

ANGLO-CHINESE JUNIOR COLLEGE
JC2 PRELIMINARY EXAMINATION

Higher 2

MATHEMATICS

9758/02

Paper 2
QUESTION PAPER

31 August 2026
3 hours

Additional Materials: Printed Answer Booklet
 List of Formulae and Results (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** questions.

Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.

You must show all necessary working clearly.

The number of marks is given in brackets [] at the end of each question or part question.

This document consists of **9** printed pages and **1** blank page.

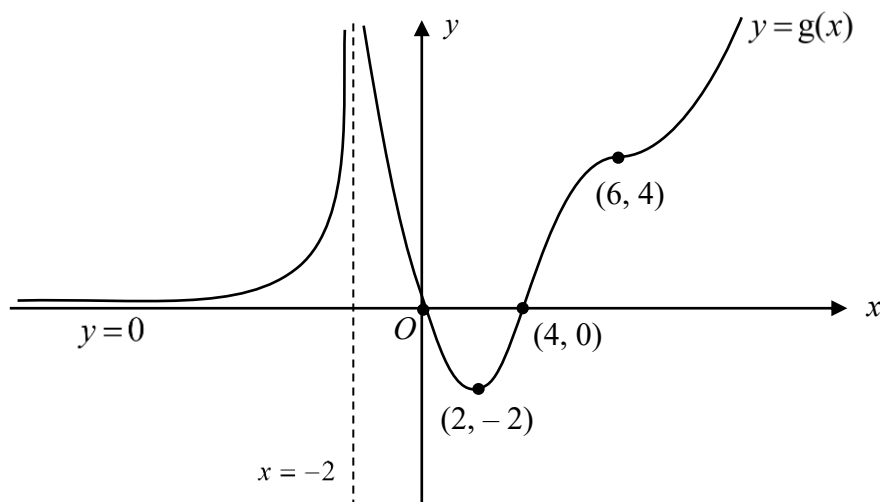


Anglo-Chinese Junior College

[Turn over

Section A: Pure Mathematics [40 marks]

- 1 The diagram below shows the curve with equation $y = g(x)$. The curve cuts the x -axis at the origin and $x = 4$. The graph has stationary points at $(2, -2)$ and point $(6, 4)$. It has asymptotes at $x = -2$ and $y = 0$.



Sketch on separate diagrams in the Printed Answer Booklet, the graphs of

- (a) $y = \frac{1}{g(x)}$, [3]
- (b) $y = g'(x)$. [3]

Your sketches should show clearly any stationary points, axial intercepts and the equations of any asymptotes where possible.

- 2 A curve C has equation $y = \frac{(x-k)^2}{x+k}$ where k is a positive constant.

- (a) Use an algebraic method to find the range of values of y in terms of k . [3]
- (b) Describe a sequence of transformations that map C onto the curve with equation $y = \frac{kx^2}{x+2k}$. [2]

3 (a) It is given that $\sum_{r=1}^n r^2 = \frac{n(n+1)(2n+1)}{6}$. Show that $\sum_{r=1}^n r(3r+1) = n(n+1)^2$. [2]

(b) Hence show that $\sum_{k=4}^{n+3} \frac{(k-3)(3k-8)}{n^3} = \left(1 + \frac{1}{n}\right)^2$. [2]

(c) An infinite geometric series has a common ratio of $\frac{1}{3}$. The first term of this series, a , is given by $a = \lim_{n \rightarrow \infty} \left[\sum_{k=4}^{n+3} \frac{(k-3)(3k-8)}{n^3} \right]$. Find the exact value of the sum to infinity of this geometric series. [3]

(d) The sequence $u_1, u_2, u_3, u_4, u_5, \dots$ is defined by $u_1 = 2$, $u_{r+1} = \frac{u_r + 1}{1 - u_r}$, for $r \geq 1$.

By considering u_1, u_2, u_3, u_4, u_5 , find $\sum_{r=1}^{4n} u_r$ in terms of n , where n is an integer. [2]

4 The plane π_1 contains the points $A(1, 3, -1)$ and $B(3, 2, 0)$, and is perpendicular to the plane π_2 with equation $3x - z = 1$.

