RAFFLES INSTITUTION
H2 Mathematics (9758)
2025 Year 5

Additional Practice Questions for Chapter 4: Functions (Solutions)

1 9758/2017/02/3b

The function g is defined by

$$\mathrm{g}: x \mapsto 1-\frac{1}{1-x}, \text{ where } x \in \mathbb{R}, x \neq a.$$

(i) State the value of a and explain why this value has to be excluded from the domain of g. **[2]**

(ii) Find $\mathrm{g}^2(x)$ and $\mathrm{g}^{-1}(x)$, giving your answers in simplified form. **[4]**

(iii) Find the values of b such that $\mathrm{g}^2(b) = \mathrm{g}^{-1}(b)$. **[2]**

Solution:

(i)	$a = 1$. This value has to be excluded from domain of g as $x=1$ will make the denominator $1-x$ zero.
(ii)	$\mathrm{g}^2(x) = \mathrm{g}\left[\mathrm{g}(x)\right]$ $= \mathrm{g}\left[1-\frac{1}{1-x}\right]$ $= 1-\left[\frac{1}{1-\left(1-\frac{1}{1-x}\right)}\right]$ $= 1-(1-x)$ $= x$ Let $y = 1-\frac{1}{1-x}$ $\Rightarrow \frac{1}{1-x} = 1-y$ $\Rightarrow 1-x = \frac{1}{1-y}$ $\Rightarrow x = 1-\frac{1}{1-y}$ $\mathrm{g}^{-1}(x) = 1-\frac{1}{1-x}$
(iii)	$\mathrm{g}^2(b) = \mathrm{g}^{-1}(b)$ $\Rightarrow b = 1-\frac{1}{1-b}$ $\Rightarrow b(1-b) = 1-b-1 = -b$ $\Rightarrow b(2-b) = 0$ $\Rightarrow b = 0$ or $b = 2$

2 RVHSPromo9740/2009/03

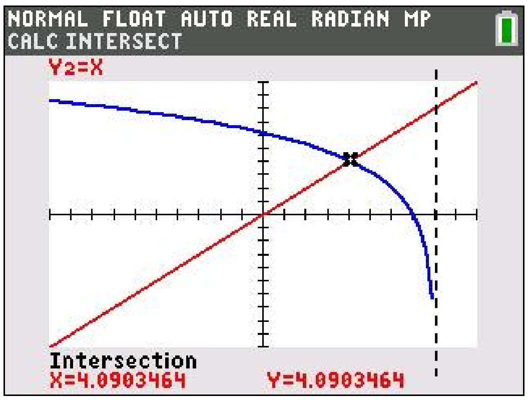
The function f is defined by

$$\mathrm{f} : x \mapsto \ln(8-x)^3, \quad x < 8.$$

(a) Solve the inequality $\mathrm{f}(x) \le x$. **[3]**

(b) Find, in a similar form, the inverse function f^{-1}. **[3]**

Solution

(a)	$\mathrm{f}(x) \le x$ $\ln(8-x)^3 \le x$  From GC, $x \in [4.09, 8)$
(b)	Let $y = \ln(8-x)^3$, then $y = 3\ln(8-x)$. $\frac{y}{3} = \ln(8-x)$ $e^{\frac{y}{3}} = 8-x$ $x = 8 - e^{\frac{y}{3}}$ $R_f = \mathbb{R}$ Therefore, f^{-1}: $x \mapsto 8 - e^{\frac{x}{3}}$, $x \in \mathbb{R}$.

3 9758/2021/02/03

(a) The function h is defined by $\mathrm{h}: x \to \frac{1}{2}x^2 + 3,$ for $x \in \mathbb{R}$.

The function g is defined by $\mathrm{g}: x \to \frac{x+1}{5x-1},$ for $x \in \mathbb{R}, x \neq 0.2$.

(i) Find $\mathrm{gh}(2)$. **[2]**

(ii) Find the value of x for which $\mathrm{g}(x) = 1.4$. **[1]**

(b) The function f is defined by $\mathrm{f}: x \to \frac{x+a}{2x+b},$ for $x \in \mathbb{R}, x \neq k$.

(i) Give an expression for k and explain why this value of x has to be excluded from the domain of f. **[2]**

The function f is such that $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$ for all x in the domain of f.

