RAFFLES INSTITUTION
H2 Mathematics (9758)
2025 Year 6

Tutorial S1A: Permutations and Combinations

Section A (Discussion Questions)

1 9 coloured balls are arranged in a row. Find the number of ways this can be done if

(i) all the balls are of different colours,
(ii) there are 4 red balls, 3 green balls and 2 yellow balls,
(iii) there are 4 red balls, 3 green balls and 2 yellow balls, **and** the arrangement is symmetrical.

[(i) 362880; **(ii)** 1260; **(iii)** 12 **]**

Solution

(i) Number of ways = 9! = 362880

9 coloured balls, 4 R, 3 G, 2 Y

(ii) Number of ways = $\dfrac{9!}{4!3!2!}$=1260

Alternative method: Out of the 9 positions, choose 4 for the 4 red balls, out of the remaining 5 positions, choose 3 for the 3 green balls : ${}^{9}C_{4} \times {}^{5}C_{3}$

(iii) there are 4 red balls, 3 green balls and 2 yellow balls, **and** the arrangement is symmetrical.

____ ____ ____ ____ __G__ ____ ____ ____ ____
2R , 1G, 1Y 2R , 1G, 1Y

Since arrangement is symmetrical, middle ball must be green. The first 4 balls must consist 2 red, 1 green and 1 yellow ball.

Number of ways = $\dfrac{4!}{2!}$=12

Alternative method: ${}^{4}C_{2} \times {}^{2}C_{1}$ Choose the slots instead.

2 How many different three figure numbers between 100 and 999 inclusive, contain two and only two consecutive identical figures?

[162]

Solution

Case 1: First 2 digits are identical.
Choices for 1st two digits = 9 (1-9, 0 excluded)
Choices for 3rd digit = 9 (any digit from 0-9 that has not been used for 1st digit)
Number of ways = 9 x 9 = 81

Case 2: Last 2 digits are identical
Choices for 1st digit = 9 (1-9, 0 excluded)
Choices for 2nd and 3rd digits = 9 (any digit from 0-9 that has not been used for 1st digit)
Number of ways = 9 x 9 = 81

Total number of three figure numbers = 81 + 81 = 162

3 NJC Prelim 9233/2005/01/Q7 (modified)

(a) A resort hotel has 3 single rooms, 5 double rooms and 4 family rooms. On a particular week, 2 individuals book a single room each, 3 couples book a double room each and 3 families book a family room each. Given that all the rooms are available for that week and each room has a distinct design theme, find the number of different possible bookings of rooms amongst the guests.

(b) A family of 6 adults and 3 children are to be seated at a round table. Find the number of possible arrangements such that

(i) all the 3 children sit together,

(ii) none of the children sit together.

[(a) 8640; **(b)(i)** 4320; **(ii)** 14400**]**

Solution

(a) Rooms : 3 single, 5 double, 4 family
People : 2 individual, 3 couples, 3 families

Number of arrangements for singles = ${}^{3}P_{2}$
Number of arrangements for couples = ${}^{5}P_{3}$
Number of arrangements for families = ${}^{4}P_{3}$

Total number of arrangements = ${}^{3}P_{2} \times {}^{5}P_{3} \times {}^{4}P_{3}$=8640

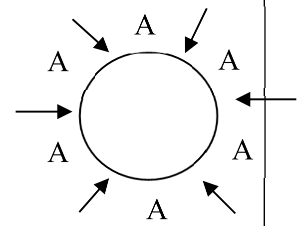
(b) 6 adults, 3 children, seated at a round table

(i) *all the 3 children sit together*

C C C A A A A A A - 7 units

Number of ways to arrange 7 units at a round table = (7 – 1)!
Number of arrangements within the "children unit" = 3!

Total number of seating arrangements = (7 – 1)! × 3! = 4320



(ii) *none of the children sit together*
Slotting Method:
Number of arrangements to first seat the 6 adults = (6 – 1)! = 120
Number of arrangements to slot in the 3 children in the 6 empty spaces between the adults = ${}^{6}P_{3}$
Total number of seating arrangements = (6 – 1)! × ${}^{6}P_{3}$ = 14400

4 **(a)** Twelve people are to make a journey in 3 cars, with 4 people in each car. Each car is driven by its owner. Find the number of ways in which the remaining 9 people may be allocated to the cars. [The arrangement of people *within* each car is not relevant.]

