Year 6 Mathematics: analysis and approaches
Higher level
MARK SCHEME
Paper 2
Preliminary Examinations

Monday 28 August 2023

2 hours

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your name and class on each answer sheet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper
- The maximum mark for this examination paper is [**110 marks**].

Section A		Section B	
Question	**Marks**	**Question**	**Marks**
1		10	
2		11	
3		12	
4			
5			
6			
7			
8			
9			

Name: ________________

Class: ________________

Index: ________________

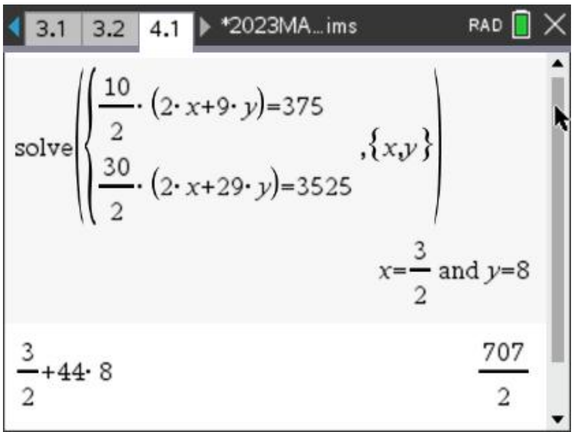
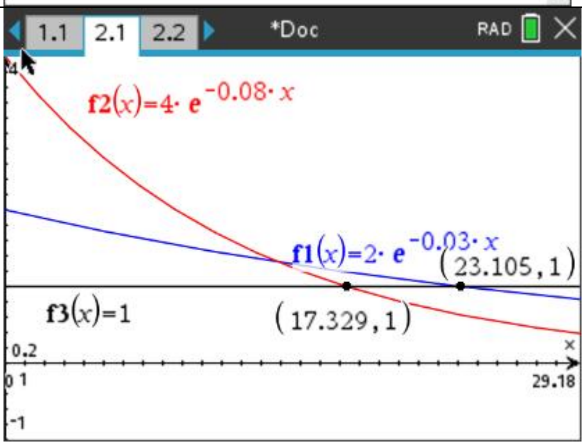
/ **110**

14 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

No	**Solutions**	**Mark scheme**
1(a)	$BC^2 = 4.4^2 + 5.8^2 - 2(4.4)(5.8)\cos 36°$ BC = 3.42166 BC = 3.42	Attempt to use cosine rule (M1) Correct substitution (A1) $BC^2 = 4.4^2 + 5.8^2 - 2(4.4)(5.8)\cos 36°$ BC = 3.42 (3s.f.) A1
1(b)	$\dfrac{\sin A\widehat{C}B}{4.4} = \dfrac{\sin 36°}{3.42166}$ $A\widehat{C}B = 49.0995°$ $A\widehat{C}B = 49.1°$	Attempt to use sine rule (M1) Correct substitution (A1) $\dfrac{\sin A\widehat{C}B}{4.4} = \dfrac{\sin 36°}{3.42166}$ $A\widehat{C}B = 49.1°$ A1
2(a)	$a = 30.4932$ a = 30.5 (3 s.f.) b = – 419.462 b = – 419 (3 s.f.)	a = 30.5 (3 s.f.) A1 b = – 419 (3 s.f.) A1
2(b)	$r = 0.93486$ $r = 0.935$	$r = 0.935$ A1
2(c)	y = 30.4932(29) – 419.462 y = 464 841 $465 000	Correct substitution of 29 into their regression equation (M1) $465 000 or 466 000 A1
2(d)	EITHER correlation does not imply causation (there is an association but not causation) OR there could be another factor involved.	EITHER correlation does not imply causation OR there could be another factor involved (eg other factors may affect profits or advertising not the sole factor affecting profits.) R1 NOT More money spent on ads will affect profits.
3	$S_{10} = \dfrac{10}{2}\left[2u_1 + (10-1)d\right]$ $375 = 5\left[2u_1 + 9d\right]$ -------------- (1) $3525 = \dfrac{30}{2}\left[2u_1 + 29d\right]$ ---------- (2) $u_1 = \dfrac{3}{2}$ $d = 8$ $u_{45} = \dfrac{3}{2} + 44 \times 8$	Attempt to use $375 = 5\left[2u_1 + 9d\right]$ or $3525 = \dfrac{30}{2}\left[2u_1 + 29d\right]$ (M1) Attempt to solve simultaneous equations (M1) $u_1 = \dfrac{3}{2}$ A1 $d = 8$ A1

