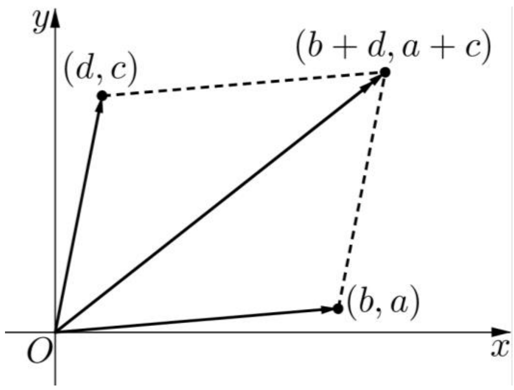
2025 NJC H3 Math Prelim Solutions

<table>
<tr><th>Qn</th><th>Solution</th></tr>
<tr><td>1(a)</td><td>


Consider the points $P(b, a)$ and $Q(d, c)$ in the x-y plane. Since $\frac{a}{b} < \frac{c}{d}$, OQ has a more positive gradient than OP. Also, the point $R(b+d, a+c)$ is such that $\overrightarrow{OR}$ is the vector sum of $\overrightarrow{OP}$ and $\overrightarrow{OQ}$. From the above diagram, we see that the gradient of OR is in between the gradients of OP and OQ. Thus,

$$\frac{a}{b} < \frac{a+c}{b+d} < \frac{c}{d}.$$
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<tr><td>1(b)</td><td>
For $0 < x < \frac{\pi}{2}$,

$0 < \cos x < 1$ and $\sin x > 0$

$\Rightarrow 1 < \frac{1}{\cos x}$ and $\sin x > 0$

$\Rightarrow \sin x < \frac{\sin x}{\cos x}$

$\Rightarrow \sin x < \tan x$

By applying the result from part (a),

$\frac{\sin x}{1} < \frac{2\sin x}{1+\cos x} < \frac{\sin x}{\cos x}$

$\frac{\sin x}{1} < \frac{4\sin\frac{x}{2}\cos\frac{x}{2}}{1+2\cos^2\frac{x}{2}-1} < \frac{\sin x}{\cos x}$

$\frac{\sin x}{1} < \frac{4\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}} < \frac{\sin x}{\cos x}$

$\sin x < 2\tan\frac{x}{2} < \tan x$ (shown)
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Qn	Solution
1(c)	By AM-GM Inequality, $ab < \frac{a^2+b^2}{2}$ $\frac{2ab}{2} < \frac{a^2+b^2}{2}$ $\frac{2ab}{2} < \frac{2ab+a^2+b^2}{2+2} < \frac{a^2+b^2}{2}$ $\frac{2ab}{2} < \frac{a^2+2ab+b^2}{4} < \frac{a^2+b^2}{2}$ $4ab < (a+b)^2 < 2(a^2+b^2)$ $4 < \frac{(a+b)^2}{ab} < \frac{2(a^2+b^2)}{ab}$ $4 < \left(\frac{a+b}{\sqrt{ab}}\right)^2 < 2\left(\frac{a^2+b^2}{ab}\right)$ $4 < \left(\frac{a}{\sqrt{ab}}+\frac{b}{\sqrt{ab}}\right)^2 < 2\left(\frac{a^2}{ab}+\frac{b^2}{ab}\right)$ $4 < \left(\sqrt{\frac{a}{b}}+\sqrt{\frac{b}{a}}\right)^2 < 2\left(\frac{a}{b}+\frac{b}{a}\right)$ (shown)