(b) Nine people are to be divided into 4 groups. Find the number of ways this can be done if 1 group consists of 3 people and the other 3 groups consist of 2 people each.

[(a) 1680; **(b)** 1260**]**

Solution

(a) Each group is distinct as there is a unique car owner.

Number of ways = ${}^{9}C_{3} \times {}^{6}C_{3} \times {}^{3}C_{3} = 1680$ OR $\dfrac{9!}{3!3!3!}$

Arrange all the people in a line. First 3 people to Car 1, next 3 to Car 2 and last 3 to Car 3. However arrangement is not needed within each car, so divide by $3!3!3!$.

(b) The 3 groups of 2 are identical, and there is a need to divide by 3!

Number of ways = $\dfrac{{}^{9}C_{2} \times {}^{7}C_{2} \times {}^{5}C_{2} \times {}^{3}C_{3}}{3!} = 1260$

5 A student has 8 different Literature books, 3 different History books and 3 different Geography books. How many ways can 5 books be selected if the selection must consist of at least one book of each subject? [*The order of selection is not relevant*]. [1128]

Solution

Lit (8)	History (3)	Geog (3)	No. of ways
3	1	1	${}^{8}C_{3} \times {}^{3}C_{1} \times {}^{3}C_{1} = 504$
2	2	1	$2({}^{8}C_{2} \times {}^{3}C_{2} \times {}^{3}C_{1}) = 504$
2	1	2	
1	3	1	${}^{8}C_{1} \times {}^{3}C_{3} \times {}^{3}C_{1} = 24$
1	2	2	${}^{8}C_{1} \times {}^{3}C_{2} \times {}^{3}C_{2} = 72$
1	1	3	${}^{8}C_{1} \times {}^{3}C_{3} \times {}^{3}C_{1} =24$

Summing up the above, total number of ways = 1128

<u>Alternative 1: Complement method</u>

Number of choices, without restriction = ${}^{14}C_5$ = 2002

Complement cases:
Case 1: All 5 books chosen are History or Geog Books ONLY
Number of choices = ${}^{6}C_5$
Case 2: All 5 books chosen are Literature or History Books ONLY
Number of choices = ${}^{11}C_5$
Case 3: All 5 books chosen are Literature or Geog Books ONLY
Number of choices = ${}^{11}C_5$
Note that in cases 2 and 3, both include the case where only literature books are chosen

Number of choices, when only literature books are chosen = ${}^{8}C_5$ =56

Hence, number of choices $= {}^{14}C_5 - \left({}^{6}C_5 + {}^{11}C_5 + {}^{11}C_5\right) + {}^{8}C_5$ = 1128

Note : ${}^{8}C_1 \times {}^{3}C_1 \times {}^{3}C_1 \times {}^{11}C_2$ is not correct. There is double counting.
You can make reference to Eg 11 of lecture notes

For eg, $L_1 \quad H_1 \quad G_1 \quad G_2\, G_3$
$L_1 \quad H_1 \quad G_2 \quad G_1\, G_3$

6 9740/2009/02/Q8

Find the number of ways in which the letters of the word ELEVATED can be arranged if

(i) there are no restrictions, **[1]**

(ii) T and D must not be next to one another, **[2]**

(iii) consonants (L, V, T, D) and vowels (E, A) must alternate, **[3]**

(iv) between any two Es there must be at least 2 other letters. **[3]**

[(i) 6720 ; **(ii)** 5040; **(iii)** 192 **(iv)** 480**]**

Solution

3 E's, L, V, A, T, D ------ 8 letters

(i) Number of arrangements without restriction $=\dfrac{8!}{3!}=6720$

(ii) *T and D must not be next to one another*

Method 1 : Slotting method

__ E __ E __ E __ L __ V __ A __

Number of arrangements where T and D are not next to one another

$=\dfrac{6!}{3!}\times{}^{7}C_{2}\times 2! = 5040$ or $\dfrac{6!}{3!}\,{}^{7}P_{2}$