(ii) Determine the possible values of a and of b. **[3]**

(iii) Find an expression for $\mathrm{f}^{-1}(-4)$. **[1]**

Solution

(a)(i)	$\mathrm{gh}(2) = \mathrm{g}\left(\frac{1}{2}(2)^2 + 3\right) = \mathrm{g}(5) = \frac{1}{4}$.
(a)(ii)	$\mathrm{g}(x) = \frac{x+1}{5x-1} = 1.4$ From GC, $x = 0.4$.
(b)(i)	When $2x + b = 0, x = -\frac{b}{2}$. This value of x has to be excluded from the domain of f as $x = -\frac{b}{2}$ will make the denominator $2x + b$ zero. $\therefore k = -\frac{b}{2}$
(b)(ii)	$\mathrm{f}(x) = \frac{x+a}{2x+b} = \frac{1}{2} + \frac{a - \frac{b}{2}}{2x+b}$ by long division and $\mathrm{D_f} = \mathbb{R} \setminus \left\{-\frac{b}{2}\right\}, \mathrm{R_f} = \mathbb{R} \setminus \left\{\frac{1}{2}\right\}$. Since $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$, and $\mathrm{D_{f^{-1}}} = \mathrm{R_f} = \mathbb{R} \setminus \left\{\frac{1}{2}\right\}, \mathrm{R_{f^{-1}}} = \mathrm{D_f} = \mathbb{R} \setminus \left\{-\frac{b}{2}\right\}$,

	So we have $\mathrm{f}^{-1}(x)=\frac{1}{2}+\frac{a-\frac{b}{2}}{2x+b},\ x\in\mathbb{R}, x\neq\frac{1}{2}$. Hence from the denominator, we have $2\left(\frac{1}{2}\right)+b=0$, so $b=-1$. Also since f is a rational function and its graph is not a horizontal line, else f^{-1} does not exist, so $a-\frac{b}{2}\neq 0$. Hence $a\neq-\frac{1}{2}$. $a\in\mathbb{R}\setminus\left\{-\frac{1}{2}\right\}$ and $b=-1$.
(b)(iii)	$\mathrm{f}^{-1}(-4)=\frac{a-b(-4)}{2(-4)-1}=\frac{4-a}{9}$ since $b=-1$.

4 HCICT8864/2006/07

The function f is defined by $\mathrm{f}: x \mapsto |2x-1|+4, \quad x \in \mathbb{R}$.

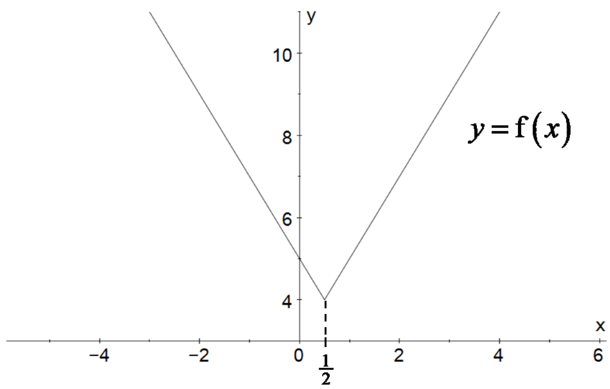
(i) Show that the composite function ff exists and give the corresponding definition of ff. State the range of ff.

(ii) Solve the equation $\mathrm{f}(x)=100$.

(iii) The function g is defined by $\mathrm{g}: x \mapsto |2x-1|+4, \quad x<k$.

Find the greatest value of k for g^{-1} to exist. Using this value of k, find g^{-1}.

Solution



(i)	$\mathrm{R_f}=[4, \infty)$ and $\mathrm{D_f}=\mathbb{R}$ Since $\mathrm{R_f} \subseteq \mathrm{D_f}$, so ff exists. $\mathrm{ff}(x)=\mathrm{f}(\|2x-1\|+4)$ $=\|2\|2x-1\|+7\|+4=2\|2x-1\|+11$ $\mathrm{ff}: x \mapsto 2\|2x-1\|+11, \quad x \in \mathbb{R}$ $\mathrm{R_{ff}}=[11, \infty)$
(ii)	$\mathrm{f}(x)=100$ $\|2x-1\|+4=100$ $\|2x-1\|=96$ $2x-1=96$ or $-(2x-1)=96 \quad \Rightarrow \quad x=\frac{97}{2}$ or $x=-\frac{95}{2}$
(iii)	Greatest value of k is $\frac{1}{2}$. For $x<\frac{1}{2}$, $\mathrm{g}(x)=-(2x-1)+4=-2x+5$. Let $y=-2x+5$, $x<\frac{1}{2} \Rightarrow x=\frac{1}{2}(5-y)$ $\mathrm{g}^{-1}: x \mapsto \frac{1}{2}(5-x), \ x>4$.