Qn	Solution
2(a)	Substituting $m=0$ into the equation, $\mathrm{f}(0)+2\mathrm{f}(n)=\mathrm{f}^2(0+n)$ $\mathrm{f}(0)+2\mathrm{f}(n)=\mathrm{f}^2(n)\cdots\cdots(1)$ Substituting $m=1$ into the equation, $\mathrm{f}(2)+2\mathrm{f}(n)=\mathrm{f}^2(1+n)$ $\mathrm{f}(2)+2\mathrm{f}(n)=\mathrm{f}^2(n+1)\cdots\cdots(2)$ Since n is arbitrary, from equation (1), $\mathrm{f}^2(n+1)=\mathrm{f}(0)+2\mathrm{f}(n+1)\cdots\cdots(3)$ Substituting equation (3) into equation (2), $\mathrm{f}(2)+2\mathrm{f}(n)=\mathrm{f}(0)+2\mathrm{f}(n+1)$ $\mathrm{f}(n+1)-\mathrm{f}(n)=\dfrac{\mathrm{f}(2)-\mathrm{f}(0)}{2}$, which is a constant. This implies that f is an **arithmetic progression**. Thus $\mathrm{f}(x)=ax+b$ for some constants a and b. Hence, $\mathrm{f}(2m)+2\mathrm{f}(n)=\mathrm{f}^2(m+n)$ $a(2m)+b+2(an+b)=a\big(a(m+n)+b\big)+b$ $2ma+b+2an+2b=a^2m+a^2n+ab+b$ $2ma-ma^2+2na-na^2+2b-ab=0$ $(ma+na+b)(2-a)=0$ $(m+n)a+b=0$ or $2-a=0$ For $(m+n)a+b=0$ for all integers m and n, $a=b=0$. Thus, f is the zero function in this case. For $2-a=0$, $a=2$ and $b\in\mathbb{R}$. But since f is an integer-valued function with domain $\mathbb{Z}$, $b\in\mathbb{Z}$. Thus either $\mathrm{f}(x)=0$ or $\mathrm{f}(x)=2x+b$, where $b\in\mathbb{Z}$.

Qn	Solution
2(b)	$x+y=4 \Rightarrow (x+y)^2=16$ $\Rightarrow x^2+2xy+y^2=16$ $\Rightarrow x^2+y^2=16-2xy \cdots\cdots (1)$ $x+y=4 \Rightarrow (x+y)^3=64$ $\Rightarrow x^3+3x^2y+3xy^2+y^3=64$ $\Rightarrow x^3+y^3+3x^2y+3xy^2=64$ $\Rightarrow x^3+y^3+3xy(x+y)=64$ $\Rightarrow x^3+y^3+3xy(4)=64$ $\Rightarrow x^3+y^3=64-12xy \cdots\cdots (2)$ From equations (1) and (2), $(x^2+y^2)(x^3+y^3)=(16-2xy)(64-12xy)$ $x^5+x^3y^2+x^2y^3+y^5=2(8-xy)\times 4(16-3xy)$ $x^5+y^5+x^2y^2(x+y)=8(8-xy)(16-3xy)$ $464+x^2y^2(4)=8(128-16xy-24xy+3x^2y^2)$ $4(116+x^2y^2)=8(128-40xy+3x^2y^2)$ $116+x^2y^2=256-80xy+6x^2y^2$ $5x^2y^2-80xy+140=0$ $x^2y^2-16xy+28=0$ $(xy-14)(xy-2)=0$ $xy=14$ or $xy=2$ If $xy=14$, then $x(4-x)=14$ $4x-x^2=14$ $x^2-4x+14=0$ $D=(-4)^2-4(14)=-40<0$ This equation has no real solution for x (and thus y). So it must be that $xy=2$. Hence $x(4-x)=2$ $4x-x^2=2$ $x^2-4x+2=0$ $x=\dfrac{4\pm\sqrt{(-4)^2-4(2)}}{2}=2+\sqrt{2}$ or $2-\sqrt{2}$ If $x=2-\sqrt{2}$, $y=4-(2-\sqrt{2})=2+\sqrt{2}$ If $x=2+\sqrt{2}$, $y=4-(2+\sqrt{2})=2-\sqrt{2}$ Since $x>y$, $x=2+\sqrt{2}$ and $y=2-\sqrt{2}$.

