

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

1 Express $\frac{x^3 - 4x^2 - 24x + 41}{x^2 - 4x - 21}$ in partial fractions.

[5]

- 2 (a)** In a quiz question, the quadratic equation $k(x^2 + 1) - 8x - 6x^2 = 0$ is given.
Find the range of values of the constant k , for which the equation has no real roots.

[4]

(b) Another question in the quiz is shown below.

Find the range of values of the constant k , for which the curve
 $y = (k - 6)x^2 - 2x$ lies completely above the line $y = 6x - k$.

Jeremy stated that the range of values of k found in part **(b)** is the same as that in part **(a)**. Is Jeremy correct? Explain your answer clearly. [2]

3 Solve the simultaneous equations.

[4]

$$2x + y = 10$$

$$\frac{3}{y} + \frac{2}{x} = 2$$

- 4 In triangle ABC , $BC = (\sqrt{7} - \sqrt{2})$ cm and angle $ABC = 30^\circ$.

The area of triangle ABC is $(4\sqrt{7} - \sqrt{2})$ cm².

Without using a calculator, express the length of AB in the form $(p + q\sqrt{14})$ cm,
where p and q are constants, and state the value of p and of q . [4]

5 A circle C_1 , with centre C , has equation $x^2 - 8x + y^2 + 2y - 17 = 0$.

(a) Find the coordinates of C and the radius of the circle.

[3]

(b) Determine whether the point $S(-1, -5)$ lies inside, outside or on the circle.
Show your working clearly.

[2]

- (c) Given that $P(7, -6)$, $Q(9, 2)$ and R are three points on the circle, and PQ is perpendicular to QR , find the coordinates of R . [3]

- (d) Another circle C_2 , is a reflection of C_1 in the x -axis.
Write down the coordinates of the centre of C_2 . [1]

- 6 (a) Prove that $\frac{\sin x}{\sec x + 1} + \frac{\sin x}{\sec x - 1} = 2 \cot x$. [4]

- (b) Hence, solve the equation $\frac{\sin(2\theta - 50^\circ)}{\sec(2\theta - 50^\circ) + 1} + \frac{\sin(2\theta - 50^\circ)}{\sec(2\theta - 50^\circ) - 1} = \sqrt{3}$,
for $-90^\circ \leq \theta \leq 90^\circ$.

[4]

7 **(a)** Find $\frac{d}{dx}\left(\frac{\ln 4x}{x}\right)$.

[2]

(b) Hence, find $\int \frac{\ln 4x}{x^2} dx$.

[3]

- (c) Find the range of values of x for which $f(x) = \frac{\ln 4x}{x}$ is increasing. [3]

8 The equation of a curve is $y = (1 - 3x)^6 (x^2 + 2)$.

- (a)** Find $\frac{dy}{dx}$ in the form $\frac{dy}{dx} = 2(1 - 3x)^p (qx^2 + kx - 18)$, where p , q and k are constants. [3]

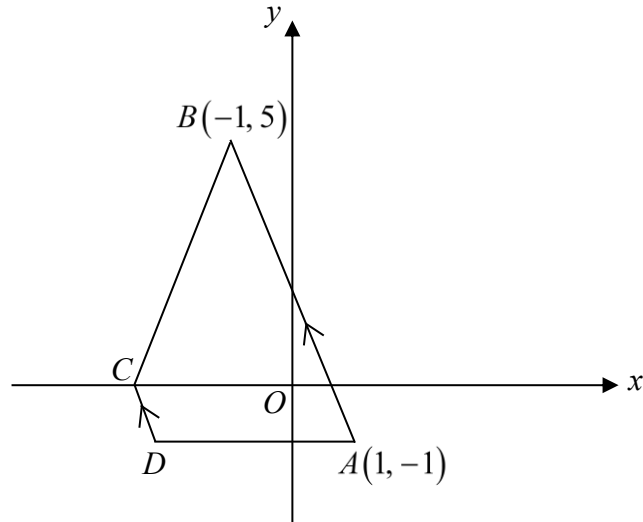
- (b)** Explain why the curve has only one stationary point and find the x -coordinate of the stationary point. [4]

Continuation of working space for question 8(b).

(c) Determine the nature of this stationary point.

[2]

- 9 The diagram shows a trapezium with vertices $A(1, -1)$, $B(-1, 5)$, C and D . AB and DC are parallel, and AD is parallel to the x -axis. The perpendicular bisector of AB crosses the x -axis at C .