5 MIPromo9740/2012/06

Functions g and h are defined by

$$g: x \mapsto x^2 + mx + 2, \quad x \in \mathbb{R} \text{ where } m \text{ is a constant,}$$

$$h: x \mapsto \ln(x-1) + 2, \quad x > 1.$$

(i) Sketch the graph of $y = h(x)$, stating the exact coordinates of the point of intersection with the axis and the equation of the asymptote. **[2]**

(ii) Find $h^{-1}(x)$ and write down the domain of h^{-1}. **[3]**

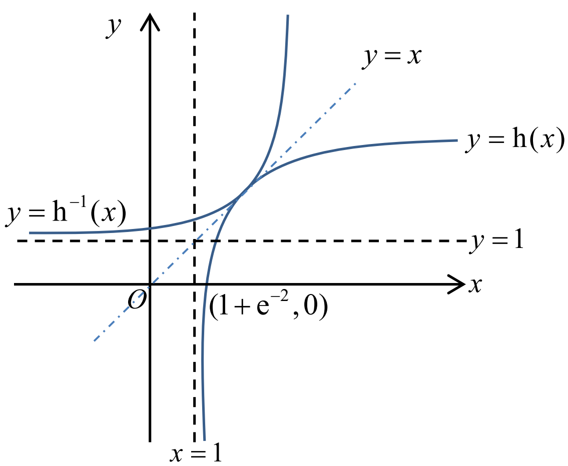
(iii) On the same diagram as in part **(i)**, sketch the graph of $y = h^{-1}(x)$.

Find the solution of the equation $h(x) = h^{-1}(x)$. **[3]**

State the condition for the composite function hg to exist.

Find the maximum range of values of m such that hg exists. **[3]**

Solution

(i)	
(ii)	Let $y = h(x) = \ln(x-1) + 2 \Rightarrow x = 1 + e^{y-2}$ $h^{-1}(x) = 1 + e^{x-2}$ with domain, $D_{h^{-1}} = \mathbb{R}$
(iii)	Using GC, $x = 2$ is the solution to the equation. $g(x) = \left(x + \frac{m}{2}\right)^2 + 2 - \frac{m^2}{4}$ For hg to exist, $\left[2 - \frac{m^2}{4}, \infty\right) = R_g \subseteq D_h = (1, \infty)$. $\Rightarrow 2 - \frac{m^2}{4} > 1$ $\Rightarrow -2 < m < 2$

6 NJCCommonTest8864/2007/Q2

The functions f , g and h are defined as follows:

$$\mathrm{f}: x \mapsto x^2+4x+1, \quad x \geq k$$

$$\mathrm{g}: x \mapsto 1-\frac{1}{x}, \quad x \in \mathbb{R} \setminus \{0\}$$

$$\mathrm{h}: x \mapsto \mathrm{e}^{2x}, \quad x \in \mathbb{R}$$

(i) State the smallest value of k such that f^{-1} exists and express f^{-1} in a similar form. **[3]**

(ii) Express each of the following functions in terms of f, g and h as appropriate.

(a) $x \mapsto \ln\sqrt{x}$, $x > 0$ **(b)** $x \mapsto 1-\frac{1}{\mathrm{e}^{2x}}$, $x \in \mathbb{R}$ **(c)** $x \mapsto x$, $x \geq -3$ **[4]**

Solution

(i)	$\mathrm{f}(x)=x^2+4x+1=(x+2)^2-3$ Least value of $k=-2$ Let $y=\mathrm{f}(x)$. $y=(x+2)^2-3$ $y+3=(x+2)^2$ $x=\sqrt{y+3}-2$ or $x=-\sqrt{y+3}-2$ (rejected, since $x \geq -2$) $\therefore$ $\mathrm{f}^{-1}: x \mapsto \sqrt{x+3}-2, \quad x \geq -3$
(ii)	**(a)** $\mathrm{h}^{-1}(x)=\ln\sqrt{x}$, $x>0$ **(b)** $\mathrm{gh}(x)=1-\frac{1}{\mathrm{e}^{2x}}$, $x \in \mathbb{R}$ **(c)** $\mathrm{ff}^{-1}(x)=x$, $x \geq -3$