	$u_{45}=\frac{707}{2}$ (353.5) [exact value]  	$u_{45}=\frac{3}{2}+44\times 8$ (M1) $u_{45}=\frac{707}{2}$ (353.5) [must be exact value] (A1)
4(a)	 Both graphs are decreasing functions. $f(t)=1$ $t=23.105$ hours $g(t)=1$ $t=17.329$ hours <23.105 hours Since Betacold took a shorter time for concentration to reach 1 unit, Betacold becomes ineffective before Anticold.	Attempt to use a graph or mention that functions are decreasing (M1) $t_A>t_B$ or find Time for Anticold $t=23.105$ hours. Time for Betacold $t=17.329$ hours. A1 AG
4(b)(i)	24 hours after 1st dose $g(24)=4\mathrm{e}^{-0.08(24)}$ $g(24)=0.586428$ units $g(24)=0.586$ units	Attempt to find $g(24)$ M1 $g(24)=0.586$ units A1
4(b)(ii)	Total concentration $g(24)+g(9)$ $=4\mathrm{e}^{-0.08(24)}+4\mathrm{e}^{-0.08(9)}$ $=2.53344$ $=2.53$ units	$24-15=9$ hours or $12-3=9$ hours for 2nd dose A1 $g(24)+g(9)$ M1 2.53 units A1

5	$(k+x)^2(3-2x)^7$ $=$ $(k^2+2kx+x^2)\left(3^7+{}^7C_1(3)^6(-2x)+\ldots+{}^7C_6(3)(-2x)^6+(-2x)^7\right)$ $x^2\cdot\left({}^7C_6(3)(-2x)^6\right)+2kx\cdot(-2x)^7=448x^8$ $\left({}^7C_6(3)(-2x)^6\right)+2k\cdot(-2)^7=448$ $k=3.5$	Valid approach for expansion for $(k+x)^2$ (M1) Valid approach for expansion for $(3-2x)^7$ (M1) Eg ${}^7C_r(3)^r(-2x)^{7-r}$ Recognizing that the term in x^8 is needed (M1) Correct term or coefficient in binomial expansion (seen anywhere) (A1) Eg $\left({}^7C_6(3)(-2x)^6\right)$ or $2k\cdot(-2)^7$ (either seen) Add their term in x^8 or coefficient of x^8 equal to $448x^8$ or 448. (M1) Eg $\left({}^7C_6(3)(-2x)^6\right)+2k\cdot(-2)^7=448$ $k=3.5$ A1

6(a)	${}^{18}C_8 = 43758$	A1
6(b)	Options $AABC \mid BBCC$ ${}^6C_2 \times {}^6C_3 \times {}^6C_3 = 15 \times 20 \times 20 = 6000$ $AABC \mid ABCC$ ${}^6C_3 \times {}^6C_2 \times {}^6C_3 = 20 \times 15 \times 20 = 6000$ $AABC \mid ABBC$ ${}^6C_3 \times {}^6C_3 \times {}^6C_2 = 20 \times 20 \times 15 = 6000$ Total $6000 \times 3 = 18000$	M1 – recognizing the options. A1 – At least one correct option from Section A, B and C (M1) finding all are the same, times by 3 A1 – Final Answer