Method 2 : Complement method

TD E E E L V A

Number of arrangements where T and D are together $=\dfrac{7!}{3!}\times 2! = 1680$

Number of arrangements where T and D are not next to one another

$= 6720 - 1680 = 5040$

(iii) *Consonants (L, V, T, D) and vowels (E, A) must alternate*,

Case 1: 1st letter is a consonant L ___ V ___ T ____ D ____

Case 2: 1st letter is a vowel E ___ E ___ E ____ A ____

The number of arrangements within each case is the same.

Number of arrangements where consonants and vowels alternate= $4!\times\dfrac{4!}{3!}\times 2 = 192$

(iv) *Between any two Es there must be at least 2 other letters.*

E___ ___ ___ E ___ ___ E

E ___ ___ E___ ___ ___ E

E ___ ___ E ___ ___ E ___

___ E ___ ___ E ___ ___ E

There are 4 cases and the number of arrangements within each case is the same.

Number of arrangements = $5!\times 4 = 480$

7 9740/2014/02/Q6

A team in a particular sport consists of 1 goalkeeper, 4 defenders, 2 midfielders and 4 attackers. A certain club has 3 goalkeepers, 8 defenders, 5 midfielders and 6 attackers.

(i) How many different teams can be formed by the club? **[2]**

One of the midfielders in the club is the brother of one of the attackers in the club.

(ii) How many different teams can be formed which include exactly one of the two brothers? **[3]**

The two brothers leave the club. The club manager decides that one of the remaining midfielders can play as either a midfielder or as a defender.

(iii) How many different teams can be formed by the club? **[3]**

[(i) 31500 **(ii)** 16800 **(iii)** 8820**]**

Solution

A team consists of 1 G, 4 D, 2 M and 4 A
A club has 3 G, 8 D, 5 M and 6 A

(i) Number of teams that can be formed $= {}^3C_1 \times {}^8C_4 \times {}^5C_2 \times {}^6C_4 = 31\,500$

(ii) One of the midfielders in the club is the brother of one of the attackers in the club.
Choose a team which includes exactly one of the two brothers
Case 1 : The brother who is the midfielder is chosen
Number of possible teams $= {}^3C_1 \times {}^8C_4 \times {}^4C_1 \times {}^5C_4 = 4200$

Case 2 : The brother who is the attacker is chosen
Number of possible teams $= {}^3C_1 \times {}^8C_4 \times {}^4C_2 \times {}^5C_3 = 12600$

$\therefore$ Number of teams which includes exactly one of the brothers =4200+12600=16800

(iii) A team consists of 1 G, 4 D, 2 M and 4 A
After the brothers leave, the club now has 3 G, 8 D, 4 M and 5 A
One of the remaining midfielders can play either midfielder or defender.

Case 1 : The midfielder plays as a midfielder
Number of possible teams $= {}^3C_1 \times {}^8C_4 \times {}^3C_1 \times {}^5C_4 = 3150$

Case 2 : The midfielder plays as a defender
Number of possible teams $= {}^3C_1 \times {}^8C_3 \times {}^3C_2 \times {}^5C_4 = 2520$

Case 3 : The midfielder does not play
Number of possible teams $= {}^3C_1 \times {}^8C_4 \times {}^3C_2 \times {}^5C_4 = 3150$

$\therefore$ Number of different teams = 3150+2520+3150=8820

Note : It is not the case that **any** of the remaining midfielders can play as midfielder or defender.