Qn	Solution
3(a)	Let S_i be the total training hours up till the *i*th day, where $1 \le i \le 140$. Then $1 \le S_1 < S_2 < ... < S_{140} \le 20(13) = 260$. Let $T_i = S_i + 19$, where $1 \le i \le 140$. Then $20 \le T_1 < T_2 < ... < T_{140} \le 279$. Each S_i and T_i is a value between 1 and 279 (inclusive), and the list $\{S_1, S_2, ..., S_{140}, T_1, T_2, ..., T_{140}\}$ has 280 elements. Consider slotting the 280 elements into boxes labelled 1 to 279. By Pigeonhole Principle, there exists S_i and $T_j\ (i > j)$ such that $S_i = T_j$, since all S_i's are distinct and all T_j's are distinct. Thus, $S_i = S_j + 19 \Rightarrow S_i - S_j = 19$. Hence, the player will have trained for exactly 19 hours successively from day $j+1$ to day *i*. **<u>Alternative:</u>** let S_i be the total training hours up till the i^{th} day , where $1 \le i \le 140$ and $1 \le S_i \le 20 \quad 13 \quad = 260$. $S_i \equiv r \quad \text{mod } 19$, where $0 \le r \le 18$, $r \in \mathbf{Z}$. Consider slotting the 140 elements S_i into 19 boxes labelled 0 to 18, depending on their corresponding value of *r* when divided by 19. Notice that $140 = 19 \quad 7 \quad + 7$. By Pigeonhole Principle, there exists $S^*_1, S^*_2, ..., S^*_8$ such that each of them is congruent to *r** modulo 19, where $0 \le r^* \le 18$, $r^* \in \mathbf{Z}$. We can therefore re-write each term in the following form: $S^*_1 = 19q_1 + r^*$ $S^*_2 = 19q_2 + r^*$ ... $S^*_8 = 19q_8 + r^*$ where $q_1, ..., q_8 \in \mathbf{Z}$ Notice that since $1 \le S_i \le 260$ and $\left\lfloor \frac{260}{19} \right\rfloor = 13$, we have $0 \le q_1, q_2, ..., q_8 \le 13$.

	Thus, there must be at least two q_i 's which are consecutive values. Let this two q_i 's be q_j and q_k where $q_k > q_j$. $19q_k + r^* - 19q_j + r^* = 19$ $\Rightarrow S^*_k - S^*_j = 19$ Hence the player would have trained for exactly 19 hours from day $j+1$ to day k.
3(b)(i)	Let $y_i = x_i$ for $i = 1,2,3,4$ and $y_5 \geq 0$, where $y_5 \in \mathbb{Z}$.
3(b)(ii)	Let $y_1 = x_1 - 3$, $y_2 = x_2$, $y_3 = x_3 + 8$, $y_4 = x_4 - 6$ and $y_5 \geq 0$. Then $x_1 + x_2 + x_3 + x_4 \leq 50$ becomes $(y_1 + 3) + y_2 + (y_3 - 8) + (y_4 + 6) + y_5 = 50$ i.e. $y_1 + y_2 + y_3 + y_4 + y_5 = 49$, where $0 \leq y_1 \leq 9$, $0 \leq y_2 \leq 7$, $0 \leq y_3 \leq 20$, $0 \leq y_4 \leq 14$ and $y_5 \geq 0$. Let A_1, A_2, A_3, A_4 be the set of integer solutions such that $y_1 \geq 10$, $y_2 \geq 8$, $y_3 \geq 21$, $y_4 \geq 15$ respectively. $\lvert A_1 \rvert = \binom{49-10+4}{4} = 123410$ $\lvert A_2 \rvert = \binom{49-8+4}{4} = 148995$ $\lvert A_3 \rvert = \binom{49-21+4}{4} = 35960$ $\lvert A_4 \rvert = \binom{49-15+4}{4} = 73815$ $\lvert A_1 \cap A_2 \rvert = \binom{49-18+4}{4} = 52360$ $\lvert A_1 \cap A_3 \rvert = \binom{49-31+4}{4} = 7315$ $\lvert A_1 \cap A_4 \rvert = \binom{49-25+4}{4} = 20475$ $\lvert A_2 \cap A_3 \rvert = \binom{49-29+4}{4} = 10626$ $\lvert A_2 \cap A_4 \rvert = \binom{49-23+4}{4} = 27405$ $\lvert A_3 \cap A_4 \rvert = \binom{49-36+4}{4} = 2380$

	$\left\|A_1 \cap A_2 \cap A_3\right\| = \binom{49-39+4}{4} = 1001$ $\left\|A_1 \cap A_2 \cap A_4\right\| = \binom{49-33+4}{4} = 4845$ $\left\|A_1 \cap A_3 \cap A_4\right\| = \binom{49-46+4}{4} = 35$ $\left\|A_2 \cap A_3 \cap A_4\right\| = \binom{49-44+4}{4} = 126$ $\left\|A_1 \cap A_2 \cap A_3 \cap A_4\right\| = 0$ By PIE, required number of integer solutions $= \binom{49+4}{4}$ $-\left(\sum_{i=1}^{4} A_i - \sum_{i \neq j}\left(A_i \cap A_j\right)\right.$ $\left.+\sum_{i \neq j \neq k}\left(A_i \cap A_j \cap A_k\right) - \left\|A_1 \cap A_2 \cap A_3 \cap A_4\right\|\right)$ $= 25199$

