ANGLO-CHINESE JUNIOR COLLEGE
JC1 PROMOTIONAL EXAMINATION

Higher 2

MATHEMATICS 9758/01

Paper 1
QUESTION PAPER

3 October 2025
3 hours

Additional Materials: Printed Answer Booklet
List of Formulae and Results (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are **not** allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.

1 Given that $y^{\cos x} = x^{\sin 2x}$, find the exact value of $\frac{dy}{dx}$ at $x = \pi$. [5]

2 It is given that $y = \sqrt{1 + \tan^{-1} 2x}$ where $-\frac{1}{2} < x < \frac{1}{2}$. Show that

$$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + \frac{8x}{(1+4x^2)^2} = 0$$

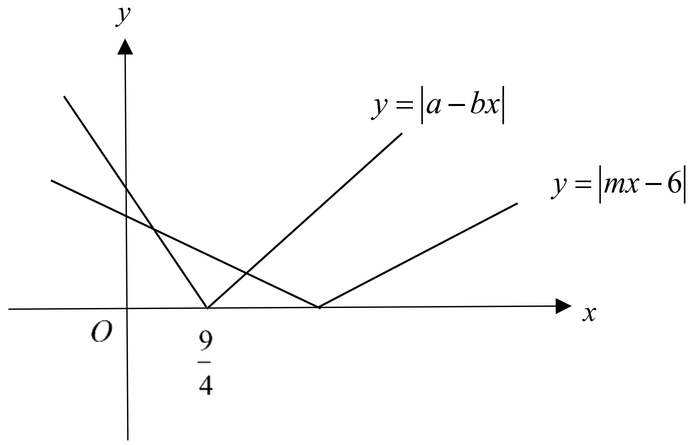
Hence find the Maclaurin series for y, up to and including the term in x^3. [5]

3 Given that $\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}$.

(a) Show that $\sum_{r=2}^{n} \frac{1}{r(r-1)} = 1 - \frac{1}{n}$. [2]

(b) Find $\sum_{r=2}^{\infty} \left(\frac{1}{2^r} + \frac{1}{r(r-1)} \right)$. [3]

4 **(a)** The diagram shows the graphs of $y = |a - bx|$ and $y = |mx - 6|$ where a, b and m are positive constants. The graph of $y = |a - bx|$ meets the x-axis at the point $\left(\frac{9}{4}, 0\right)$.



The solution set of the inequality $|mx - 6| < |a - bx|$ is given as $\{x : x \in \mathbb{R}, x < 1 \text{ or } x > 3\}$.

Find the values a, b and m. [4]

(b) Given that k is a constant and $k > 1$, solve the inequality $\frac{kx^2 + 1}{x^2 + (k+1)x + k} > -1$, giving your answer in terms of k. [4]

5 A curve C_1 has parametric equation

$$x = 3\sec^2\theta, \quad y = 2\tan\theta, \quad 0 \le \theta < \frac{\pi}{2}.$$

(a) Sketch the curve C_1, stating the coordinates of the endpoint(s). [1]

(b) Show that $\dfrac{dy}{dx} = \dfrac{1}{3\tan\theta}$. [2]

L_1 and L_2 are tangents to curve C_1.

(c) Find

(i) the equation of L_1 which is parallel to the y-axis. [2]

(ii) the gradient of L_2 which is tangent to the curve C_1 at the point $(3.75, 1)$. [2]