- (a) Find the coordinates of C .

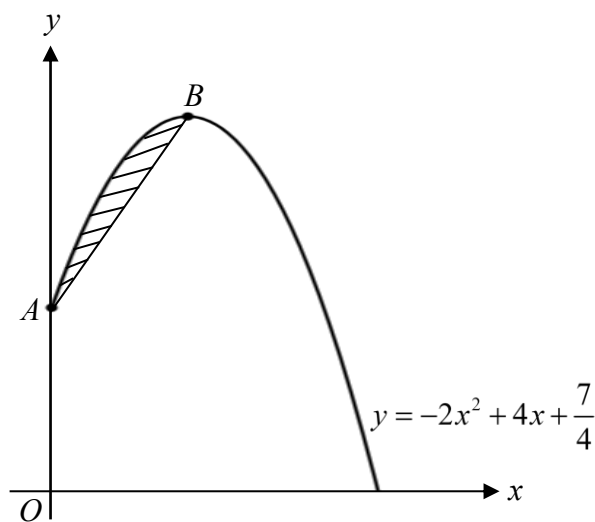
[4]

(b) Find the area of the trapezium $ABCD$.

[4]

- 10 (a) By expressing $-2x^2 + 4x + \frac{7}{4}$ in the form $a(x+b)^2 + c$, where a , b and c are constants, explain why the value of $-2x^2 + 4x + \frac{7}{4}$ cannot be 4. [4]

The diagram shows part of the curve $y = -2x^2 + 4x + \frac{7}{4}$. The curve intersects the y -axis at A , and the highest point of the curve is B .



- (b) Find the area of the shaded region bounded by the curve and the line AB . [5]

- 11** The table shows the values of two variables x and y from an experiment.

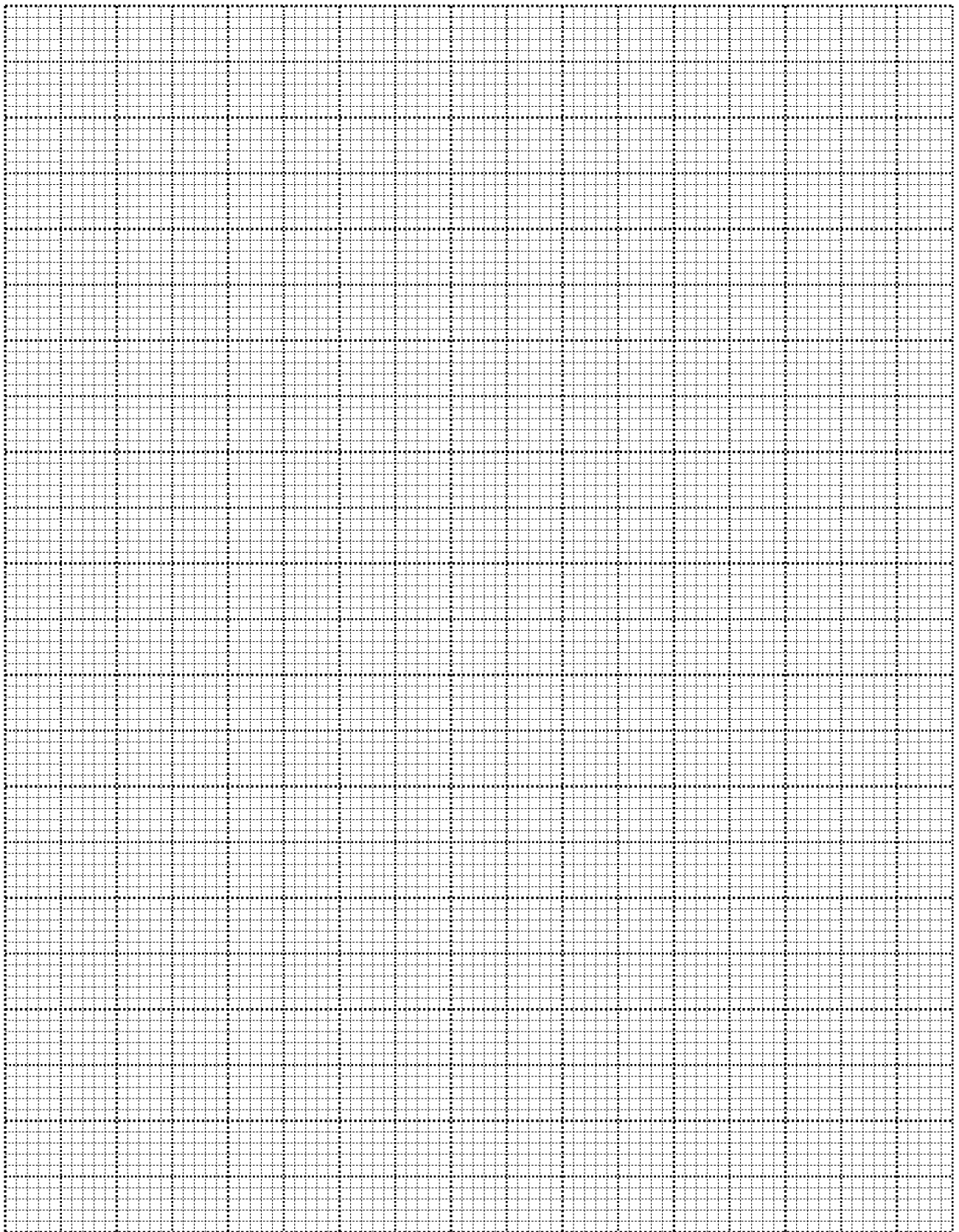
The two variables are connected by the equation $\frac{\sqrt{x}}{y} = px + q\sqrt{x}$, where p and q are constants. The values of y are corrected to 2 decimal places.