7	Finding the intercept P. $2\mathrm{e} - x = \mathrm{e}^{\frac{1}{2}x}$ $x = 2.2916$ $y = 3.14496$ $P(2.29, 3.14)(3\mathrm{sf})$	(A1) – or seen as a limit on the integral.
	Region 1 Either Method 1 – Cone $V_1 = \frac{1}{3}\pi r^2 h$ $V_1 = \frac{1}{3}\pi(2.2916)^2(2\mathrm{e} - 3.145)$ $V_1 = 12.60196..$ $V_1 = 12.602$ OR Method 2 – Integration $V_1 = \pi\int_{y_1}^{2\mathrm{e}}(2\mathrm{e} - y)^2\mathrm{d}y$ $V_1 = 12.6016$ $V_1 = 12.602$ Region 2 Integration $V_2 = \pi\int_1^{y_1}(2\ln(y))^2\mathrm{d}y$ $V_2 = 15.2281$ Total Volume $V_T = V_1 + V_2$ $V_T = 12.6 + 15.2281 = 27.8303$ $V_T = 27.8\text{units}^3$	M1 Finding the volume of the cone A1 M1 - Using the volume equation on either part (condone the lack of limits) M1 – arranging either equation into $x = f(y)$ A1 A1 A1

8(a)	$\int_0^5 kxe^{\sin x}dx = 1$ $k = 0.073643$ If using DEG on GDC $k = 0.0755$	M1 for integral within limits =1 A1 A1
8(b)	$\int_0^5 x \times kxe^{\sin x}dx =$ $\text{mean} = 2.53189$ $\int_0^m kxe^{\sin x}dx = 0.5$ $\text{median} = 2.39507$ $\int_0^5 x^2 \times kxe^{\sin x}dx = 7.65031$ $\text{SD} = \sqrt{7.65031-(2.53189)^2}$ $\text{SD} = 1.11348$ $\gamma = \frac{3(2.53189\text{-}2.39507)}{1.11348}$ $\gamma = 0.368623$ The data is NOT symmetrical For GDC in DEGREE mode mark correct with values below. $\text{mean} = 3.3589$ $\text{median} = 3.56452$ $E(X^2) = 12.6504$ $\text{sd} = 1.1697$ $\gamma = -0.513$ The data is symmetrical	M1 Attempt to find 1 of the values required. A1 A1 A1 A1

9	$x^2+\frac{(2\sin\gamma)}{(\sec\gamma)}x-\frac{24\cos\gamma\sin^2\gamma}{(\sec\gamma)}=0$ $\alpha\beta=\frac{-24\cos\gamma\sin^2\gamma}{\sec\gamma}=-24\cos^2\gamma\sin^2\gamma=-6\sin^2 2\gamma$ $\alpha+\beta=-\frac{2\sin\gamma}{\sec\gamma}=-2\cos\gamma\sin\gamma=-\sin 2\gamma$ Solving for the roots. $\alpha=-\sin 2\gamma-\beta$ $(-\sin 2\gamma-\beta)\beta=-6\sin^2 2\gamma$ $\beta^2+\sin 2\gamma\beta-6\sin^2 2\gamma=0$ $(\beta-2\sin 2\gamma)(\beta+3\sin 2\gamma)=0$ $\beta=2\sin 2\gamma$ $\beta=-3\sin 2\gamma(\text{rej})$ $\alpha=-\sin 2\gamma-2\sin 2\gamma$ $\alpha=-3\sin 2\gamma$ $\frac{2\alpha}{\beta}=\frac{-6\sin 2\gamma}{2\sin 2\gamma}=-3$ $\frac{\beta}{2\alpha}=\frac{2\sin 2\gamma}{-6\sin 2\gamma}=-\frac{1}{3}$ $y=(x+3)\left(x+\frac{1}{3}\right)=x^2+\frac{10}{3}x+1$	A1 – product of roots A1 – sum of roots M1 – attempting to solve for the roots A1 one of the roots correct. M1- using their roots to find the equation. A1