Qn	Solution
4(a)	The line passing through point $\left(x_0, y_0\right)$ with gradient m, has an equation $y-y_0=m\left(x-x_0\right)$, where $m\neq 0$. For its axial intercepts: When $x=0$, $y=y_0-mx_0$ … y-intercept When $y=0$, $x=x_0-\dfrac{y_0}{m}$ … x-intercept For the y-intercept $\left(0, y_0-mx_0\right)$ to be the midpoint of $\left(x_0, y_0\right)$ and the x-intercept $\left(x_0-\dfrac{y_0}{m}, 0\right)$: $\dfrac{x_0+\left(x_0-\dfrac{y_0}{m}\right)}{2}=0$ and $\dfrac{y_0+0}{2}=y_0-mx_0$ $x_0=\dfrac{y_0}{2m}$ and $y_0=2mx_0$ $\dfrac{y_0}{2x_0}=m$ For $Q\left(x, y\right)$ on C, $-\dfrac{\mathrm{d}x}{\mathrm{d}y}=\dfrac{y}{2x}$ $-2x\dfrac{\mathrm{d}x}{\mathrm{d}y}=y$ Integrating both sides with respect to y: $\int -2x\,\mathrm{d}x=\int y\,\mathrm{d}y$ $-x^2=\dfrac{y^2}{2}+c$ $2x^2+y^2=D, D=-2c>0$
4(b)	$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{3\sin x}v\right)=\left(\dfrac{\mathrm{d}}{\mathrm{d}x}\mathrm{e}^{3\sin x}\right)\left(\dfrac{\mathrm{d}v}{\mathrm{d}x}\right)$ $\mathrm{e}^{3\sin x}\dfrac{\mathrm{d}v}{\mathrm{d}x}+\left(3\mathrm{e}^{3\sin x}\cos x\right)v=3\mathrm{e}^{3\sin x}\cos x\dfrac{\mathrm{d}v}{\mathrm{d}x}$ $\left(3\mathrm{e}^{3\sin x}\cos x-\mathrm{e}^{3\sin x}\right)\dfrac{\mathrm{d}v}{\mathrm{d}x}=\left(3\mathrm{e}^{3\sin x}\cos x\right)v$ $\mathrm{e}^{3\sin x}\left(3\cos x-1\right)\dfrac{\mathrm{d}v}{\mathrm{d}x}=\left(3\mathrm{e}^{\sin x}\cos x\right)v$ $\dfrac{1}{v}\dfrac{\mathrm{d}v}{\mathrm{d}x}=\dfrac{3\cos x}{3\cos x-1}$

	Integrating both sides with respect to x: $\frac{1}{v}\frac{\mathrm{d}v}{\mathrm{d}x}=1+\frac{1}{3\cos x-1}$ $\int \frac{1}{v}\,\mathrm{d}v=\int 1+\frac{1}{3\cos x-1}\,\mathrm{d}x$ Let $\cos x=\frac{1-t^2}{1+t^2}$. Then $-\sin x\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{\left(1+t^2\right)\left(-2t\right)-\left(1-t^2\right)2t}{\left(1+t^2\right)^2}$ $-\frac{2t}{1+t^2}\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{-4t}{\left(1+t^2\right)^2}$ $\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{2}{1+t^2}$ $\int \frac{1}{v}\,\mathrm{d}v=x+\int \frac{1}{3\cos x-1}\,\mathrm{d}x$ $=x+\int \frac{1}{3\left(\frac{1-t^2}{1+t^2}\right)-1}\frac{2}{1+t^2}\,\mathrm{d}t$ $=x+\int \frac{1}{\frac{3-3t^2-1-t^2}{1+t^2}}\frac{2}{1+t^2}\,\mathrm{d}t$ $=x+\int \frac{1}{1-2t^2}\,\mathrm{d}t$ $=x+\frac{1}{2\sqrt{2}}\ln\left\lvert\frac{1+\sqrt{2}t}{1-\sqrt{2}t}\right\rvert+c$ $\ln\lvert v\rvert=x+\frac{1}{2\sqrt{2}}\ln\left\lvert\frac{1+\sqrt{2}\tan^{-1}\frac{x}{2}}{1-\sqrt{2}\tan^{-1}\frac{x}{2}}\right\rvert+c$ $=x+\frac{1}{2\sqrt{2}}\ln\left\lvert\frac{1+\sqrt{2}\tan^{-1}\frac{x}{2}}{1-\sqrt{2}\tan^{-1}\frac{x}{2}}\right\rvert+c$ $v=A\mathrm{e}^{x}\left\lvert\frac{1+\sqrt{2}\tan^{-1}\frac{x}{2}}{1-\sqrt{2}\tan^{-1}\frac{x}{2}}\right\rvert^{\frac{1}{2\sqrt{2}}}$, $A=\pm\mathrm{e}^{c}$