Alternative 1:
Noting that the choices for the goalkeepers and attackers do not change, we have

$$ {}^{3}C_{1}\times{}^{5}C_{4}\times\Big(\underbrace{{}^{8}C_{4}\times{}^{3}C_{1}}_{\text{M plays as M}}+\underbrace{{}^{8}C_{3}\times{}^{3}C_{2}}_{\text{M plays as D}}+\underbrace{{}^{8}C_{4}\times{}^{3}C_{2}}_{\text{M does not play}}\Big)=8820 $$

<u>Alternative 2:</u>

Case 1 : The midfielder remains as a midfielder (he may get to play; or he may not get to play)

Number of possible teams $= {}^{3}C_{1}\times{}^{8}C_{4}\times{}^{4}C_{2}\times{}^{5}C_{4}=6300$

Case 2 : The midfielder as a defender (he may get to play; or he may not get to play)

Number of possible teams $= {}^{3}C_{1}\times{}^{9}C_{4}\times{}^{3}C_{2}\times{}^{5}C_{4}=5670$

Note that in the above 2 case, they both include the number of possible teams with the mid-fielder NOT playing.

When the midfielder does not play.

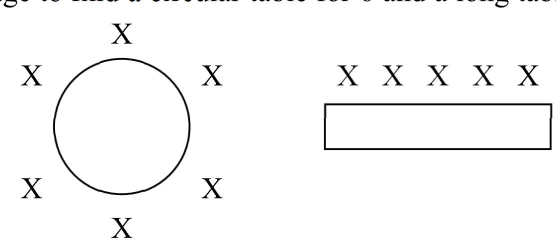
Number of possible teams $= {}^{3}C_{1}\times{}^{8}C_{4}\times{}^{3}C_{2}\times{}^{5}C_{4}=3150$

$\therefore$ Number of different teams = 6300 + 5670 − 3150 = 8820

8 MI Prelim 9740/2011/02/Q8(b)

The Mathematics Department of M Institute consists of 11 teachers which includes Mr Tan and Mr Lin. At a meeting, all 11 teachers sit at a round table with numbered seats. How many ways are there of arranging the department if Mr. Tan and Mr. Lin must sit next to each other?

After the meeting, the department heads to a nearby food court for lunch. Due to the lunch crowd they only manage to find a circular table for 6 and a long table with a row of 5 seats as shown below.



Find the number of arrangements of seating all 11 teachers at the two tables such that Mr. Tan is seated next to Mr. Lin. [7983360; 1209600]

Solution

Consider Mr Tan and Mr Lin as 1 unit.

During the meeting, there are 10 units altogether.

Number of ways to arrange 10 units at a round table = (10 – 1)!

Number of ways Mr Tan and Mr Lin can arrange amongst themselves = 2!

Since the seats are numbered, there is 11 ways to rotate the people, within the same arrangement.

Hence, total number of arrangements = (10 – 1)! x 2 x 11 = 7983360

After the meeting,

Case 1 : Mr Tan and Mr Lin are seated at the round table
Number of ways to choose 4 more people to sit at the round table = ${}^{9}C_{4}$
Number of ways to arrange the Mr Tan and Mr Lin and 4 others at the round table
=(5 – 1)! 2!
Number of ways to arrange the remaining 5 people at the long table = 5!
Number of arrangements $= {}^{9}C_{4} \times (5-1)! \times 2! \times 5!$

Case 2 : Mr Tan and Mr Lin are seated at the long table
With a similar way of counting,
Number of arrangements $= {}^{9}C_{3} \times 4! \times 2! \times (6-1)!$

Hence, total number of ways $= {}^{9}C_{4} \times (5-1)! \times 2! \times 5! + {}^{9}C_{3} \times 4! \times 2! \times (6-1)! = 1209600$

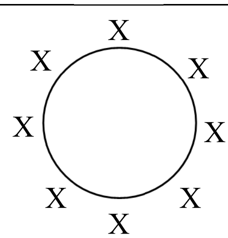
9 TJC Prelim 9740/2011/02/Q6(b)

Six people sit at a round table with eight identical chairs. Find the number of seating arrangements if

(i) there are no restrictions,

(ii) the seats are numbered. **(i)** 2520 **(ii)** 20160 **]**

Solution



(i) Since there are 6 people and 8 chairs, 2 chairs would not be occupied.