Section B

10(a)	$X \sim \mathrm{N}(2040, 210^2)$ $\mathrm{P}(X > 2300) = 0.10784$ $\mathrm{P}(X > 2300) = 0.108$ (3 s.f.)	Recognise P(X > 2300) (M1) $\mathrm{P}(X > 2300) = 0.108$ (3 s.f.) A1
10b)	Expected days $= 0.10784 \times 365$ $= 39.3616$ $= 39$ 39 days	Attempt to multiply P(X > 2300) and 365 (M1) Accept 39 or 39.4 or 40 days A1
10(c)	$X \sim \mathrm{N}(2250, \sigma^2)$ $\mathrm{P}(X > 2300) = 0.4$ $\mathrm{P}(X < 2300) = 0.6$ $\frac{2300-2250}{\sigma} = 0.253347$ $\sigma = 197.358$ $\sigma = 197$ (3 s.f.)	Valid approach (M1) Eg 1 – 0.4, 0.6 z = 0.253347 A1 attempt to standardize (M1) Eg $\sigma = \frac{2300-2250}{z}$ *or* $z = \frac{2300-2250}{\sigma}$ Correct substitution with their z (do not accept a probability) A1 Eg $\frac{2300-2250}{\sigma} = 0.253347$ $\sigma = 197$ (3 s.f.) A1
10(d)(i)	P(X<2500 | X>2300) $\frac{\mathrm{P}(2300 < X < 2500)}{\mathrm{P}(X > 2300)}$ $= \frac{0.297375}{0.4}$ $= 0.743438$ $= 0.743$ (3 s.f.)	Recognition of conditional probability (M1) P(X<2500 | X>2300) (accept P(X>2300|X<2500) (do not accept P(A| B)) $\frac{\mathrm{P}(2300 < X < 2500)}{\mathrm{P}(X > 2300)}$ (A1) $= \frac{0.297375}{0.4}$ (A1) $\mathrm{P}(X < 2500 \mid X > 2300) = 0.743438$ $= 0.743$ (3 s.f.) (A1)
10(d)(ii)	Let Y be the number of special days that the daily revenue is less than \$2500. $Y \sim \mathrm{B}(10, 0.743438)$ $\mathrm{P}(Y < 5)$ $= \mathrm{P}(Y \le 4)$ = 0.022408 = 0.0224	Recognizing binomial probability (M1) $Y \sim \mathrm{B}(n, p)$ n = 10, p =0.743438… (A1) (using their p) P(Y<5) $= \mathrm{P}(Y \le 4)$ A1 = 0.0224 (3s.f.) A1 (0.0226 using p = 0.743)

<table>
<tr><td>11(a) (i)</td><td>$e^{2x}y - y^2 = \sin x - x$
$x = 0$
$e^{2(0)}y - y^2 = \sin 0 - 0$
$y^2 - y = 0$
$y(y-1) = 0$
$y = 0 \to (\text{rej})$
$y = 1$</td><td>A1 condone lack of rejection or 0</td></tr>
<tr><td>11(a) (ii)</td><td>$e^{2x}y - y^2 = \sin x - x$
$2e^{2x}y + e^{2x}\frac{dy}{dx} - 2y\frac{dy}{dx} = \cos x - 1$
$\frac{dy}{dx}\left(e^{2x} - 2y\right) = \cos x - 1 - 2e^{2x}y$
$\frac{dy}{dx} = \frac{\cos x - 1 - 2e^{2x}y}{\left(e^{2x} - 2y\right)}$</td><td>M1 - product rule on first term
M1 - Implicit on the y^2
A1 – differentiating RHS