<table>
<tr><th>Qn</th><th>Solution</th></tr>
<tr><td>5(i)</td><td>$F_0 = 2^1 + 1 = 3$
$F_1 = 2^2 + 1 = 5$
$F_2 = 2^4 + 1 = 17$
$F_3 = 2^8 + 1 = 257$</td></tr>
<tr><td>5(ii)</td><td>
<table>
<tr><td>k</td><td>$F_0F_1F_2...F_{k-1}$</td><td>F_k</td></tr>
<tr><td>1</td><td>$F_0 = 3$</td><td>5</td></tr>
<tr><td>2</td><td>$F_0F_1 = 15$</td><td>17</td></tr>
<tr><td>3</td><td>$F_0F_1F_2 = 255$</td><td>257</td></tr>
</table>

Conjecture:
$F_0F_1F_2...F_{k-1} = F_k - 2, \quad k \in \mathbb{Z}^+$</td></tr>
<tr><td>5(iii)</td><td>Let $\mathrm{P}(k)$ be the statement: $F_0F_1F_2...F_{k-1} = F_k - 2$ for $k \in \mathbb{Z}^+$.

When $k = 1$,
$\text{LHS} = F_0 = 3$
$\text{RHS} = F_1 - 2 = 5 - 2 = 3$.

Since LHS = RHS, $\mathrm{P}(1)$ is true.

Assume that $\mathrm{P}(m)$ is true for some $m \in \mathbb{Z}^+$ i.e. $F_0F_1F_2...F_{m-1} = F_m - 2$.

(Need to prove that $\mathrm{P}(m+1)$ is true i.e. $F_0F_1F_2...F_m = F_{m+1} - 2$.)

$F_0F_1F_2...F_{m-1}F_m$
$= (F_m - 2)F_m$
$= \left(2^{2^m} - 1\right)\left(2^{2^m} + 1\right)$
$= \left(2^{2^m}\right)^2 - 1$
$= \left(2^{2^{m+1}} + 1\right) - 2$
$= F_{m+1} - 2$