7 SAJCPromo9740/2012/Q6

Functions f and g are defined by

$\mathrm{f}: x \mapsto x^2 - 2x + a \quad \text{for } x \in \mathbb{R}, \text{ where } a \text{ is an unknown constant}$

$\mathrm{g}: x \mapsto \tan x \qquad \text{for } x \in \mathbb{R},\ -\dfrac{\pi}{2} < x < \dfrac{\pi}{2}.$

(i) Find the range of f in terms of a. **[2]**

(ii) Show that the composite function fg^{-1} exists. Find the range of fg^{-1} in terms of a. **[3]**

(iii) It is given that $a = -3$. The domain of f is now restricted to $\{x \in \mathbb{R} : x \geq 1\}$.

For the new domain, give a definition (including the domain) of f^{-1}. **[3]**

Solution

(i)	$\mathrm{f}(x) = x^2 - 2x + a$ $= (x-1)^2 + a - 1$ From the graph, range of f $= [a-1, \infty)$
(ii)	$\mathrm{R}_{\mathrm{g}^{-1}} = \mathrm{D}_{\mathrm{g}} = \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$ and $\mathrm{D}_{\mathrm{f}} = \mathbb{R}$ Since $\mathrm{R}_{\mathrm{g}^{-1}} \subseteq \mathrm{D}_{\mathrm{f}}$, the composite function fg^{-1} exists Finding range of fg^{-1}: $\mathrm{D}_{\mathrm{fg}^{-1}} = \mathrm{D}_{\mathrm{g}^{-1}} \xrightarrow{\mathrm{g}^{-1}} \mathrm{R}_{\mathrm{g}^{-1}} = \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right) \xrightarrow{\mathrm{f}} \mathrm{R}_{\mathrm{fg}^{-1}} = \left[a-1, \mathrm{f}\left(-\dfrac{\pi}{2}\right)\right)$ Therefore, range of $\mathrm{R}_{\mathrm{fg}^{-1}} = \left[a-1, \dfrac{\pi^2}{4} + \pi + a\right)$
(iii)	When $a = -3$, $y = x^2 - 2x - 3, \text{where } x \geq 1$ $y = (x-1)^2 - 4$ $x = \sqrt{y+4} + 1 \quad \text{or} \quad x = -\sqrt{y+4} + 1 \quad (\text{rejected, since } x \geq 1)$ $\mathrm{f}^{-1}: x \to \sqrt{x+4} + 1 \quad \text{for } x \in \mathbb{R},\ x \geq -4$ Domain of f^{-1} is $[-4, \infty)$

8 RVHSPrelim9740/2010/02/Q5

The functions f and g are defined by

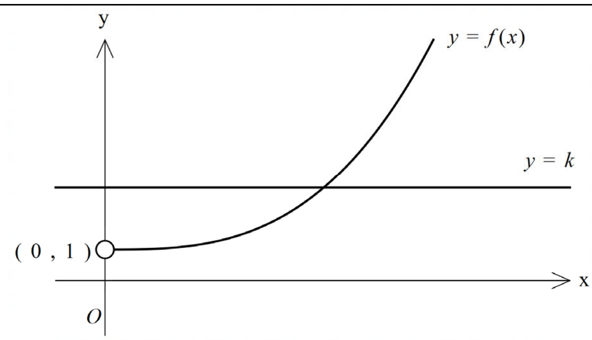
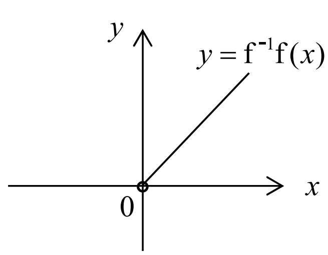
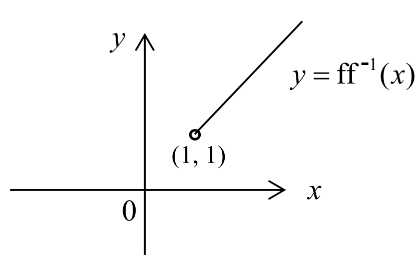
$$\mathrm{f} : x \mapsto 2x^3 + 1, \qquad x > 0,$$
$$\mathrm{g} : x \mapsto \ln(x - a), \qquad x > a.$$

(i) Give a reason why f^{-1} exists. Hence find $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. **[3]**

(ii) Only one of the composite functions fg and gf exists. State the greatest possible value of *a* such that this composite function exists and using this value of *a*, give a definition of this composite function. Write down the range of the composite function. Explain why the other composite function does not exist. **[5]**