(a) Show that the cartesian equation of plane π_1 is $x + 5y + 3z = 13$. [2]

(b) Find the equation of the line of intersection of π_1 and π_2 . [1]

(c) Find the length of projection of $\overline{AB}$ onto π_2 . [2]

The line l has equation $\mathbf{r} = p\mathbf{i} - \mathbf{j} + \lambda(3\mathbf{i} + q\mathbf{j} + 4\mathbf{k})$, where λ is a parameter, p and q are unknown constants.

(d) What can you say about the values of p and q for which the line l does not intersect plane π_1 . [3]

(e) Given that $p = 18$ and $q = 0$, find the acute angle between line l and plane π_1 . [2]

5 In this question $\sin^{-1}(1-x)$ refers to the principal value.

(a) (i) Use the substitution $z = \sin^{-1}(1-x)$ to show that $\int_0^1 \sin^{-1}(1-x) \, dx = \int_a^b z \cos z \, dz$, where a and b are constants to be determined. [2]

(ii) Hence show that $\int_0^1 \sin^{-1}(1-x) \, dx = \frac{\pi}{2} - 1$. [2]

(b) A curve C is given by the equation
 $y = \sin^{-1}(1-x)$ for $0 \leq x \leq 2$.

(i) Find exactly $\int_0^2 |\sin^{-1}(1-x)| \, dx$. [2]

(ii) The region R is bounded by the curve C , the line $x = 2$ and the part of the x -axis between 1 and 2. Find the exact volume generated when R is rotated 360° about the y -axis. [4]

Section B: Probability and Statistics [60 marks]

- 6 It is found that the delivery time, in minutes, of orders from a restaurant is normally distributed. 20% of the orders are delivered in less than 13 minutes and 26% of the orders take more than 25 minutes.

Find the mean and standard deviation of the delivery time. [4]

- 7 The events A and B are such that $P(A) = 0.6$, $P(B) = 0.5$ and $P(A \cap B) = k$.

(a) State the exact range of possible values for k . [2]

A third event C is introduced such that A and C are mutually exclusive, with $P(C) = 0.2$ and $P(B|C) = 0.4$. It is also given that $k = 0.25$.

(b) Explain whether A and B are independent events. [1]

(c) Find $P(A' \cap B' \cap C')$. [3]

(d) Given that at least one of A or C occurs, find the probability that B occurs. [2]

- 8 A pop group consists of 3 rappers and 4 vocalists.

(a) During a variety show, the 7 members are to be allocated into 3 distinct rooms, labelled X , Y and Z , and the following conditions are met.

- Room X must contain exactly 2 members.
- Room Y must contain at least 2 members.
- Each room must contain exactly 1 rapper.

Find the number of ways the 7 members can be allocated among the 3 rooms. [3]

(b) During a formal dinner, the 7 members and their manager are seated in a round table with 8 identical seats, and the following conditions are met.

- The manager and the 3 rappers must be seated adjacent to each other.
- The manager must be seated between 2 rappers.
- Two particular vocalists must not be seated next to each other.

Find the number of ways these 8 people can be seated in a round table. [3]

- 9 The mean mass of packets of Oolong tea produced by a manufacturer is claimed to be 600 grams. An auditor wishes to test whether this claim is true. She took a random sample of packets and carry out a hypothesis test.

- (a) State null and alternative hypotheses for the auditor's test, defining any parameters you used. [2]

She weighs 50 randomly chosen packets and records the mass, x grams, of each packet. The results are summarised as follows.

$$\sum x = 30685 \quad \sum x^2 = 18925681$$

- (b) Calculate unbiased estimates of the population mean and variance of the mass of the packets, leaving your answers to 1 decimal place. [2]
- (c) Test, at the 5% significance level, whether the population mean mass is 600 grams. [3]

A quality control supervisor took another random sample of 47 packets and the mean weight is k grams. It is now given that the population variance is 49 grams². The above test was conducted again and the quality control supervisor concludes that there is no reason to reject the null hypothesis.