$\therefore \mathrm{P}(m)$ is true $\Rightarrow \mathrm{P}(m+1)$ is true.
Since $\mathrm{P}(1)$ is true, and $\mathrm{P}(m)$ is true $\Rightarrow \mathrm{P}(m+1)$ is true, by mathematical induction, $\mathrm{P}(k)$ is true for all $k \in \mathbb{Z}^+$.</td></tr>
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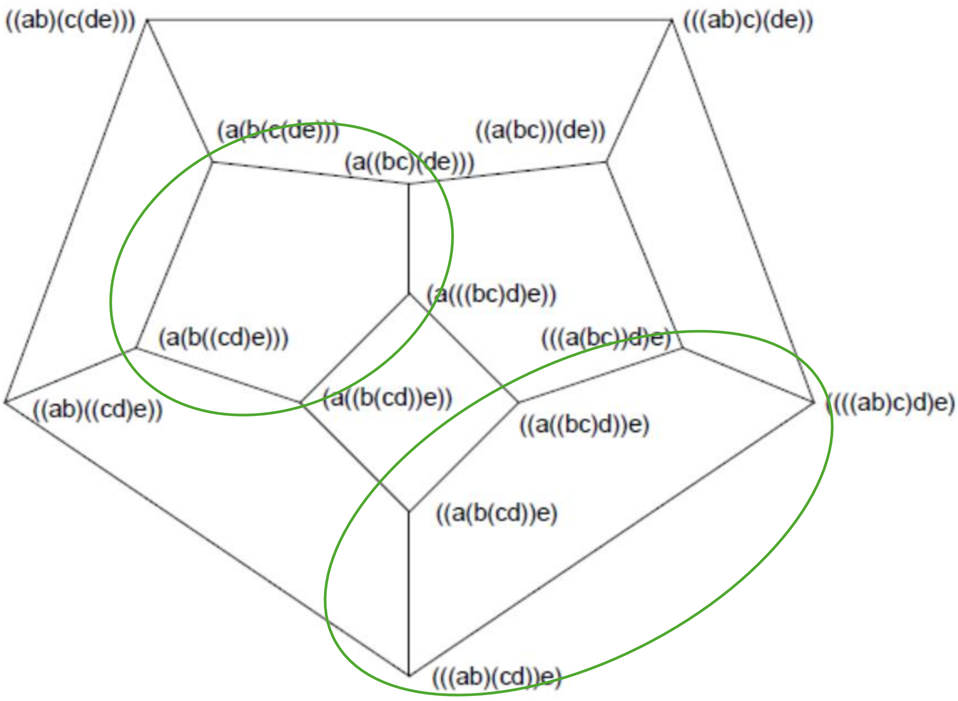
Qn	Solution
5(iv)	Suppose $p > 1$ is a common divisor of two distinct Fermat numbers F_i and F_j, where $i < j$. Then $p \mid F_i$ and $p \mid F_j$. Since $F_0F_1...F_i...F_{j-1} = F_j - 2$, $p \mid F_0F_1...F_i...F_{j-1}$ and hence, $p \mid F_j - 2$. Given $p \mid F_j$, we have $p \mid F_j - \left(F_j - 2\right) \Rightarrow p \mid 2$. Thus, $p = 2$ (since $p > 1$) which is a contradiction because all Fermat numbers $2^{2^n} + 1$ are odd and hence, not divisible by $p = 2$. $\therefore$ the only common divisor of any two distinct Fermat numbers is 1 and hence, they are coprime.
5(v)(a)	$5\times 2^{7}\ +1 \equiv 0 \mod 641$ $5\times 2^{7} \equiv -1 \mod 641$ $5\times 2^{7}\ {}^{4} \equiv\ -1\ {}^{4} \mod 641$ $5^{4}\times 2^{28} \equiv 1 \mod 641$ Remainder when $5^4 \times 2^{28}$ is divided by 641 is 1.
5(v)(b)	From part (v)(a), $5^4 \times 2^{28} = 641q_1 + 1,\ q_1 \in \mathbb{Z}$ -----(1) $2^4 + 5^4 \equiv 0 \left(\mod 641\right)$ $\Rightarrow 5^4 \equiv -2^4 \left(\mod 641\right)$ $\Rightarrow 5^4 = 641q_2 - 2^4,\ q_2 \in \mathbb{Z}$ -----(2) Substitute (2) into (1), $\left(641q_2 - 2^4\right)\times 2^{28} = 641q_1 + 1$ $\Rightarrow 2^{32} + 1 = 641\left(q_2 - q_1\right),\ q_2 - q_1 \in \mathbb{Z}$ $\Rightarrow F_5 \equiv 0 \left(\text{mod } 641\right)\ \left(\text{Shown}\right)$

Qn	Solution
6(i)	For a five-fold product: P_5 $= P_1P_4 + P_2P_3 + P_3P_2 + P_4P_1$ $= 1(5) + 1(2) + 2(1) + 5(1) = 14$ $C_4 = C_4 = \frac{1}{4+1}\binom{2(4)}{4} = \frac{1}{5}\binom{8}{4} = 14$
6(ii)	Let $k = 2i+1$ where $i \in \mathbb{Z}^+$. P_k $= P_1P_{k-1} + \ldots + P_{\lfloor \frac{k}{2} \rfloor} P_{\lfloor \frac{k}{2} \rfloor + 1} + P_{\lfloor \frac{k}{2} \rfloor + 1} P_{\lfloor \frac{k}{2} \rfloor} + \ldots + P_{k-1}P_1$ $= 2\left(P_1P_{k-1} + \ldots + P_{\lfloor \frac{k}{2} \rfloor} P_{\lfloor \frac{k}{2} \rfloor + 1} \right)$ Since $P_k = P_{2i+1}$ is even, $P_{2i+1} \equiv 0 \pmod 2$ for $i \in \mathbb{Z}^+$.
6(iii) (a)	$g_1 = (a(b(c(de))))$ $g_2 = (a((bc)(de)))$
6(iii) (b)	 B1: I will look for groupings in the associahedron where *a* is the last number to be associated or where *e* is the last number to be associated. Note: The circled sub-associahedron counts p_4. Not needed as an answer. But students can circle to illustrate their point.