(iii) By sketching suitable graphs or otherwise, find the set of values of *x* such that $\mathrm{f}^{-1}\mathrm{f}(x) = \mathrm{ff}^{-1}(x)$. **[2]**

Solution

(i)	 Since every horizontal line $y = k,\ k > 1$, cuts the graph of f at one and only one point, f is a one-one function . Hence f^{-1} exists. $\mathrm{f} : x \mapsto 2x^3 + 1, \qquad x > 0.$ Let $y = 2x^3 + 1$ $x = \sqrt[3]{\dfrac{y-1}{2}}$ $\mathrm{f}^{-1} : x \to \sqrt[3]{\dfrac{x-1}{2}},\ x > 1$ with domain of $\mathrm{f}^{-1} = (1, \infty)$
(ii)	For gf to exist, $(1,\infty) = \mathrm{R_f} \subseteq \mathrm{D_g} = (a, \infty)$. Hence largest possible $a = 1$. For a=1, $\mathrm{gf}(x) = g(2x^3+1) = \ln(2x^3+1-1) = \ln(2x^3), x > 0$. $\mathrm{R_{gf}} = \mathbb{R}$. Since $\mathbb{R} = \mathrm{R_g} \not\subset \mathrm{D_f} = (0, \infty)$, so fg does not exist.
(iii)	$\mathrm{f}^{-1}\mathrm{f}(x) = x, x \in \mathrm{D_f} = (0,\infty)$, $\mathrm{ff}^{-1}(x) = x, x \in \mathrm{D}_{\mathrm{f}^{-1}} = (1, \infty)$  The set of values of x such that $\mathrm{f}^{-1}\mathrm{f}(x) = \mathrm{ff}^{-1}(x)$ is $(1, \infty)$.

9 HCIPrelim9740/2009/01/08 (modified)

The functions f and g are defined as follows:

$$\mathrm{f}: x \mapsto \frac{5-x}{1-x}, \qquad x \in \mathbb{R}, x \neq 1,$$

$$\mathrm{g}: x \mapsto 2x^2 + 4x + \lambda, \quad x \in \mathbb{R}, x > -2.$$

It is given that range of f is $(-\infty, 1) \cup (1, \infty)$.

(i) Show that $\mathrm{f}^{-1} = \mathrm{f}$.

(ii) Evaluate $\mathrm{f}^{51}(4)$.

(iii) Find the range of values of λ such that fg exists.
For these values of λ, find the range of fg in terms of λ.

Solution

(i)	$\mathrm{f}(x) = \frac{5-x}{1-x} = 1 + \frac{4}{1-x}$ Let $y = 1 + \frac{4}{1-x} \Rightarrow x = 1 - \frac{4}{y-1}$. $\mathrm{f}^{-1}(x) = 1 - \frac{4}{x-1} = 1 + \frac{4}{1-x} = \mathrm{f}(x)$ and $\mathrm{D}_{\mathrm{f}^{-1}} = \mathrm{R}_\mathrm{f} = \mathbb{R} \setminus \{1\} = \mathrm{D}_\mathrm{f}$ $\therefore \mathrm{f}^{-1} = \mathrm{f}$.
(ii)	$\mathrm{f}^2(x) = \mathrm{f}^{-1}\mathrm{f}(x) = x$ $\mathrm{f}^{51}(4) = \mathrm{f}^2 \ldots \mathrm{f}^2\left[\mathrm{f}(4)\right] = \mathrm{f}(4) = \frac{5-4}{1-4} = -\frac{1}{3}$
(iii)	$\mathrm{g}(x) = 2x^2 + 4x + \lambda$ $= 2(x+1)^2 - 2 + \lambda \geq -2 + \lambda$ for all $x \in \mathbb{R},\ x > -2$ $\mathrm{R}_\mathrm{g} = [\lambda - 2,\ \infty)$ For fg to exist, $\mathrm{R}_\mathrm{g} \subseteq \mathrm{D}_\mathrm{f} = \mathbb{R} \setminus \{1\}$ So $\lambda - 2 > 1 \ \Rightarrow\ \lambda > 3$ $(-2, \infty) \xrightarrow{\mathrm{g}} [-2+\lambda, \infty) \xrightarrow{\mathrm{f}} \left[\frac{7-\lambda}{3-\lambda},\ 1\right)$

10 TJCPromo9740/2010/12

The function f is defined by $\mathrm{f} : x \mapsto (ax + b)^2,\ x \in \mathbb{R}, x \geq -\dfrac{b}{a}$ where a and b are positive constants.