Number of ways to arrange 6 people and 2 empty slots (*treated as distinct empty seats*) at a round table = (8 – 1)!
However, the 2 empty slots are actually identical.
Hence, number of arrangements $= \frac{7!}{2} = 2520$

(ii) If the seats are numbered, number of arrangements $= \frac{7!}{2} \times 8$ (OR ${}^{8}P_{6}$) = 20160

10 9205/1999/01/Q18

(a) Eight people go to the theatre and sit in a particular group of eight adjacent reserved seats in the front row. Three of the eight belong to one family and sit together.

(i) If the other five people do not mind where they sit, find the number of possible seating arrangements for all eight people. **[4]**

(ii) If the other five people do not mind where they sit, except that two of them refuse to sit together, find the number of possible seating arrangements for all eight people. **[5]**

(b) The salad bar at a restaurant has 6 separate bowls containing lettuce, tomatoes, cucumber, radishes, spring onions and beetroot respectively. John decides to visit the salad bar and make a selection. At each bowl, he can choose to take some of the contents or not.

(i) Assuming that John takes some of the contents from at least one bowl, find how many different selections he can make. **[2]**

(ii) John decides he is going to have 4 salad items, and one of them will be tomatoes. How many different selections can he make? **[1]**

[(a)(i) 4320 **(ii)** 2880 **(b)(i)** 63 **(ii)** 10**]**

Solution

(a) 8 people, to be seated in a row ; family of 3 to be seated together

(i) If the other five people do not mind where they sit, find the number of possible seating arrangements for all eight people.

X X X X X X X X

family

Number of ways to arrange 6 units = 6!
Number of arrangements within the family unit = 3!
Total number of arrangements = 6! x 3! = 4320

(ii) If the other five people do not mind where they sit, except that two of them refuse to sit together, find the number of possible seating arrangements for all eight people.

Leave out Y & Z first ___ X X X ___ X ___ X ___ X ____

<u>Method 1 : Slotting method</u>

Number of ways to arrange the 1 unit of family and 3 other people = 4!
Number of arrangement within the unit of family = 3!

Number of spaces to slot in the 2 people who needs to be separated $= {}^5C_2$
Number of ways to arrange these 2 people = 2!

Total number of ways = 4! x 3! x ${}^5C_2$ x 2! = 2880

<u>Method 2 : Complement method</u>

X X X Y Z X X X

Letting the 2 people form a unit, then there are 5 units altogether.

Number of ways to arrange 5 units = 5!
Number of arrangements within the family unit = 3!
Number of arrangements within the 2 people = 2!
Number of arrangements where family and the 2 specific people sit together
$= 5!\times 3!\times 2! = 1440$

Total number of ways where 2 people do not sit together, and family sits together
= 4320 – 1440 = 2880

(b) 6 types of vegetables to choose from

(i) Assuming that John takes some of the contents from at least one bowl, find how many different selections he can make.

John can choose to take or not take from each bowl
Number of selections = $2^6 = 64$
However, we take away the case where he takes nothing.
Total number of selections = 64 – 1 = 63

<u>Alternative method</u>
He can choose to have 1 or 2 or 3 or 4 or 5 or 6 veg.
${}^6C_1 + {}^6C_2 + {}^6C_3 + {}^6C_4 + {}^6C_5 + {}^6C_6 = 63$

(ii) John decides he is going to have 4 salad items, and one of them will be tomatoes. How many different selections can he make?

Since tomatoes are already chosen, John needs 3 more salad items, and he can choose from the remaining 5 bowls.
Number of selections = ${}^5C_3 = 10$

11 EJC JC2CT/2021/02/Q9 (modified)

Mary holds a party for her son's birthday. She has invited her son's friends consisting of 5 boys and 4 girls to the party. She wants to take a picture of the 10 children standing in a row.

(i) How many ways can the 10 children be arranged in a row if the 4 girls are next to each other and the 6 boys are next to each other? **[1]**

(ii) Mary then asks the 6 boys, all of different heights, to arrange themselves in order of their heights, ascending from left to right, but they need not stand together. The girls are free to stand anywhere. In how many ways can the 10 children be arranged? **[2]**

To play a game, the 10 children are divided into two equal groups of five each. Find the number of ways that this can be done

(iii) without restriction, **[1]**

(iv) if there must be at least one boy and one girl in each group. **[2]**

In the game, the children must arrange the nine letters in the word CHALLENGE in a row randomly.