x	5	15	25	30	40
y	0.37	0.30	0.26	0.25	0.23

- (a)** Draw, on the grid provided on the next page, a straight line graph of $\frac{1}{y}$ against $\sqrt{x}$. [3]

- (b)** Use your graph to estimate the value of p and of q . [2]

- (c)** Use your graph to estimate the value of y when $x = 19$. [2]



12 The gradient of a curve is given as $\frac{dy}{dx} = Ae^{3x} + e^{-x}$, where A is a constant.

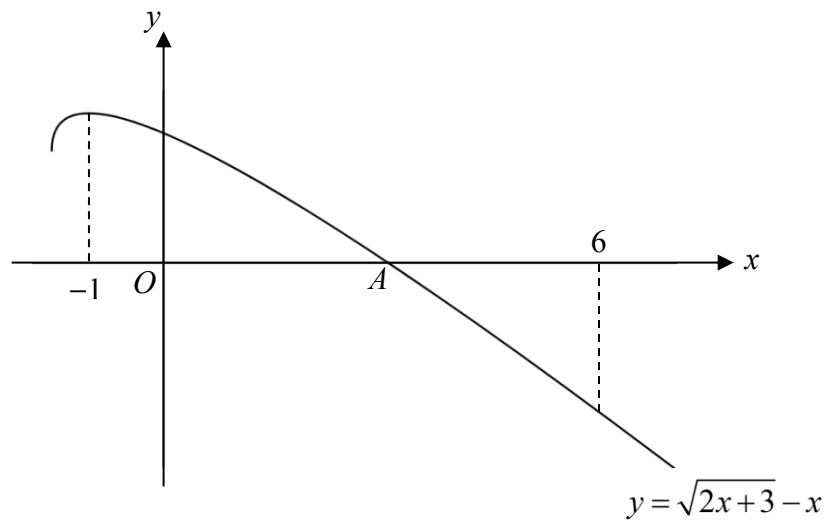
- (a)** The curve passes through the point $(0, -1)$. Find the equation of the curve in terms of A .

[3]

- (b)** Given that $9y + 8e^{-x} + 12 = \frac{d^2y}{dx^2}$, find the value of A .

[2]

- 13 The diagram shows part of the curve $y = \sqrt{2x+3} - x$, and two vertical lines $x = -1$ and $x = 6$.



- (a) Explain why the curve intersects the x -axis at only one point, A .
Find the x -coordinate of point A .

[3]

- (b) Point B is moving along the curve such that its y -coordinate is decreasing at a constant rate of 0.6 units per second. Find the rate of change of point B 's x -coordinate at where the curve intersects the x -axis.

[3]

- (c) Jenny claimed that $\int_{-1}^6 (\sqrt{2x+3} - x) dx$ can be used to find the area bounded by the curve, the two vertical lines and x -axis.

Do you agree with her? Explain your answer clearly.

[2]