(i) Show that f^{-1} exists. **[2]**

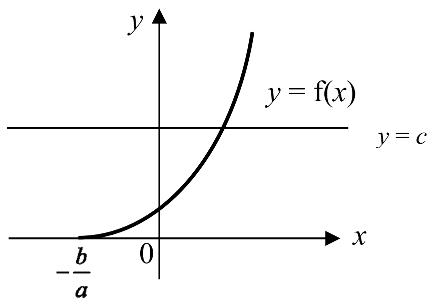
(ii) State the geometrical relationship between the graphs of f and f^{-1}.

Given that the graphs of f and f^{-1} intersect, show that $a \leq \dfrac{1}{4b}$. **[4]**

(iii) Show that the composite function ff is defined. **[2]**

(iv) Solve the equation $\mathrm{ff}(2) = b^2$. **[3]**

Solution

(i)	 Every horizontal line $y = c,\ c \in \mathbb{R}_0^+$, cuts the graph of $y = \mathrm{f}(x)$ at one and only one point. $\therefore$ f is one-one and so f^{-1} exists.
(ii)	The graphs of f and f^{-1} are reflections of each other in the line $y = x$. At point of intersection, $\mathrm{f}(x) = x \Rightarrow (ax+b)^2 = x \Rightarrow a^2x^2 + (2ab-1)x + b^2 = 0$. Since f and f^{-1} intersect, the quadratic equation has real roots. Discriminant $= (2ab-1)^2 - 4a^2b^2 \geq 0$ $-4ab + 1 \geq 0$ $4ab \leq 1 \quad \Rightarrow a \leq \dfrac{1}{4b}$ (shown)
(iii)	Range of f $= [0, \infty) \subseteq [-\dfrac{b}{a}, \infty) =$ Domain of f. Thus ff exists as a function.
(iv)	$\mathrm{ff}(2) = b^2 \Rightarrow \mathrm{f}((2a+b)^2) = b^2$ $\left[a(2a+b)^2 + b\right]^2 = b^2$ $a(2a+b)^2 + b = b$ since $a > 0$ and $b > 0$ $2a + b = 0$ $b = -2a$ (Reject as $a > 0$ and $b > 0$) Thus there is no solution.

11 RIPromo9740/2012/10

The functions f and g are defined by

$$\mathrm{f}: x \mapsto 50-(2x-4)^2, \quad -k \le x \le k,$$

$$\mathrm{g}: x \mapsto \frac{3x-3}{x^2-8x+18}, \quad x \in \mathbb{R},$$

where k is a positive real number.

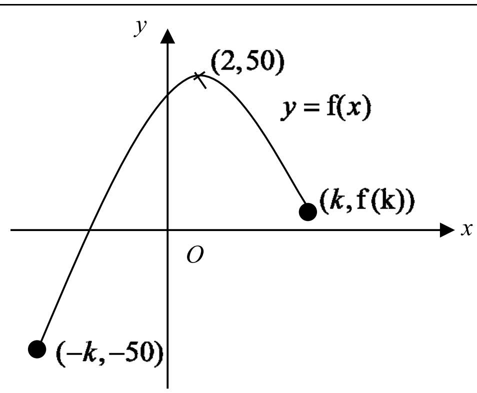
(a) Without using a calculator, solve the inequality $\mathrm{g}(x) > 2$. **[3]**

(b) It is given that the range of f is $[-50, 50]$.

(i) Find the value of k. **[2]**

(ii) Without the use of a calculator, solve the inequality $\mathrm{gf}(x) > 0$. **[4]**

Solution

(a)	$\mathrm{g}(x) > 2$ $\Rightarrow \dfrac{3x-3}{x^2-8x+18} > 2$ $\Rightarrow \dfrac{3x-3-2(x^2-8x+18)}{x^2-8x+18} > 0$ $\Rightarrow \dfrac{2x^2-19x+39}{x^2-8x+18} < 0.$ Since $x^2-8x+18=(x-4)^2+2>0$ for all $x \in \mathbb{R}$, the inequality reduces to $2x^2-19x+39<0$ $\Rightarrow (2x-13)(x-3)<0$ $\Rightarrow 3 < x < \dfrac{13}{2}.$
(b)(i)	 Given that the range of f is $[-50, 50]$, we have $\mathrm{f}(-k)=50-(2(-k)-4)^2=-50$ $\Rightarrow k=-\dfrac{4\pm 10}{2}$ $\quad \therefore k=3$ (since $k>0$)
(b)(ii)	Consider the inequality $\mathrm{g}(x) > 0$