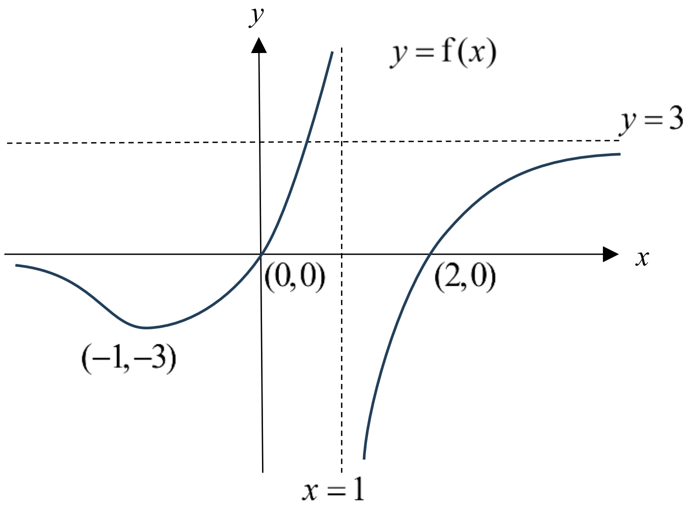
(d) β is the acute angle between L_1 and L_2. State the value of $\tan\beta$. [1]

(e) Describe a transformation that will map C_1 onto the curve C_2 with equation

$$x = 3\sec^2\theta, \quad y = 2\tan(\pi - \theta), \quad 0 \le \theta < \frac{\pi}{2}.$$ [1]

[Turn over

6 **(a)** The figure below shows the sketch of the graph $y = \mathrm{f}(x)$. The graph has asymptotes at $x = 1$, $y = 3$ and the x-axis. The graph has stationary point at $(-1, -3)$ and cuts the axes at $(0,0)$ and $(2,0)$.



Sketch, on separate diagrams, the following graphs, indicating clearly any asymptotes, axial intercepts and stationary points, where possible.

(i) $y = -2\mathrm{f}\left(x\right) + 1$ [3]

(ii) $y = \dfrac{1}{\mathrm{f}(x)}$ [3]

(iii) $y = \mathrm{f}\left(|x| + 1\right)$ [3]

(b) Describe fully a sequence of transformations that transform the graph of $y = x^2$ onto the graph of $y = 4x^2 - 4x + 1$. [2]

7 Referred to the origin O, points A, B and C have position vectors **a, b** and **c** respectively. It is given that $\mathbf{a}+\mathbf{b}-\mathbf{c}$ and $\mathbf{a}-\mathbf{b}+\mathbf{c}$ are parallel.

(a) Using vector product, show that $\overrightarrow{OA}$ and $\overrightarrow{BC}$ are parallel. [4]

It is now given that **a** and **b** are unit vectors, $\overrightarrow{BC}=2\overrightarrow{OA}$ and angle $OBC=60^\circ$.

(b) Using scalar product, show that $\overrightarrow{OB}$ and $\overrightarrow{OC}$ are perpendicular. [3]

(c) The point H has position vector **h**. Given that $\mathbf{h}=\mathbf{b}\times\mathbf{c}$ and $|\mathbf{c}|=\sqrt{3}$, find the exact volume of the pyramid $HOBC$. [3]

[The volume of a pyramid is $\frac{1}{3}\times$ base area $\times$ height.]

8 The function f is defined by

$$\mathrm{f}: x \mapsto \frac{ax}{x-a}, \quad \text{for } x \in \mathbb{R},\ -a \le x < a,$$

where a is a positive constant.

(a) Sketch the curve $y=\mathrm{f}(x)$, stating the equation of the asymptote and the coordinates of the endpoint. Explain, stating the reason clearly, why f has an inverse. [3]

(b) Find $\mathrm{f}^{-1}(x)$ and state its domain. [3]

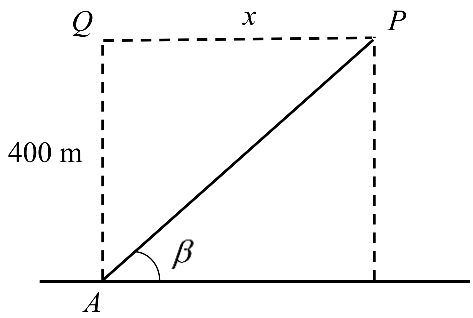
(c) Find the range of values of x in terms of a, for which $\mathrm{f}(x)=\mathrm{f}^{-1}(x)$. [1]

