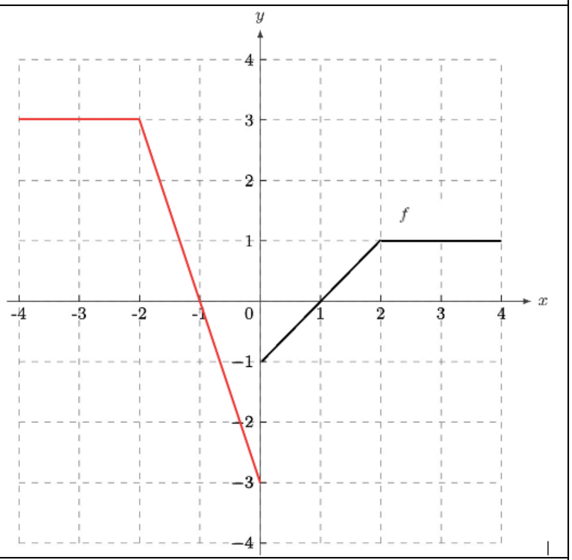
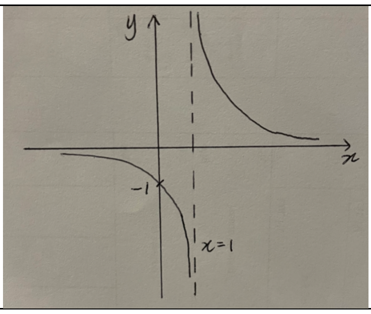
2023 Prelim MAA HL Paper 1 Student Solutions
Section A

No	Solutions
1(a)	$(3n+1)^2-(n+1)^2$ $=9n^2+6n+1-(n^2+2n+1)$ $=8n^2+4n$
1(b)	$8n^2+4n$ $=4(2n^2+n)$ So $(3n+1)^2-(n+1)^2$ a multiple of 4.
2(a)(i)	$f'(3)=0$
2(a)(ii)	$(f\circ f)(1)$ $=f(0)$ $=-1$
2(b)	
3(a)	$2x^2-mx+m-2=0$ $b^2-4ac=(-m)^2-4(2)(m-2)=0$ $m^2-8m+16=0$ $(m-4)(m-4)=0$ $m=4$
3(b)	Let $m-2>0$ $m>2$

4	$f(x) = \int \frac{6}{(1-x)^4} dx$ $= \int 6(1-x)^{-4} dx$ $= \frac{6(1-x)^{-3}}{(-3)(-1)} + c$ $= 2(1-x)^{-3} + c$ $0 = 2(1-3)^{-3} + c$ $c = \frac{1}{4}$ or $\frac{2}{8}$ $f(x) = 2(1-x)^{-3} + \frac{1}{4}$ or $\frac{2}{(1-x)^3} + \frac{1}{4}$
5(a)	$\mathrm{P}(X = 2) > 0$ $\frac{2}{3} - \cos 2\theta > 0$ $\cos 2\theta < \frac{2}{3}$
5(b)	$\frac{1}{3} + \sin\theta + \frac{2}{3} - \cos 2\theta = 1$ $\frac{1}{3} + \sin\theta + \frac{2}{3} - \left(1 - 2\sin^2\theta\right) = 1$ $\frac{1}{3} + \sin\theta + \frac{2}{3} - 1 + 2\sin^2\theta = 1$ $2\sin^2\theta + \sin\theta - 1 = 0$ $(2\sin\theta - 1)(\sin\theta + 1) = 0$ $2\sin\theta - 1 = 0 \quad or \quad \sin\theta + 1 = 0$ $\sin\theta = \frac{1}{2} \quad or \quad \sin\theta = -1 (rej)$ Hence, $\sin\theta = \frac{1}{2}$
5(c)	$\mathrm{E}(X) = 0 \times \frac{1}{3} + 1 \times \frac{1}{2} + 2 \times \left[\frac{2}{3} - \left(1 - 2\left(\frac{1}{2}\right)^2\right)\right]$ $= \frac{1}{2} + 2 \times \frac{1}{6}$ $= \frac{5}{6}$

6(i)	Recognise that $v=\frac{ds}{dt}$ $3s^2\frac{ds}{dt}+\frac{ds}{dt}-2=0$ $v=\frac{ds}{dt}=\frac{2}{1+3s^2}$
6(ii)	**Either** $a=\frac{dv}{dt}=\frac{dv}{ds}\frac{ds}{dt}$ $\frac{dv}{ds}=\frac{-12s}{(1+3s^2)^2}$ $a=\frac{-12s}{(1+3s^2)^2}\frac{ds}{dt}$ **Or** $6s\left(\frac{ds}{dt}\right)^2+3s^2\frac{d^2s}{dt^2}+\frac{d^2s}{dt^2}=0$ $\frac{d^2s}{dt^2}=\frac{-6s\left(\frac{ds}{dt}\right)^2}{1+3s^2}$ $a=\frac{-24s}{(1+3s^2)^3}$
7(a)	
7(b)	Attempt to show reflection about y-axis $p=1$
7(c)	$x\in\mathbb{R}, x\neq 0$
8(a)	Eliminate one variable and obtain 2 equations e.g. $3y+6z=10-b \quad and \quad y+(1-a)z=1$ Eliminate another variable $a=-1$
8(b)	$b=7$
8(c)	Attempt to let one of the variable be the parameter e.g. let $z=t, t\in\mathbb{R}$ $y=1-2t$ $x=3+3t$