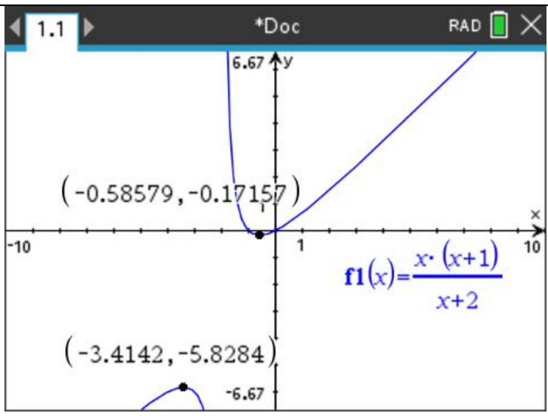
AG</td></tr>
<tr><td>11(a)(iii)</td><td>$\frac{dy}{dx} = \frac{\cos x - 1 - 2e^{2x}y}{\left(e^{2x} - 2y\right)}$
$x = 0, y = 1$
$\frac{dy}{dx} = \frac{\cos(0) - 1 - 2e^{2(0)}(1)}{\left(e^{2(0)} - 2(1)\right)}$
$\frac{dy}{dx} = \frac{1 - 1 - 2}{(1-2)} = 2$</td><td>M1 input x=0 and y=1
Or
M1 attempt to find $\frac{dy}{dx}$ as a $f(y)$

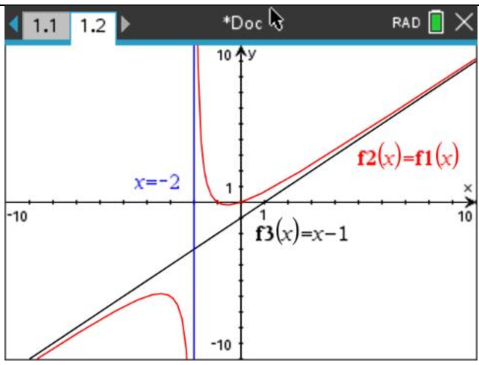
A1</td></tr>
<tr><td></td><td colspan="2">For the rest of the question only penalise once for answers not to 6sf.</td></tr>
<tr><td>11(b)</td><td colspan="2">

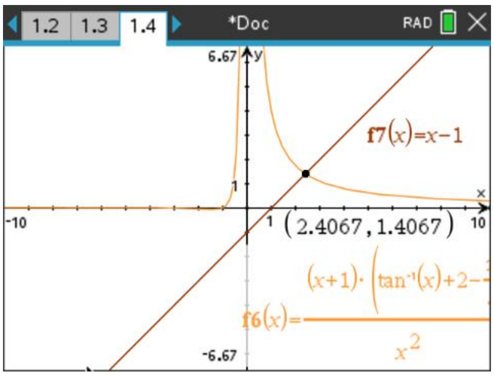
x	y	dy/dx
0	1.00	2
0.05	1.1	2.221922855
0.1	1.211096143	2.46793306
0.15	1.334492796	2.739678001
0.2	1.471476696	3.039223188
0.25	1.623437855	3.36906152

(M1) – Attempt at Euler's evidence of the first term
(A1) – y=1.1
(A1) – any other correct intermediary values
A1 Final value of y = 1.47148 penalise if not to 6 sf
Use of DEG on the GDC M1,A1,A1,
Interim y values are 1.21104, 1.33423,
Final = <u>1.47082 A0</u></td></tr>
<tr><td>11(c)(i)</td><td>$y = 1 + 2x + 2x^2 + \frac{19x^3}{6}$</td><td>A1- first 3 terms
A1 third term</td></tr>
</table>

11(c)(ii)	$y=1+2x+2x^2+\frac{19x^3}{6}$ $y=1+2(0.2)+2(0.2)^2+\frac{19(0.2)^3}{6}$ $y=1.50533$	A1- Answer only.
11(d)(i)	$y=1.49272$	(M1) A1
11(d)(ii)	Euler Error $=\lvert 1.49272-1.47147\rvert$ $=0.02125$ Maclaurin's Error $=\lvert 1.49272-1.50533\rvert$ $=0.01261$ Maclaurin's is a better estimate as closer to the actual value	A1 for both correct values. R1

