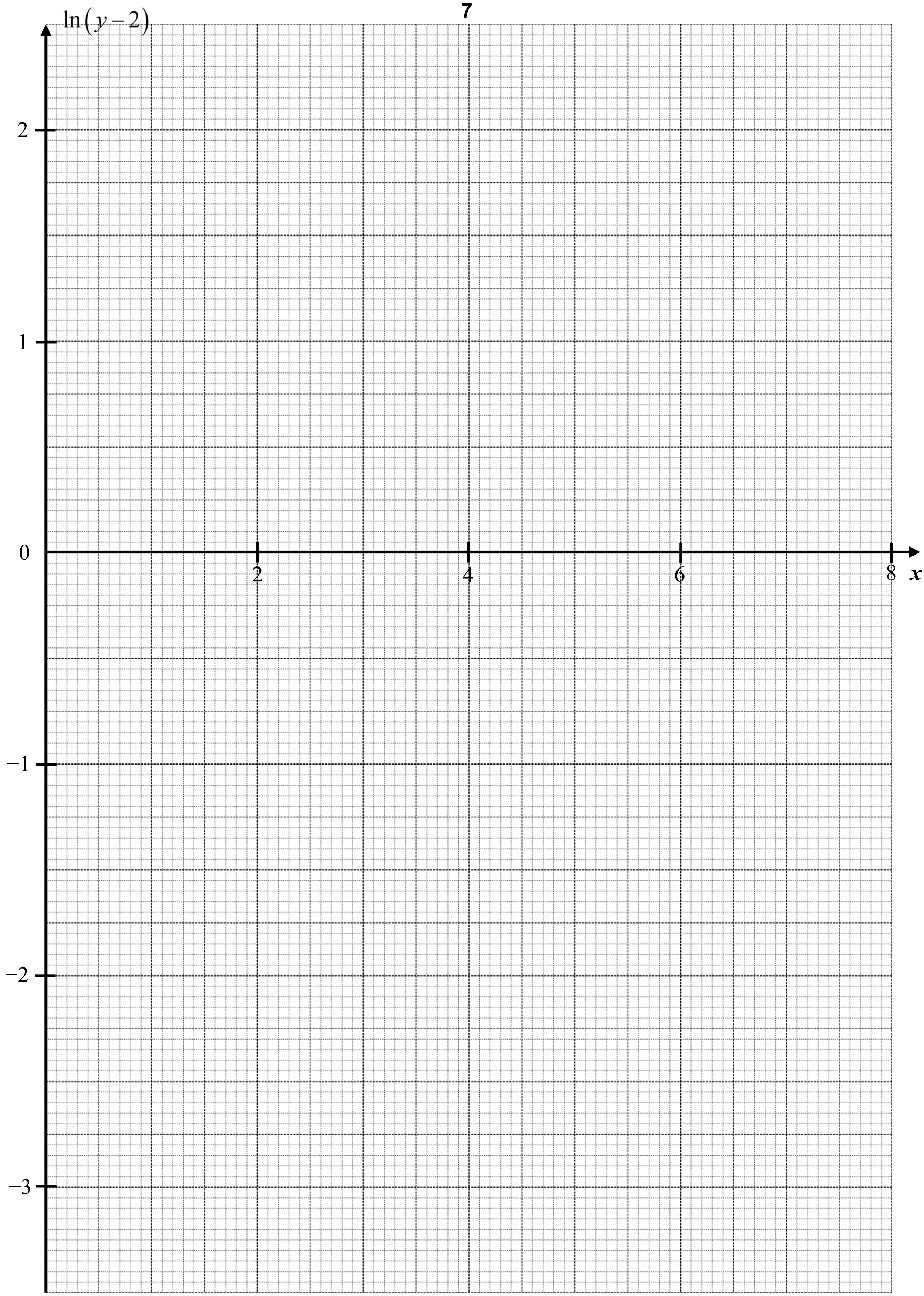
5 The table shows experimental values of two variables, x and y, which are connected by the equation $y = A\mathrm{e}^{\frac{x}{k}} + 2$, where A and k are constants.

x	2	4	6	8
y	2.135	2.368	3	4.718

(a) On the grid provided on the next page, draw a graph of $\ln(y-2)$ plotted against x. [3]

(b) Use your graph to estimate the value of A and of k. [4]

(c) On the same diagram, draw the straight line representing the equation $y - 2 = \mathrm{e}^{-\frac{x}{5}}$. Hence find the value of x for which $\frac{x}{k} + \ln A = -\frac{x}{5}$. [2]



END OF PAPER