(v) How many ways are there with no two adjacent letters being identical? **[3]**

[(i) 34560 **(ii)** 5040 **(iii)** 126 **(iv)** 120 **(v)** 55440**]**

Solution

(i) No. of ways $= 2! \times 4! \times 6! = 34560$

(ii) There is only 1 way to arrange the boys in order. However, there are ${}^{10}C_6$ ways to choose the 6 slots the boys will stand in. That leaves us with 4 slots for the 4 girls to stand in.

No of ways $= {}^{10}C_6 \times 4! = 5040$

(iii) No of ways without restriction= $\dfrac{{}^{10}C_5}{2!} = \dfrac{252}{2!} = 126$

(iv) No of ways such that there is not at least one boy and one girl in each group

= No. of ways there is a 'all-boys' group

$= {}^{6}C_5 = 6$

No of ways such that there is at least one boy and one girl in each group

$= 126 - 6 = 120$

(v) <u>Method 1</u>

The number of ways with no two adjacent letters being identical

= Number of arrangements of these 9 letters

− Number of arrangements with both LL together and both EE together

− Number of arrangements with both LL together and EE not adjacent

− Number of arrangements with both EE together and LL not adjacent

$= \dfrac{9!}{2!2!} - 7! - 2 \times 6! \times {}^{7}C_{2}$
$= 55440$

<u>Method 2</u>
The number of ways with no two adjacent letters being identical
= Number of arrangements of these 9 letters
 $-$ Number of arrangements with both LL together (EE may or may not be together)
 $-$ Number of arrangements with both EE together (LL may or may not be together)
 $+$ Number of arrangements with both LL together and both EE together

$= \dfrac{9!}{2!2!} - \dfrac{8!}{2!} \times 2 + 7!$
$= 55440$

<u>Method 3</u>
Case 1:

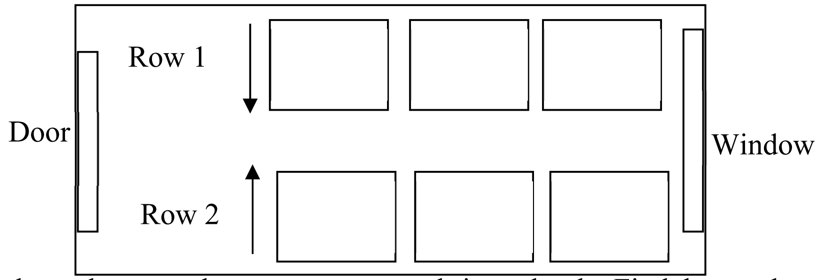
_ C _ H _ A _ N _ G _

Slotting 2 L into the above 6 possible slots, and then 2 E into 8 possible slots
$= 5! \times {}^{6}C_{2} \times {}^{8}C_{2} = 50400$

Case 2: Arrange C, H, A, N, G, E, and with LEL as a grouped unit
$= 7! = 5040$
Total number of ways = 50400 + 5040 = 55440

12 NYJC Prelim 9233/2006/01/Q5

The passenger compartment of a train has 2 rows of 3 seats each. The 2 rows are facing each other, as shown by the diagram below.



Three married couples enter the compartment and sit randomly. Find the number of seating arrangements in which

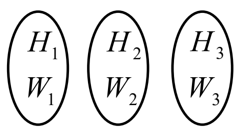
(i) each man sits opposite and facing his wife,

(ii) no man sits on the same row as his wife.