9(a)	$\left(\frac{1}{3}\right)\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)=\frac{1}{9}$
9(b)	Recognise that Amanda can win in her first shot, second, third, e.g. $\frac{2}{3}+\left(\frac{1}{3}\right)\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)+\left(\frac{1}{3}\right)\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)+\cdots\cdots$ Recognise that it is a geometric series First term is $\frac{2}{3}$ and common ratio is $\frac{1}{6}$ Correct substitution into the sum to infinity formula i.e. $\frac{\frac{2}{3}}{1-\frac{1}{6}}$ $\frac{4}{5}$
9(c)	Use of conditional probability, e.g. $\frac{\frac{1}{9}}{\frac{4}{5}}$ $\frac{5}{36}$

Section B

No	Solutions
10(a)(i)	Amplitude = 0.5
10(a)(ii)	Minimum value $= 2 - 2\times 0.5$ $= 1$
10(a)(iii)	$2 = 0.5\sin\left[3\left(\frac{\pi}{6}\right)\right] + q$ $2 = 0.5 + q$ $q = 1.5$
10(b)(i)	Period = $\frac{2\pi}{3}$
10(b)(ii)	$x = \frac{\pi}{6} + \left(\frac{2\pi}{3} \div 2\right)$ $= \frac{\pi}{2}$
10(b)(iii)	$p = 2\pi \div \frac{2\pi}{3}$ $= 3$
10(c)(i)	$f'(x) = 1.5\cos(3x)$
10(c)(ii)	$f'(x) = -1.5$ $1.5\cos(3x) = -1.5$ $\cos(3x) = -1$ $3x = \cos^{-1}(-1)$ $3x = \pi$ $x = \frac{\pi}{3}$

11(a)	$\begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix} \times \begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$
11(b)	$\frac{1}{2}\left\|\overrightarrow{AB} \times \overrightarrow{AC}\right\| = \frac{1}{2}\sqrt{2^2 + 2^2 + (-1)^2} = \frac{3}{2}$

11(c)	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$ $2x + 2y - z = 11$
11(d)	$\boldsymbol{r} = \begin{pmatrix} -3 \\ -3 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}, \quad \lambda \in \mathbb{R}$
11(e)	(i) Substitute the line into the plane $\lambda = 3$ Substitute into the line (3,3,1) (ii) Attempt to find the distance between the point of intersection and D e.g. $\left\| \begin{pmatrix} -3 \\ -3 \\ 4 \end{pmatrix} - \begin{pmatrix} 3 \\ 3 \\ 1 \end{pmatrix} \right\| = \sqrt{(-6)^2 + (-6)^2 + 3^2}$ 9
11(f)	Valid approach e.g. $\left(\frac{-3+x}{2}, \frac{-3+y}{2}, \frac{4+z}{2} \right)$ (9,9,–2)
11(g)	$2 \times \frac{1}{3} \times base\ area \times height$ 9

12(a)	Attempt to expand Change to polar form $e^0 = 1$ $z^2 - z(\cos\theta - i\sin\theta + \cos\theta + i\sin\theta) + 1$ $z^2 - 2z\cos\theta + 1$
12(b)	$-1 = cis\pi$

	$z = cis\left(\dfrac{\pi + 2k\pi}{4}\right), k \in \mathbb{Z}$ $e^{i\frac{\pi}{4}}, \quad e^{i\frac{3\pi}{4}}$ $e^{i(-\frac{\pi}{4})}, \quad e^{i(-\frac{3\pi}{4})}$
12(c)	$z^4 + 1 = (z - e^{i\frac{\pi}{4}})(z - e^{i(-\frac{\pi}{4})})(z - e^{i\frac{3\pi}{4}})(z - e^{i(-\frac{3\pi}{4})})$ $(z^2 - 2z\cos(\dfrac{\pi}{4}) + 1)(z^2 - 2z\cos(\dfrac{3\pi}{4}) + 1)$
12(d)(i)	Attempt to solve $z^8 = -1$ $e^{i\frac{\pi}{8}}, \quad e^{i\frac{3\pi}{8}}, e^{i\frac{5\pi}{8}}, \quad e^{i\frac{7\pi}{8}}$ $e^{-i\frac{\pi}{8}}, \quad e^{-i\frac{3\pi}{8}}, e^{-i\frac{5\pi}{8}}, \quad e^{-i\frac{7\pi}{8}}$ $z^8 + 1 = (z^2 - 2z\cos(\dfrac{\pi}{8}) + 1)(z^2 - 2z\cos(\dfrac{3\pi}{8}) + 1)$ $(z^2 - 2z\cos(\dfrac{5\pi}{8}) + 1)(z^2 - 2z\cos(\dfrac{7\pi}{8}) + 1)$
12(d)(ii)	$i^8 + 1 = (-2i\cos(\dfrac{\pi}{8}))(-2i\cos(\dfrac{3\pi}{8}))(-2i\cos(\dfrac{5\pi}{8}))(-2i\cos(\dfrac{7\pi}{8}))$ $2 = 16\cos(\dfrac{\pi}{8})\cos(\dfrac{3\pi}{8})\cos(\dfrac{5\pi}{8})\cos(\dfrac{7\pi}{8})$ $\cos(\dfrac{\pi}{8})\cos(\dfrac{3\pi}{8})\cos(\dfrac{5\pi}{8})\cos(\dfrac{7\pi}{8}) = \dfrac{1}{8}$