12(a) (i)	 Minimum (-0586,-0.172) Maximum (-3.41-5.83)	A1 A1
12 (a) (ii)	Method 1 $\dfrac{(px+q)(x+2)}{x+2}+\dfrac{B}{x+2}$ $(px+q)(x+2)+B \equiv x(x+1)$ $px^2+2px+qx+2q+B \equiv x^2+x$ $p=1$ $2p+q=1 \to q=-1$ $2q+B=0 \to B=2$ $x-1+\dfrac{2}{x+2}$ Asymptote = $y=x-1$ or Method 2 $$\begin{array}{r} x-1 \\ (x+2)\overline{\big)\, x^2+x} \\ (-)\ \underline{x^2+2x} \\ -x \\ (-)\ \underline{-x-2} \\ +2 \end{array}$$ $x-1+\dfrac{2}{x+2}$ Asymptote $g(x)=x-1$	M1 put into two parts A1 - Solving to find p and q or equivalent. AG M1 Attempting to Divide A1 – one correct process of division AG

12 (a) (iii)		A1 – Sketch correct, no touching of asymptotes A1 – Asymptotes drawn with labels A1 intercepts at (-1,0) and (0,0)
(b)	$\frac{1}{f(x)}=\frac{x+2}{x(x+1)}$ $\frac{A}{x}+\frac{B}{x+1}\equiv\frac{x+2}{x(x+1)}$ $A(x+1)+Bx\equiv x+2$ $B=0\to A=2$ $A=-1\to B=-1$ $\frac{2}{x}-\frac{1}{x+1}$	M1 Attempt to solve for partial fraction. A1 – for both Can be seen in the equation.
12(c)(i)	$IF=e^{\int P(x)dx}$ $\int P(x)\mathrm{d}x=\int\frac{1}{f(x)}\mathrm{d}x=\int\frac{2}{x}-\frac{1}{x+1}\mathrm{d}x$ $\int\frac{2}{x}-\frac{1}{x+1}\mathrm{d}x=2\ln(x)-\ln(x+1)$ $\ln\left(\frac{x^2}{x+1}\right)$ $IF=e^{\ln\left(\frac{x^2}{x+1}\right)}$ $IF=\frac{x^2}{(x+1)}$	M1 - identifying P(x) is from part (b) M1 Attempt to integrate. A1 Correct integral Condone the presence of +c M1 Log laws to simplify. AG

12 (c) (ii)	$y \times IF = \int Q(x) \times IF \mathrm{d}x$ $IF = \dfrac{x^2}{(x+1)}$ $y\dfrac{x^2}{(x+1)} = \int \dfrac{x+1}{x^4+x^2} \times \dfrac{x^2}{(x+1)} \mathrm{d}x$ $y\dfrac{x^2}{(x+1)} = \int \dfrac{x+1}{x^2(x^2+1)} \times \dfrac{x^2}{(x+1)} \mathrm{d}x$ $y\dfrac{x^2}{(x+1)} = \int \dfrac{1}{x^2+1} \mathrm{d}x$ $y\dfrac{x^2}{(x+1)} = \arctan(x) + c$ OR $y = \dfrac{x+1}{x^2}(\arctan(x) + c)$	M1 input all parts into the equation. M1 simplify LHS A1 -correct integration A1 answer must include +c, no need to simplify to y=
12 (c) (iii)	$y = \dfrac{x+1}{x^2}(\arctan(x) + c)$ $4 = \dfrac{1+1}{1^2}(\arctan(1) + c)$ $2 = \left(\dfrac{\pi}{4} + c\right)$ $c = 2 - \dfrac{\pi}{4}$ $h(x) = \dfrac{x+1}{x^2}\left(\arctan(x) + 2 - \dfrac{\pi}{4}\right)$	M1 Values substituted in A1- value of c. A1 correct equation.
12 (d)	 $x = 2.41$ Full marks can be awarded with their equation in (c)(iii) and GDC intercept. Even if incorrect. But only this equation for FT.	(M1) attempt to make $g(x) = h(x)$, can be seen algebraically. A1