Qn	Solution
6(iv)	We're looking for a way to pair each non-good sequence of n 1's and n (-1)'s with a unique sequence of $(n+1)$ 1's and $(n-1)$ (-1)'s. $A=\left\{\begin{matrix} s\text{: sequence of } n \text{ 1's and } n\ (-1)\text{'s where there exists an} \\ \text{index } m \text{ such that } x_1+x_2+...+x_m<0, \text{ for } 1\le m\le 2n. \end{matrix}\right\}$. $B=$ {s*: sequence of $(n+1)$ 1's and $(n-1)$ (-1)'s} 1. **Identify the First "Negative" from *s***: For a non-good sequence of n 1's and n (-1)'s i.e. $\{x_1,x_2,...,x_{2n}\}$, find the smallest index i such that $x_1+x_2+...+x_i<0$. Note that the sequence $\{x_1,x_2,...,x_{i-1}\}$ must have an equal number of 1's and (-1)'s . So $i=2j+1$ must be odd. 2. **Transform the sequence *s***: Let $s^*=\{y_1,y_2,...,y_{2n}\}$ where $y_k=-x_k$ for $k\le 2j+1$ and $y_k=x_k$ otherwise. s* is a sequence of $(n+1)$ 1's and $(n-1)$ (-1)'s.
	1 mark: Definition of A. 1 mark: Definition of B. 1 mark: Identify the First Negative. 1 mark: Description of the transformation.
6(v)	C_n counts the number of ways p_{n+1} to associate the product $x_1x_2...x_{n+1}$ into products of two expressions using parenthesis, which is exactly the number of good sequences of n 1's and n (-1)'s. By the bijection in part (iv), the set of non-good sequences of n 1's and n (-1)'s and the set of all sequences of $(n+1)$ 1's and $(n-1)$ (-1)'s has the same size. Since every sequence of n 1's and n (-1)'s is either good or non-good, and no such sequence is both, $\lvert\text{set of sequences of } n \text{ 1's and } n\ (-1)\text{'s}\rvert$ $=\lvert\text{set of good sequences of } n \text{ 1's and } n\ (-1)\text{'s}\rvert$ $+\lvert\text{set of non-good sequences of } n \text{ 1's and } n\ (-1)\text{'s}\rvert$ $\binom{2n}{n}=C_n+\binom{2n}{n+1}$

	C_n $=\binom{2n}{n}-\binom{2n}{n+1}$ $=\frac{(2n)!}{n!n!}-\frac{(2n)!}{(n+1)!(n-1)!}$ $=\frac{(2n)!}{n!n!}-\frac{(2n)!}{(n+1)n!(n-1)!}$ $=\frac{(2n)!}{n!n!}\left(1-\frac{n}{(n+1)}\right)$ $=\frac{(2n)!}{n!n!}\left(1-\frac{n}{(n+1)}\right)$ $=\left(\frac{1}{n+1}\right)\binom{2n}{n}$
6(vi)	$(\Leftarrow)$ For $n=1,2,3$, $C_1=1, n+2=3$ $C_2=2, n+2=4$ $C_3=5,\ n+2=5$ Clearly $C_n \mid n+2$ for $n=1,2,3$. $(\Rightarrow)$ Using contrapositive: Let $n>3$ $C_n=\frac{(n+2)(n+3)...(2n)}{n!}$ $\frac{n!}{(n+3)(n+4)...(2n)}=\frac{n+2}{C_n}$ $\frac{1(2)...(n-1)(n)}{(n+3)(n+4)...(2n-1)(2n)}=\frac{n+2}{C_n}$ $\frac{(3)(4)...(n-1)}{(n+3)(n+4)...(n+(n-1))}=\frac{n+2}{C_n}$ $\frac{3}{n+3}\frac{4}{n+4}\frac{n-1}{n+(n-1)}=\frac{n+2}{C_n}$

	LHS is a product of rational numbers smaller than 1. So $\frac{n+2}{C_n}$ is not an integer, that is $C_n \nmid n+2$. Thus if $n>3$, $C_n \nmid n+2$.