[(i) 48; **(ii)** 288**]**

Solution

(i) each man sits opposite and facing his wife

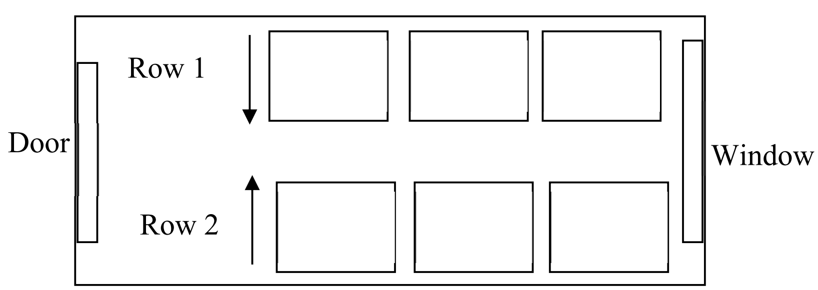


Number of ways to arrange 3 couples = 3!
Number of arrangements within each couple = 2!
Number of arrangements = $3! \times (2!)^3 = 6 \times 8 = 48$

<u>Alternative method:</u>
Number of choices the 1st husband has = 6
Number of choices the 1st wife has = 1
Number of choices the 2nd husband has = 4
Number of choices the 2nd wife has = 1
Number of choices the 3rd husband has = 2
Number of choices the 3rd wife has =1

Hence, number of arrangements = $(6\times1)\times(4\times1)\times(2\times1) = 48$

(ii) no man sits on the same row as his wife



Each arrangement in (i) is a situation with spouses not in the same row.
From the arrangement in (i), we only need to arrange the people in one of the rows.
Number of arrangements $=48\times3! = 288$

<u>Alternative 1:</u>

The wife of couple A has 6 seats to choose from.
Thereafter, her husband has 3 seats, which is in the opposite row to choose from.

The wife of couple B has now 4 remaining seats to choose from, and thereafter the husband has 2 seats in the opposite row to choose from.

The wife of couple C has now 2 remaining seats to choose from and her husband is left with 1 choice.

Hence, number of arrangements $= (6\times3)\times(4\times2)\times(2\times1) = 288$

<u>Alternative 2:</u>

We have 3 couples, couple A, B and C
In each row, there can only be 1 of A, B and C.
Number of arrangements $= 3!\times3! = 36$

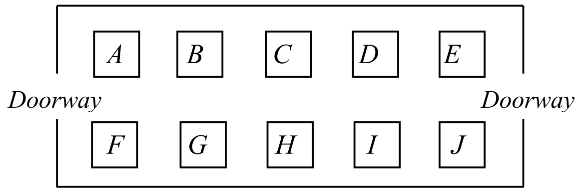
However, in each pair of As, Bs, Cs, we can interchange the man and the lady.

Total of arrangements $= 36\times\left(2!\right)^3 = 288$

13 9233/2004/02/Q4

A rectangular shed, with a door at each end, contains ten fixed concrete bases marked *A, B, C, ..., J,* five on each side (see diagram). Ten canisters, each containing a different chemical, are placed with one canister on each base.

In how many different ways can the canisters be placed on the bases? **[1]**



Find the number of ways in which the canisters can be placed

(i) if 2 particular canisters must not be placed on any of the 4 bases *A, E, F* and *J* next to a door, **[3]**

(ii) if 2 particular canisters must not be placed next to each other on the same side of the shed. **[3]**

[3628800; **(i)** 1209600; **(ii)** 2983680**]**

Solution

Arranging 10 canisters where order matters, so
Number of arrangements = 10! = 3628800

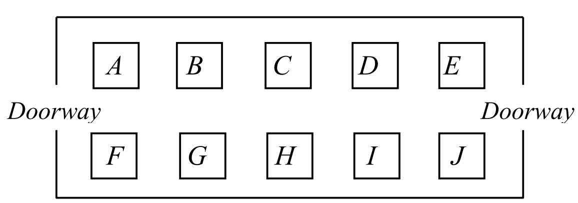
(i) if 2 particular canisters must not be placed on any of the 4 bases A, E, F and J next to a door

Number of ways of arranging the 2 particular canisters in the remaining possible bases = ${}^6P_2$
Number of ways of arranging the remaining 8 canisters = 8!
Total number of arrangements = ${}^6P_2 \times 8! = 1209600$

Alternative 1:
Number of ways of arranging 4 out of the remaining 8 canisters in bases *A, E, F* and *J* = ${}^8P_4$
Number of ways of arranging the remaining 6 canisters = 6!
Total number of arrangements = ${}^8P_4 \times 6! = 1209600$

Alternative 2:
Base A has 8 choices, Base E has 7 choices, Base F has 6 choices, Base J has 5 choices
Arrange the rest of the 6 canisters = 6!
Total number of arrangements = $8 \times 7 \times 6 \times 5 \times 6! = 1209600$

(ii) if 2 particular canisters must not be placed next to each other on the same side of the shed.