- (d) Find the set of possible values of k . [3]

- 10** Air defence batteries are deployed to intercept incoming drones. Only one missile is fired at each drone. Under severe electronic jamming, the average probability that a drone is successfully intercepted by a missile is p , where $0 < p < 1$.

- (a) State, in the context of the question, two assumptions needed for the number of successfully intercepted drones by a defence battery to be well-modelled by a Binomial distribution. [2]

Two defence batteries, Battery Alpha and Battery Delta, operate independently and are deployed to intercept the drones. Battery Alpha fires at a wave of 10 drones, while Battery Delta fires at a separate wave of 15 drones.

It is assumed that the number of successfully intercepted drones by each battery can be modelled by binomial distributions. The total number of successfully intercepted drones by both batteries is denoted by W and has the distribution $B(25, p)$.

- (b) The probability that W is at most one is twice the probability that Battery Alpha did not intercept any drone and Battery Delta intercepted exactly one drone. Show algebraically that $p = \frac{1}{6}$. [3]
- (c) A random sample of 60 independent defence batteries is monitored, where each battery faces a wave of 40 drones. The number of successfully intercepted drones by a battery is denoted by X and follows $B(40, 0.15)$. Find the probability that the mean number of successfully intercepted drones is between 5.5 and 6.5. [3]

11 In this question you should state the parameters of any distributions used.

The dentist at a clinic attends to patients with appointments only and do not accept walk-in patients. From past records, 84% of the patients turn up for their appointment. On a particular day, 25 patients each made an appointment to see the dentist. It should be assumed that the number of patients turning up for their appointment follows a binomial distribution.

- (a) Find the probability that at least 21 of these patients turn up for their appointments. [2]
- (b) Given that at least 21 of the 25 patients turned up for their appointments, find the probability that exactly 21 patients turned up. [2]

On any day, the dentist can attend to at most 50 patients. As some patients do not turn up for their appointments, the clinic decides to allow more than 50 appointments to be made every day. The clinic aims for a probability of at least 97% that the doctor attends to all patients who turn up for their appointment.

- (c) Find the greatest number of appointments that can be made to achieve the clinic's aim. [2]

In another dental clinic, the durations of general consultation, in minutes, have the distribution $N(30, 2.7^2)$ and the durations of cosmetic consultation have the distribution $N(42, 0.8^2)$.

- (d) Find the probability that the total duration of 6 randomly selected general consultations is more than 170 minutes. [3]

The clinic upgrades its ultrasonic scaler which reduces the durations of each general consultation and each cosmetic consultation by 15%.

- (e) Find the probability that the total duration of 4 randomly selected general consultations differs from 3 times the duration of a randomly selected cosmetic consultation by at most 10 minutes. [4]

- 12 (a) Lina is investigating the relationship between exercise and resting heart rate. She obtains a random sample of nine 17-year-old students and records their resting heart rate, h beats per minute, and their exercise time per day, t minutes. The table below shows the data.

h	79	74	73	68	66	64	64	60	58
t	20	25	30	35	40	45	50	55	60

- (i) Sketch a scatter diagram of the data. [1]

The following two models are proposed, where a , b , d and e are constants.

$$(A) h = a + b \ln t \quad (B) h = d + et^2$$

- (ii) State the product moment correlation coefficient for each model. [2]
- (iii) Explain which model gives the better fit to the data, and find the equation of the regression line for this model. [2]
- (b) Lina continues to investigate the sleep duration, y hours, and the screen duration, x hours of another random sample of five 18-year-old students.

y	8.2	p	6.9	6.6	6.1
x	0.9	1.9	3.3	3.9	4.7

Lina proposed to use the least squares regression line $y = -0.55x + 8.72$ to model the above data.

- (i) Interpret, in context to the question, the value -0.55 in Lina's model. [1]
- (ii) Calculate the value of p , giving your answer correct to 1 decimal place. [2]
- (iii) Use the above regression equation to estimate the sleep duration of a student if he has a total screen duration of 7 hours. Comment on the reliability of this estimate. [2]

Lina verifies her data and realises that she has made an error by overstating each recorded sleep duration by one hour.

- (iv) Rewrite Lina's proposed model so that it clearly represents the amended data. [1]