	$\Rightarrow 3x-3>0$ $\Rightarrow x>1.$ So to solve $\mathrm{gf}(x)>0$, we replace x with $\mathrm{f}(x)$ to obtain $\mathrm{f}(x)>1$ $50-(2x-4)^2>1$ $(2x-4)^2<49$ $-7<2x-4<7$ $-\frac{3}{2}<x<\frac{11}{2}.$ Since $\mathrm{D_{gf}}=\mathrm{D_f}=[-3,3]$, $-\frac{3}{2}<x\le 3.$

12 The functions f and fg^{-1} are defined as follows:

$$\mathrm{f}: x \mapsto x+5, \quad x \in \mathbb{R},$$
$$\mathrm{fg}^{-1}: x \mapsto x^2, \quad x \in \mathbb{R}, \quad x > \sqrt{5}$$

(i) Find the functions g and fg.

(ii) The function h is defined such that its inverse exists and the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{h}^{-1}(3x+1)$ are symmetrical about the line $y = x$. Find $\mathrm{h}(x)$.

Solution

(i)	$\mathrm{fg}^{-1}(x) = x^2 = \mathrm{g}^{-1}(x)+5$ $\mathrm{g}^{-1}(x) = x^2 - 5$ To find $\mathrm{g}(x)$, let $y = x^2 - 5, x > \sqrt{5}$. $x = \sqrt{y+5} \quad \because x > \sqrt{5}$ $\mathrm{g}(x) = \sqrt{x+5}, x > 0$ $\mathrm{fg}(x) = \sqrt{x+5}+5, x > 0$
(ii)	Graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{h}^{-1}(3x+1)$ are symmetrical about line $y = x$ $\Rightarrow \mathrm{h}^{-1}(3x+1) = \mathrm{f}^{-1}(x)$ but $\mathrm{f}^{-1}(\mathrm{f}(x)) = x$ So $\mathrm{h}^{-1}(3\mathrm{f}(x)+1) = x$ $\mathrm{h}\left(\mathrm{h}^{-1}(3\mathrm{f}(x)+1)\right) = \mathrm{h}(x)$ $3\mathrm{f}(x)+1 = \mathrm{h}(x)$ $\mathrm{h}(x) = 3(x+5)+1 = 3x+16$

13 CJCPromo9740/2013/Q12

Functions f and g are defined by

$$\mathrm{f}: x \mapsto (4+2x)^{\frac{1}{2}}, \quad x \in \mathbb{R},\ 0 \le x \le 16$$

$$\mathrm{g}: x \mapsto 3x+1, \quad x \in \mathbb{R}$$

(i) State the range of f. **[1]**

(ii) With the aid of a diagram, show that f^{-1} exists and define f^{-1} in a similar form. **[4]**

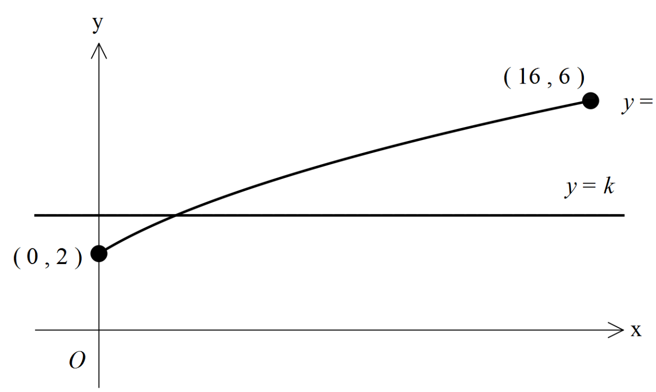
(iii) On the same diagram as in part **(ii)**, sketch the graphs of f^{-1} and $\mathrm{f}^{-1}\mathrm{f}$, indicating their endpoints. **[3]**

(iv) Explain why the x-coordinates of the point(s) of intersection between the graphs in part **(iii)** satisfies the equation $x^2-2x-4=0$. **[1]**

(v) State whether the composite function fg exists, justifying your answer. **[2]**

(vi) **Not in syllabus (For enrichment)**
Find the largest possible domain of g in the form [m, n], m, n $\in \mathbb{R}$, for which the composite function fg exists.