Number of *positions* where the 2 canisters are placed next to each other on the same side

= 8 (AB, BC, etc)
Number of *arrangements* where the 2 canisters are placed next to each other on the same side $=8\times\left(8!\times 2!\right)=645120$

Number of arrangements where 2 canisters are not placed next to each other on the same side = 3628800 – 645120 = 2983680

Alternative method:
The two particular canisters may be placed on different sides or on the same side but not next to each other.

Case 1: Two particular canisters placed on different sides
If they are on different sides, number of positions for 1 particular canister is 10, and number of positions for the 2nd particular canister is 5 (since it cannot be on the same side as the 1st one), and the remaining 8 canisters may be arranged in 8! Ways.
Total number of arrangements is 10(5)(8!) = 2 016 000

Case 2: Two particular canisters placed on the same side but not next to each other.
No of ways= (3+2+1)(2)(2!)8!=967680

So total number of arrangements where 2 canisters are not placed next to each other on the same side =2 016 000+967680= 2983 680

14 AJC JC2 Mid-Year CT 9758/2018/02/Q3

Four double-decker beds are arranged in a row. Eight boys, three of whom are Andrew, Ben and Perry, are to be allocated to these beds.

(i) Find the number of ways of allocating the beds to the eight boys. **[1]**

(ii) If Perry does not want to share the same double decker bed as Andrew, find the number of ways of allocating the beds to the eight boys. **[2]**

(iii) Besides not sharing the same double decker bed as Andrew, Perry who is afraid of height, has also requested to be allocated on the lower deck. Ben, who is Perry's best friend, has requested to be allocated on a lower deck bed next to Perry. Find the number of ways of allocating the beds to the eight boys based on these requests. **[2]**

The eight boys decide to hold a meeting at a round table with eight seats.

(iv) Find the number of ways of arranging the boys at the round table if Perry is to be seated next to Andrew or Ben but not between them. **[3]**

[(i) 40320 **(ii)** 34560 **(iii)** 3600 **(iv)** 2400**]**

Solution

(i) No of ways of allocating the eight beds to the boys = 8!=40320

Alternatively, ${}^{8}C_{4}\times 4!\times 4!=40320$

(ii) No. of choices for Perry $= {}^{8}C_{1}$

No. of choices for Andrew = ${}^{6}C_{1}$ (cannot take same double decker)

Rest of 6 boys allocated beds in 6! ways

No. of ways to allocate the beds = ${}^{8}C_{1}\times {}^{6}C_{1}\times 6!=34560$

(iii) Group Perry and Ben together as 1 unit

No. of choices for Perry and Ben (together) on lower deck = ${}^{3}C_{1}\times 2!$

No. of choices for Andrew = ${}^{5}C_{1}$ (cannot take same double decker as P)

Rest of 5 boys allocated beds in 5! ways

No. of ways to allocate the beds = ${}^{3}C_{1}\times 2!\times {}^{5}C_{1}\times 5!=3600$

(iv) <u>Method 1</u>

a) Perry next to Ben only (or Andrew only)

A'	P	B

+ 5 others

${}^{5}C_{1}$ ways of putting a boy (not Andrew) next to Perry

No of ways of arranging the boys at the round table $= {}^{5}C_{1}\times(6-1)!\times 2!\times 2 = 2400$

(A' and B interchange) (same no of ways if P is next to A)

<u>Method 2</u>

No of ways Perry next to Ben + No of ways Perry next to Andrew − 2 No of ways Perry between A and B

$= (7-1)!\times 2! + (7-1)!\times 2! - 2\left((6-1)!\times 2!\right)$

$= 2880 - 480 = 2400$