(d) For positive integers k, find $\mathrm{f}^{2k+1}\left(-\frac{1}{2}a\right)$ in terms of a. [2]

(e) Given that $\mathrm{fg}(x)=x+a$ for $x\in\mathbb{R}$, $x\le -a$, find $\mathrm{g}(x)$ stating its domain. [2]

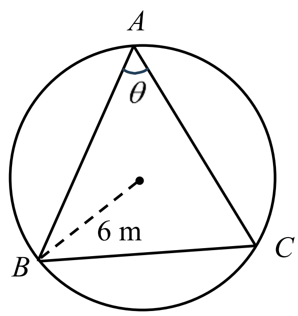
[Turn over

9(a)



In the diagram, point Q is 400 m vertically above point A. Initially, a balloon is at point Q and moves horizontally in a straight line at a constant speed of 10 m per second away from an observer stationed at point A. At time t seconds, after the start, the balloon is at point P where QP is x m and the angle of elevation of the balloon from the observer at A is β radians. Find exactly the rate of change of β at the instant when x is 100 m. [4]

9(b)



A gardener designs a flower bed ABC in the shape of an isosceles triangle inscribed in a circular plot of land with radius 6 meters, as shown in the diagram. The vertices of the triangle lie on the circumference of the circle and $AB = AC$. The angle BAC is θ, where θ is acute. Express the area of the flower bed ABC in terms of θ. [2]

As θ varies, use differentiation to find the exact maximum possible area of the flower bed, and show that it is a maximum. [4]

10 The plane p_1 has equation $x - y + z = 0$ and the plane p_2 has equation $x + y + z = 2$.

(a) Find the acute angle between planes p_1 and p_2. [2]

(b) Find the vector equation of the line of intersection of planes p_1 and p_2. [1]

With reference to the origin O, the points A and B are such that $\overrightarrow{OA} = -2\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\overrightarrow{OB} = \mathbf{i} + \mathbf{j}$. The line l has equation $\mathbf{r} = -2\mathbf{i} + \mathbf{j} - \mathbf{k} + \mu(\mathbf{i} - \mathbf{k})$, where μ is a parameter.

(c) Find the shortest distance between line l and the line with cartesian equation $x - 3 = -z + 2, y = 1$. [2]

(d) The point C lies on l such that angle $ABC = 90^\circ$. Find the position vector of point C. [3]

(e) Find the position vector of the foot of perpendicular from point A to p_2. [3]

(f) A plane p_3 is equidistant to both point A and plane p_2. Find the cartesian equation of plane p_3. [2]

[Turn over

11 Caleb is training for a vertical marathon, by running up the stairs of a building with 62 floors. He decides to train **three days a week** starting with 10 floors on his first day of training. On each subsequent training day, he runs 2 floors more than the previous training day.

(a) Find the number of floors he completes on his 10^{th} training day. [2]

Once he can complete 62 floors in a training day, he is considered as ready for the vertical marathon. Then for subsequent training days, he will continue to complete 62 floors each day.

(b) He is ready for the vertical marathon at the end of *k* weeks of his training. Find *k*. [2]

(c) Find the total number of floors he completes at the end of 20 weeks of his training. [3]

After training, Caleb's heart rate decreases by 10% at the start of every *n*th minute. When $n=1$, his heart rate at the start of the first minute is 180 beats per minute (bpm).

(d) Find the least value of *n* for Caleb's heart rate to drop below 82 bpm. [3]

Caleb's friend, Alex also tracks his own heart rate after training. His heart rate at the start of the first minute is 170 bpm and his heart rate can be modelled by the recurrence relation $H_{n+1}=0.85H_n+10.5$, where H_n is his heart rate at the start of the *n*th minute after training.

(e) Find the least value of *n* for Alex's heart rate to drop below 90 bpm. [1]

(f) Find his heart rate after he has rested long enough. [2]

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Summary of Areas for Improvement			
Knowledge (K)	**Careless Mistakes (C)**	**Read/Interpret Qn wrongly (R)**	**Presentation (P)**