Solution

(i)	$f(0) = (4)^{½} = 2$ and $f(16) = (36)^{½} = 6$ Range of f, $R_f = [2,6]$
(ii)	 Since every horizontal line $y = k,\ 2 \le k \le 6$, cuts the graph of f at one and only one point, f is a one-one function . Hence f^{-1} exists. $y = \mathrm{f}(x) = (4+2x)^{\frac{1}{2}}$ $y^2 = 4+2x$ $x = \frac{1}{2}\left(y^2-4\right)$ $\mathrm{f}^{-1}(x) = \frac{1}{2}\left(x^2-4\right),\ \mathrm{D}_{\mathrm{f}^{-1}} = \mathrm{R}_\mathrm{f} = [2,6]$ Hence $\mathrm{f}^{-1}: x \to \frac{1}{2}\left(x^2-4\right),\ 2 \le x \le 6$

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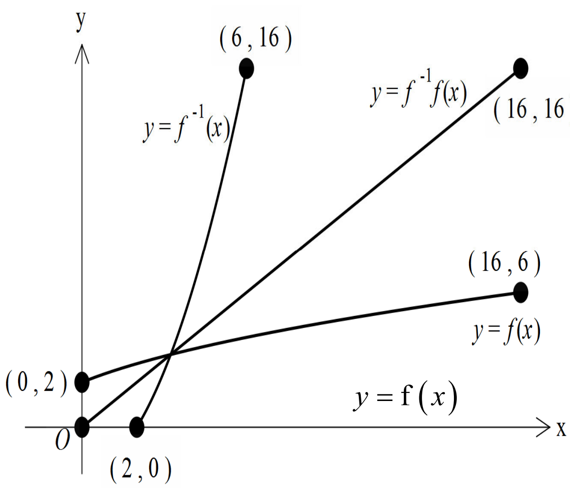
<tr><td>(iii)</td><td>
</td></tr>
<tr><td>(iv)</td><td>The point of intersection of the graphs lie on $y = \mathrm{f}(x)$ and $y = x$.
By considering $\mathrm{f}(x) = x$, $\quad (4+2x)^{\frac{1}{2}} = x \quad \Rightarrow \quad x^2 - 2x - 4 = 0$
The x-coordinates of the points of intersection satisfy the equation $x^2 - 2x - 4 = 0$.</td></tr>
<tr><td>(v)</td><td>$\mathrm{R_g} = \mathbb{R}$ and $\mathrm{D_f} = [0, 16]$
Since $\mathrm{R_g} \not\subset \mathrm{D_f}$, fg does not exist.</td></tr>
<tr><td>(vi)</td><td>Not in syllabus (For enrichment)
For fg to exist, $\mathrm{R_g} \subseteq \mathrm{D_f} = [0, 16]$
$3x + 1 = 0 \quad \Rightarrow \quad x = -\frac{1}{3}$
$3x + 1 = 16 \quad \Rightarrow \quad x = 5$
Hence $\left[-\frac{1}{3}, 5\right]$ is the largest possible domain of g for fg to exist.</td></tr>
</table>

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The function f and g are defined by

$$\mathrm{f}(x) = \mathrm{e}^{2x} - 4, \quad x \in \mathbb{R},$$
$$\mathrm{g}(x) = x + 2, \quad x \in \mathbb{R}.$$

(i) Find $\mathrm{f}^{-1}(x)$ and state its domain **[3]**

(ii) Find the exact solution of $\mathrm{fg}(x) = 5$, giving your answer in its simplest form. **[3]**

Solution

(i)	$\mathrm{f}(x) = \mathrm{e}^{2x} - 4,\ x \in \mathbb{R}$ $\mathrm{g}(x) = x + 2,\ x \in \mathbb{R}$ $y = \mathrm{e}^{2x} - 4$ $\mathrm{e}^{2x} = y + 4$ $2x = \ln(y+4)$ $x = \frac{1}{2}\ln(y+4)$ $\mathrm{f}^{-1}(x) = \frac{1}{2}\ln(x+4)$ Domain of f^{-1} = Range of f $= (-4, \infty)$
(ii)	$\mathrm{fg}(x) = 5$ $\mathrm{f}^{-1}(\mathrm{fg}(x)) = \mathrm{f}^{-1}(5)$ $\mathrm{g}(x) = \mathrm{f}^{-1}(5)$ $x + 2 = \frac{1}{2}\ln(5+4)$ $x = \ln 3 - 2$